







A TREATISE  
ON  
LAND  
AND  
LEVELLING,  
ILLUSTRATED BY  
COPIOUS FIELD NOTES, PLANS, AND DIAGRAMS.  
IN FOUR PARTS,

BY THE  
CHAIN, THEODOLITE, CIRCUMFERENTOR, AND SPIRIT LEVEL,  
*With Drawings,*  
EXPLAINING THEIR USE, AND EXHIBITING THEIR ADJUSTMENTS,  
TOGETHER WITH INTRODUCTORY CHAPTERS  
ON GEOMETRY, LOGARITHMS, MENSURATION, AND TRIGONOMETRY,  
AND AN APPENDIX OF  
TABLES OF LOGARITHMS, SINES, COSINES, TANGENTS, &c., TO SIX PLACES,  
AND  
A TRAVERSE TABLE  
TO ANY DISTANCE, AND TO THREE MINUTES OF BEARING.

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TO  
LIEUT. COL. SIR R. BONNYCASTLE, R.E.,

COMMANDING OFFICER OF ENGINEERS,  
UPPER CANADA,

AS AN HUMBLE TOKEN OF RESPECT FOR HIS VIRTUES,  
OF ADMIRATION FOR HIS TALENTS,

AND

AS A SINCERE, THOUGH INADEQUATE, OFFERING OF ACKNOWLEDGMENT

OF THE MANY

PROFESSIONAL SERVICES RECEIVED AT HIS HANDS,

WHILE EMPLOYED UNDER HIM IN THE FIELD,

THIS WORK

IS, WITH EVERY WISH FOR HIS HEALTH AND HAPPINESS,

RESPECTFULLY INSCRIBED BY HIS SINCERE FRIEND,

AND OBLIGED SERVANT,

THE AUTHOR.



It was from the want of any sufficient treatise, that I could put into my pupils' hands, on the subject of LAND SURVEYING and LEVELLING, and the inconvenience I experienced in consequence, while engaged in my professional duties, as Lecturer at King's College, London, that I was induced to compile the following pages.

I found many parts of the subject ably discussed, scattered indifferently among several authors, but none sufficiently consecutive, or in detail, to suit my purpose.

Most of them were too elementary—confined to Chain Surveying—and sometimes not referring, even in that, to the modern system of “tyeing in,” as it is called, or triangulating.

Others, on the contrary, were either purely military, or soared so highly into the depths of analytical calculations, that, though of invaluable assistance to the professional man of science, were, from the omission altogether of the more humble details of operation, both of the Chain and Theodolite, unfitted for the use of the civil surveyor.

and not one of them contained any information on the subject of the Circumferentor, or seemed sufficiently illustrated with plans and field notes.

From the direction also, that education had now taken towards the arts and sciences of life, in most public and private schools, I was induced to think that the present would be found a desirable *school book*, among the higher forms, as a full and corrected treatise of the theory and practice of Surveying. At the same time I trusted, that, by supplying the previous deficiencies, I should be furnishing the Surveyor with a complete *vade mecum* of reference.

To fulfill both these objects, I have introduced preliminary chapters on the most useful Problems and Theorems—on the Nature and Use of Logarithms—on Mensuration of Planes—and sufficient of Trigonometry as to enable the reader to understand fully any of the subsequent Trigonometrical Problems.

That portion, which refers to Surveying, has been, for facility of reference, divided into three Parts—by the Chain, Theodolite, and Circumferentor—containing copious field notes, plans, and diagrams, of each kind, together with drawings of the Theodolite and Circumferentor, and ample descriptions of their uses and adjustments.

In the Third Part, on Surveying, a special treatise has been introduced on the uses of the Circumferentor, in its application to the surveying of new countries. And the various methods, in actual operation in America—in the survey, by the needle, of extensive tracts of country—with the nature and application of the Traverse Table, to that purpose, are fully entered into and explained. The method of using the needle in this country, holding, as it does, but a subordinate part in surveying here, bears about an equal proportion to the same instrument in its perfection and accuracy in

new Countries, as the bubble of the Theodolite does to the same principle, when it is perfected in the Spirit Level.

In that part of the book that treats upon Levelling, in addition to the mere description of the nature and object of LEVELLING generally, and the use and adjustments of the *Instrument* and the reading of the Staff, have been introduced Field Notes of three miles of levels, taken on a line of railway, with the several Cross Sections. The method of computing the lowering and raising of the approaches have been given; and Drawings of the sections and cross sections, conformably to the regulations required for plans, that are intended for parliamentary deposit, have been added for illustration.

The several methods, whether correct or incorrect, adopted in practice, for calculating the CUTTINGS AND EMBANKMENTS, have been carefully investigated and examined, and corrections given for those that are wrong; and practical examples have been annexed, fully worked out, by the *Prismoidal Formula*—by Bidder's tables—and checked by a new Formula, which is specially adapted for cuttings of *any* length and height, for which I am indebted to Professor Moseley, of King's College.

And, lastly, there is an Appendix of tables of Logarithms, Sines, and Cosines; and a Traverse Table of Latitudes and Departures to any distance, and to three minutes of bearing.

I have now to acknowledge my obligation to the works of several authors, to whom I am indebted for much valuable assistance: among others, to Hall, Bridges, Keith, Hutton, Gummere, &c., &c.

Having intended these pages, as much for those who may have no opportunity of obtaining the assistance of a master, as for those who have, I have, in every case, annexed examples for practice;—to some of them have been given answers; to others, when the same result could be obtained by two or three methods, as additional practice for the Student, the answers have been omitted.

Before concluding, I have only to appeal to the kind indulgence of my readers for any mistakes that may have crept into the work, assuring them, that, as I shall have full opportunity of checking every example among my own Pupils, those mistakes shall be duly corrected in the next Edition.

30, *Queen Square, Bloomsbury,*

*March 2, 1842.*

P.S.—On further consideration, I have given, at the end of the work, the *corrected and omitted Answers to all the Examples.*

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AND

ONE HUNDRED AND FORTY

**WOOD CUTS.**

# INTRODUCTION.

## GEOMETRY.

### GEOMETRICAL DEFINITIONS.

A point has neither length, breadth, nor thickness.

A line has length.

A plane, or superficies, has length and breadth.

A solid has all three—length, breadth, and thickness.

An angle is the inclination of two straight lines meeting in a point.

A right angle is the inclination of one straight line to another, when, if either be produced, the second angle will be equal to the former.

An obtuse angle is greater: an acute angle, less: than a right angle.

Parallel lines never meet.

A parallelogram is a four parallel sided figure.

A rectangle is a right angle parallelogram.

A square is an equal sided rectangle.

A rhombus is equilateral, but not equiangular.

A rhomboid is neither equiangular nor equilateral.

A circle is a plane figure bounded by its circumference, every part of which is equidistant from the centre. The radius is this distance from the centre.

An arc is a portion of this circumference.

The chord of an arc is the straight line, joining the extremities of the arc.

A segment is the space, included between the chord and the arc.

The sector of a circle is the space, included between the two sides, subtending the angle, and the arc; therefore the sector of a right angle is a quadrant.



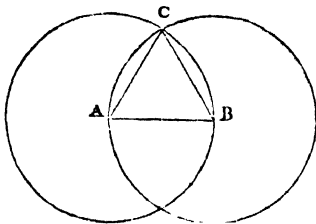
## GEOMETRICAL PROBLEMS.

1. *To describe an equilateral triangle upon a given line.*

Let AB be the given line.

From A and B, with the radius AB, describe two circles intersecting in C; join AC and BC.

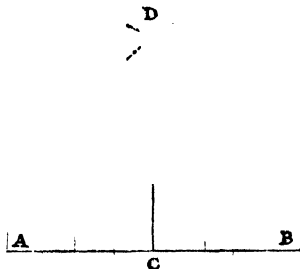
ABC is the equilateral triangle required.



2. *From a point, within a given line, to erect a perpendicular.*

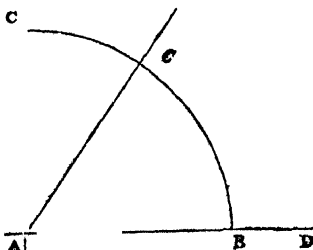
Let AB be the given line, and C be the given point.

From C, as a centre, take any distance, CA, and make  $CB = CA$ . Then from A, and B, as centres, at the distance AB, describe arcs intersecting at D; join the point of intersection at D with the point C. CD will be perpendicular to AB.



At a given point, in a given line, to construct a right angle, or an angle of any number of degrees, by means of a scale of chords; let AD be the given line, and A the given point.

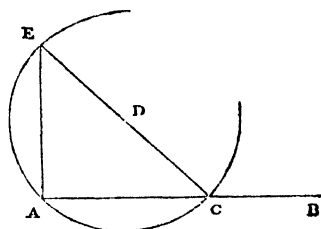
Take off with your compasses, on any scale of chords, AB, the chord of 60 degrees (*radius*); and from the given point A, with that distance, describe a circle, intersecting the given line at B; then, from the point of intersection, with the distance of the chord of 90 degrees, BC, or of the chord of any other angle that may be required, BC, on the same scale, describe another



circle, intersecting the former. Join the points of intersection with the given point, and the lines will be perpendicular to, or making the required angle, with the given line. A protractor, which is a semi-circle graduated into 180 degrees, and numbering, both from right to left, and from left to right, may be used, but not so accurately, for the same purpose.

3. *From a point at the end of a given straight line to erect a perpendicular.*

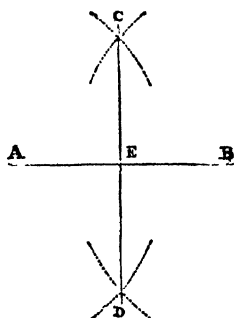
Let AB be the given straight line, and A the point at the end. Take any point D and from D as centre, at the distance DA, describe the circle EAC, and join CD, and produce it to E. Join EA.



EA shall be the perpendicular required.

4. *To bisect a given straight line.*

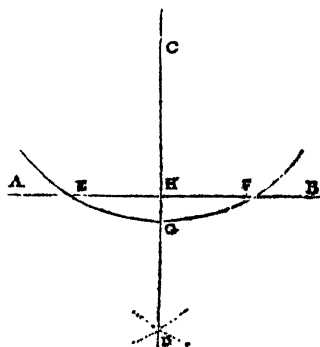
Let AB be the given straight line; it is required to bisect it. Upon AB describe the equilateral triangles, ACB and ADB; join the vertices C and D by the line CD, intersecting AB at E. E shall be the point of bisection.



5. *To let fall a perpendicular upon a given straight line, from a given point above it.*

Let AB be the given straight line, and C the given point. From C take any distance CG, and describe the circle EGF, intersecting AB in E and F; bisect EF in H, and join CH.

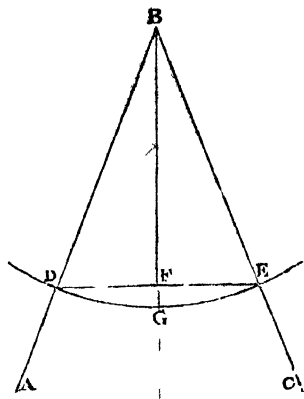
CH is the perpendicular required.



6. To bisect a given angle.

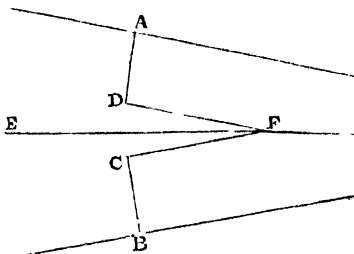
Let  $ABC$  be the given angle ; take any point  $D$  in  $AB$ , and from  $B$ , as centre, at the distance  $BD$ , describe the circle  $DGE$  ; join  $DE$ , and bisect it in  $F$  ; join  $BF$ .

$BF$  shall bisect the angle  $ABC$ .



7. To bisect a given angle, when the inclination of the two sides can only be obtained, and not the vertex of the angle, included between them.

Let  $A$  and  $B$  be the two sides, such, that they cannot be produced. Take any points,  $A$  and  $B$ , and draw the equal perpendiculars  $AD$  and  $BC$  ; through  $D$  and  $C$  draw  $DF$  and  $CF$  parallel to  $A$  and  $B$  respectively ; the angle  $DFC$  will be equal and similarly situated to the angle at the vertex, which will be bisected by the line  $EF$  (*produced*), that bisects the angle  $DFC$ .

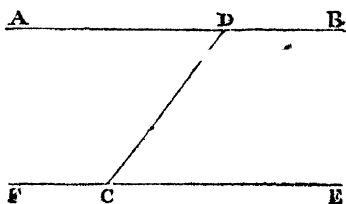


8. To draw a line parallel to a given line.

Through the point  $C$  to draw a line parallel to the given straight line  $AB$ .

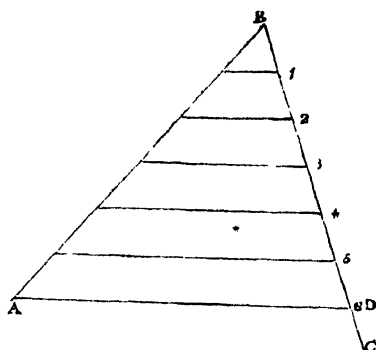
Take any point  $D$  in  $AB$ , join  $CD$ , and at the point  $C$  make the angle  $DCE$  equal to angle  $ADC$  ; produce  $EC$  to  $F$ .

$EF$  shall be parallel to  $AB$ .



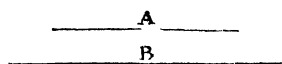
9. To divide a given line,  $AB$ , into any number of equal parts.

At the point  $B$ , making any angle with  $AB$ , draw the unlimited straight line  $BC$ ; take the required number of (six) equal measurements of any length from  $B$  towards  $C$ , ending at  $D$ ; join  $DA$ , and through the several points on  $BD$ , draw lines, parallel to  $AD$ , to the line  $AB$ ; these lines will intersect  $AB$  equally, and will be of the required number.

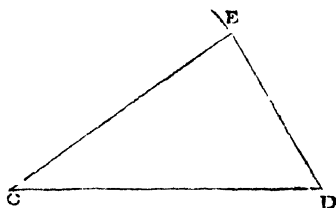


10. Upon a given base, to describe a triangle, whose other two sides shall be respectively equal to two given lines, any two of which lines, however, must be greater than the third.

Let  $A$  and  $B$  be the given lines, and  $CD$  be the given base; any two of them being greater than the third. It is required to describe upon  $CD$  a triangle, whose other two sides shall be equal to  $A$  and  $B$ .



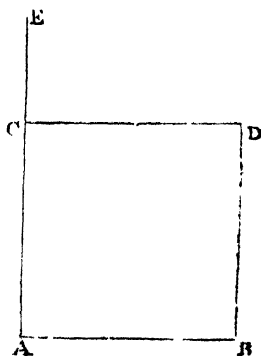
At the centre  $C$ , with the distance  $CE$ , equal to  $B$ , describe a circle; and at the centre  $D$ , with the distance  $DE$ , equal to  $A$ , describe another circle, intersecting the first in  $E$ ; join  $EC$  and  $ED$ . The triangle  $CED$  shall be the triangle required.



11. To describe a Square on a given line  $AB$ .

From the point  $A$  erect a perpendicular to  $AB$ ; make  $AC$  equal to  $AB$ ; and through the point  $C$  draw  $CD$  parallel to  $AB$ ; make  $CD$  equal to  $AB$ , and join  $DB$ .

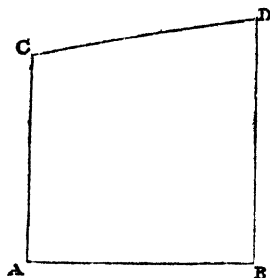
$ACDB$  shall be the square required.



12. To construct a trapezoid, having its two perpendiculars and its base given.

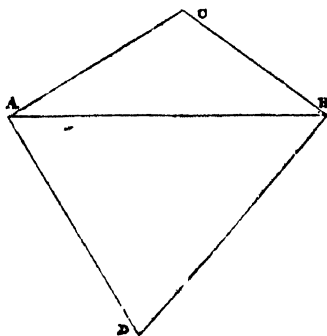
At the points A and B, of the base AB, erect <sup>of unequal lengths</sup> two perpendiculars of the length required, and join CD. †

ACDB shall be the figure required.



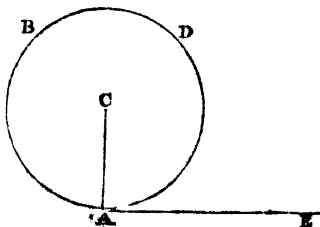
13. To construct a trapezium, ACBD, having the sides and the diagonal given.

Conceive the trapezium divided into two triangles, ~~ACB~~ <sup>ACD</sup> and ADB, having the common base AB, which is the given diagonal. Draw the base AB, and upon it, on their respective sides, construct the required triangles (prop. 10) ADB, ACB; having the sides AD, DB; AC, BC, of the required length.



14. From a given point A, in the circumference, to draw a tangent to the circle ABD, whose centre is C.

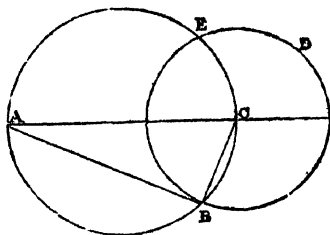
Join CA, and make EA perpendicular to AC; EA shall be the tangent required.



# GEOMETRICAL PROBLEMS.

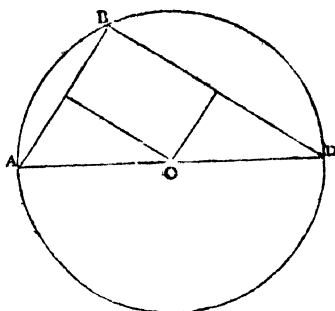
15. *From a given point A, without the circumference, to draw a tangent to the circle BED.*

Join AC, and upon AC describe the circle AECB. Join AB. AB shall be the tangent required; for ABC is a right angle, being in a semicircle, and therefore AB is at right angles to CB, which is the radius, and is therefore the tangent required.



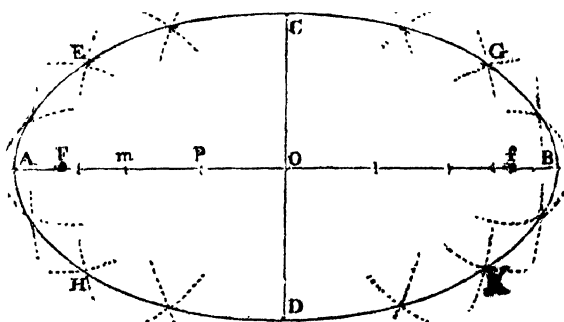
16. *Through the three given points A, B, D, not in a straight line, to describe a circle.*

Join AB and BD, and bisect them; erect perpendiculars till they meet in C; C will be the centre of the circle.



17. *To construct an ellipse, whose transverse and conjugate diameters are given.*

## METHOD 1.



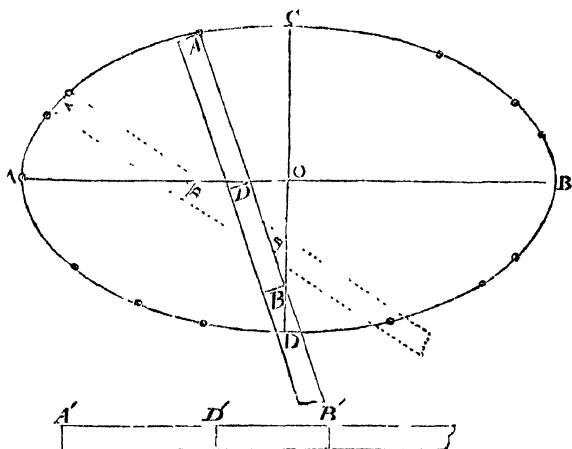
Let AB be the transverse diameter; bisect it in O; erect OC and OD perpendicular to AB, and equal each of them to the semi-conjugate; from C, as a centre, at the distance of the semi-transverse, describe the arcs intersecting AB in F and f. F, f are the foci. Then taking any point whatever m, and from F and f as centres, with distances Am, Bm, describe the arcs intersecting at E, G, H, K; thes

four points of intersection are points in the curve; again, by taking any other point  $p$ , and proceeding in the same way, four other points may be obtained.

By increasing the number of points taken upon the semi-transverse  $Ao$ , any number of points in the curve, that may be required, can be determined.

Through these points the circumference of the ellipse must be afterwards drawn.

#### METHOD 2.\*

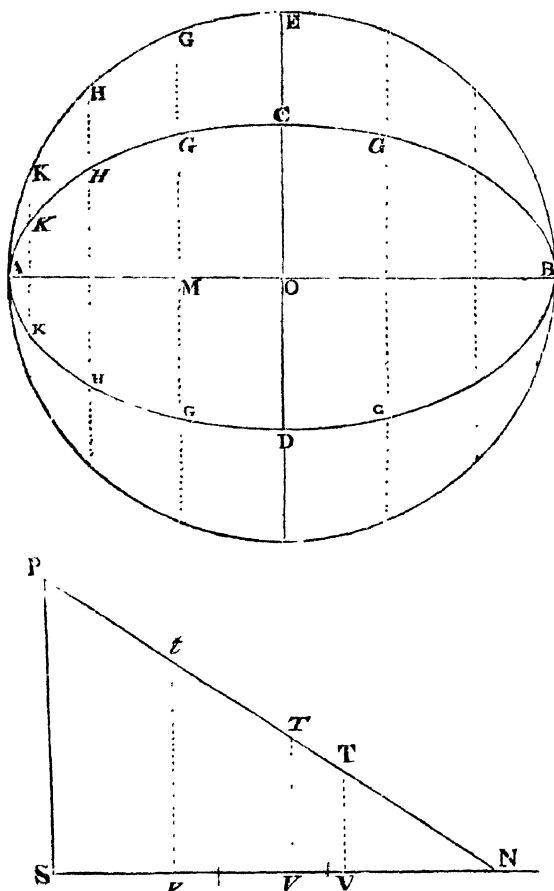


Having constructed the transverse and conjugate diameters, take a piece of card-board, with a fine clear edge, about the length of the transverse, and mark off upon it, from the same end, both the semi-transverse and semi-conjugate, so that  $AB=AO$ , and  $AD=CO$ ; then, by keeping the semi-transverse mark  $B$  always in the semi-conjugate line  $CD$ , and the semi-conjugate mark  $D$  always, at the same time, in the semi-transverse line  $AB$ , and moving the point  $B$  upwards from  $D$  to  $C$ , and the point  $D$  sideways from  $O$  to  $A$ , the end of the card  $A$  will describe points of the curve between  $C$  and  $A$ .

By adopting the same method in the other quarters of the ellipse, the whole of the curve can be determined.

\* This is a purely mechanical process, called striking the ellipse by the trammel. It is, however, a very correct and simple method, and one very much in use among Architects and Engineers.

## METHOD 3.



Construct the transverse and conjugate diameters as before, and upon the transverse, as diameter, describe a circle; then, because the ordinates of this circle are, to the corresponding ordinates in the ellipse, as the semi-transverse is to the semi-conjugate, we obtain a very simple method of determining the points in the curve.

Describe any right-angled triangle PNS, such that PN shall be to PS as the semi-transverse is to the semi-conjugate, then all lines TV,  $TV$ ,  $tv$ , drawn parallel to PS, will have the same proportion to all lines TN,  $TN$ , as PS has to PN, or as the semi-conjugate has to the semi-transverse. Draw any ordinate to the circle, GM, and in the triangle PNS, cut off NT equal to GM; let fall the perpendicular TV, (practically, place one leg of your compasses in T, and

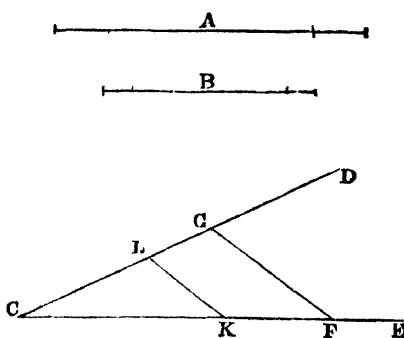


*expand them till, describing an arc, the other leg touches the lines  $SN$ ,) and measure off this distance ( $MG$ ) on the line  $MG$ ;  $MG$  is the corresponding ordinate of the ellipse, and  $G$  is a point in the curve. Produce  $GM$  to  $g$ ; making  $Mg$  equal to  $MG$ ; this is also another point. By taking equal abscissa from  $O$  to  $B$ , two other points,  $G$  and  $g$ , on the other side of  $DC$ , may be obtained also.*

The other points  $H, K, L$ , are determined in the same way, by measuring the ordinates of the circle on  $NP$ , and letting fall perpendiculars, for the corresponding ordinates of the ellipse.

18. *To find a third proportional to two given lines.*

Let  $A, B$  be two given lines.  
Draw any two unlimited lines,  $CD$  and  $CE$ , making any angle between them. From  $CE$  cut off  $CF$ , equal to  $A$ ; and from  $CD$ ,  $CG$ , equal to  $B$ , join  $GF$ ; again, take  $CK$ , equal to  $CG$ , and through  $K$  draw  $KL$  parallel to  $GF$ .  $CL$  is a third proportional to  $A$  and  $B$ , because, by similar triangles,  
 $CF : CG :: CK$  or  $CG : CL$ .



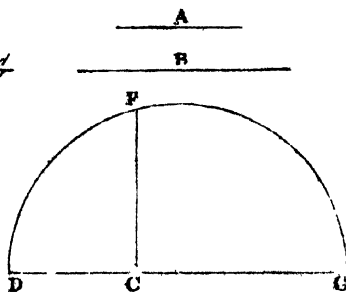
19. *To find a fourth proportional to three lines.*

Proceed as in the above, but instead of taking  $CK$ , equal to  $CG$  or  $B$ , take  $CK$  equal to the third line; then  $CL$  in this case also becomes the fourth proportional.

20. *To find a mean proportional between two lines  $A$  and  $B$ .*

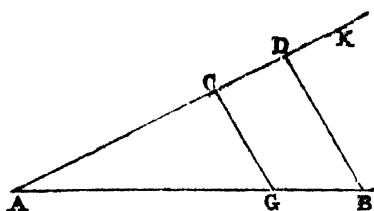
Draw any line  $DG$ ; make  $DC$  equal  $A$ , and  $CG$  equal  $B$ ; upon  $DG$  as diameter, describe a circle; erect the perpendicular  $CF$ , which is the mean proportional required. For  $DC \cdot CG = CF^2$ .

$\therefore DC : CF :: CF : CG$ .



21. To divide a given line,  $AB$ , into proportional parts.

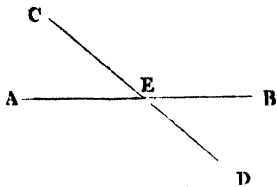
Through  $A$  draw any unlimited line  $AK$ , and take  $AC$  and  $CD$  of the required proportions; join  $DB$ , and through  $C$  draw  $CG$  parallel to  $DB$ .  $AG$  and  $GB$ , are the parts required.



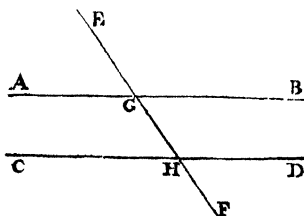
For, by similar triangles,  $AC : AD :: AG : AB$ ,  
*convertendo*.  $AC : (AD - AC) :: AG : (AB - AG)$ ;  
*i. e.*  $AC :: CD :: AG : GB$ .

## USEFUL THEOREMS.

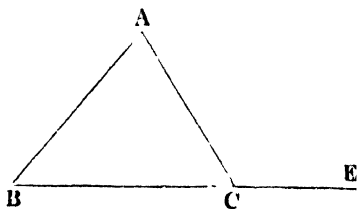
1. If two straight lines cut one another, the opposite angles are equal, and the four angles are together equal to four right angles; *i.e.*, the angle AEC is equal to the angle DEB, and the angle CEB to the angle AED. (*Euclid I, 15.*)



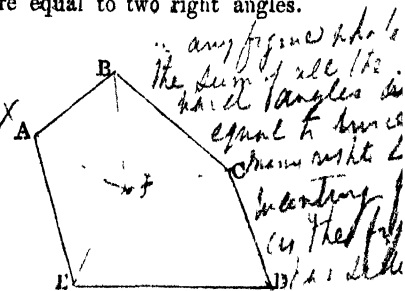
2. The alternate angles, made by a line cutting two parallel lines, are equal; and the exterior and interior angles are equal; and the two interior angles are, together, equal to two right angles; *i.e.*, the alternate angles AGH, GHD are equal; the exterior EGB equal to the interior GHD; and the two interior, BGH, GHD, equal to two right angles. (*Euclid I, 29.*)



3. If any side of a triangle be produced, the exterior angle is equal to the two interior opposite angles; and the three interior angles are equal to two right angles; *i.e.*, the angle ACE is equal to the two at A and B; and the angle ACB, together with the two at A and B, are equal to two right angles. (*I, 32.*)

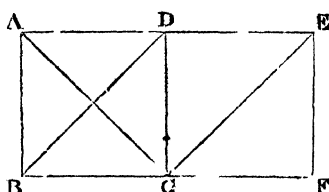


4. All the interior angles of any rectilineal figure are equal to four less than twice as many right angles as the figure has sides; *i.e.*, in the five-sided figure ABCDE, all the interior angles at A, B, C, D, and E, are equal to four less than twice five right angles; that is, are equal to six right angles or 540 degrees, (*Euclid I, 32, Cor.*)



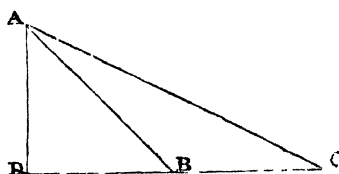
5. The greatest angle of every triangle is opposite the greatest side. (*Euclid I, 18.*)

6. Parallelograms, upon equal bases and between the same parallels, that is, having the same perpendicular height, are equal to each other; *i.e.*, the parallelogram  $ABCD$  = the parallelogram  $DBCE$ .



Triangles, alike situated, are also equal; *i.e.*, the triangle  $ABC$  = the triangle  $CFE$ . (*Euclid I, 35—38.*)

7. In every triangle, the square of the side, subtending any angle, is greater than, equal to, or less than, the square of the sides containing the angle, according as the angle is obtuse, right-angled, or acute; and the excess or deficiency is equal to twice the rectangle of one of the sides containing the angle, and that portion of it intercepted between the angle and a perpendicular, drawn to it from the opposite angle; *i.e.*,—



In the *obtuse*-angled triangle ;

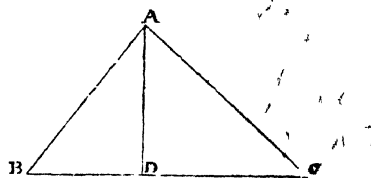
$$AC^2 = AB^2 + BC^2 + 2CB \cdot BD.$$

In the *right*-angled triangle ;

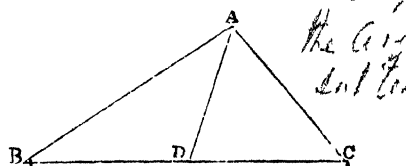
$$AC^2 = AB^2 + BC^2.$$

In the *acute*-angled triangle ;

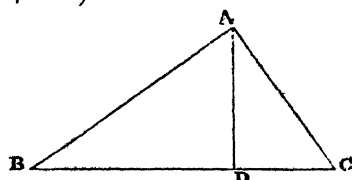
$$AC^2 = AB^2 + BC^2 - 2CB \cdot BD.$$



8. In any triangle, the sum of the squares of any two adjacent sides is equal to twice the square of half the other side, together with twice the square of the line drawn from the vertex to the point of bisection, *i.e.*,  $AB^2 + AC^2 = 2(BD^2 + DA^2)$ .



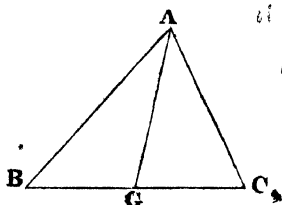
9. In any right-angled triangle, if a line be drawn at right angles to the hypotenuse, this line is a mean proportional between the segments of the hypo-



thenuse; and the base and perpendicular are, respectively, mean

proportionals between the hypotenuse and the segment adjacent to them; *i.e.*,  $BD \cdot DC = DA^2$ ,  $CB \cdot BD = BA^2$ , and  $BC \cdot CD = CA^2$ . (*Euclid*—VI, 8).

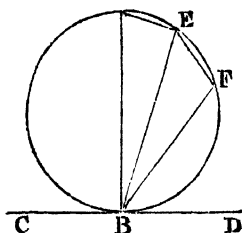
10. The rectangle of any two sides of a triangle is equal to the rectangle of the segments of the base, together with the square of the line, bisecting the included angle; *i.e.*,  $BA \cdot AC = BG \cdot GC + GA^2$  (*Euclid* VI, B), and equal to the rectangle of a perpendicular drawn from this angle and the diameter of the circumscribing circle. (*Euclid* VI, C.)



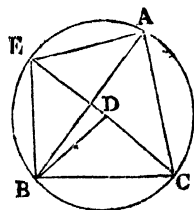
11. In every parallelogram, the squares of the two diagonals are together equal to the squares of all the sides.

12. Of any quadrilateral inscribed in a circle, the rectangle of the diagonals is equal to the two rectangles contained by its opposite sides. (*Euclid* VI, D.)

13. The angle, in a semicircle, is a right angle; in a segment, less than a semicircle, greater than a right angle; in a segment, greater, less; *i.e.*, AB being the diameter, the angle AEB is a right angle; the angle EFB, being in a segment less than a semicircle, is greater than a right angle; and the angle BAE is less. (*Euclid* III, 31.)



14. Angles, standing upon equal circumferences, are equal, whether they be in the centre or the circumference; *i.e.*, if the circumference BE be equal to the circumference BC, the angles EAB, BAC, in the circumference; and the angles EDB, BDC at the centre, are equal; and the angles EDB, BDC are respectively double the angles EAB, BAC. (*Euclid* III, 20 and 27.)



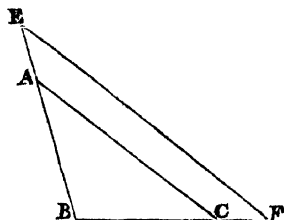
15. The two opposite angles of any quadrilateral figure, inscribed in a circle, are equal to two right angles; *i.e.*, the angles BEA and ACB, and the angles EBC, EAC, are equal to two right angles (*Vide* figure in Theorem 14). (*Euclid* III, 22.)

16. If a line touch a circle, any angle made between this line at the point of contact, and a line cutting the circle, is equal to the angle in the alternate segment; *i.e.* (See Fig 13), the angle CBE equals the

angle EFB, and the angle CBA equals the angle AEB. (*Euclid* III, 32.)

17. Parallelograms, and triangles of equal altitude, are as their bases; and of equal bases, are as their altitudes. (*Euclid* VI, 1.)

18. Equiangular triangles are similar, or have their homologous sides proportional. (*Euclid* VI, 4.) Let the triangles ABC, EBF, be equiangular;  $AB : BC :: EB : BF$  and  $BA : AC :: BE : EF$  and  $BC : CA :: BF : FE$ . (*Euclid* VI, 4)



19. Similar triangles have their areas, proportional to the square of their homologous sides. (*Euclid* VI, 19.)

20. Circumferences of circles are proportional to their radii. Areas of circles, to the squares of their radii. And, generally, all lineal measurements are proportional to their respective units of measurement.

21. All superficial areas are proportional to the compound ratio of the sides, that measure their areas; and, when these sides are similar, to the duplicate ratio of any one of them.

22. All solids also are proportional to the compound ratio of their measuring sides, and, when similar, to the triplicate ratio of any one of them.

but one integer, the index is 0: if they are all decimals, bring the first significant decimal figure into the fraction of an integer, and the power of 10, contained in the denominator, will in all cases be the index, with a negative sign—or, the index will be the number of figures, that this significant figure is removed from the unit place of the whole number, thus—

log. 5151	=	3.711892
log. 515.1	=	2.711892
log. 51.51	=	1.711892
log. 5.151	=	0.711892
log. .5151	=	—1.711892
log. .05151	=	—2.711892
log. .005151	=	—3.711892

EXAMPLES.—What are the logarithms of the following numbers?

log. 3.1420 x	=	0.497206	+ then 0 in 1st
log. 41.36	=	1.616581	
log. 8.910	=	0.949878	
log. 647.1	=	2.810971	
log. .000672	=	—4.827369	
log. 3.004	=	—0.477700	

x If it should be found that the logarithms of the given number cannot be exactly found in the tables, the simple rule of proportion will obtain it.

Thus—As the difference between the next lower and the next higher number: the difference between the next lower and next higher logarithm :: so is the difference between the next lower and the given number: the difference between the next lower logarithm and the logarithm required.

For example—to find the logarithm of 235.756:—

The log. of the next lower number 2357	is	.372360
And of the next greater 2358	is	.372544
Their difference being		.000176

Then say, as 100 :: .000176, :: 56 : .000098 which, added to the log. of 235.7, which is 2.37236, is equal to 2.372458, the logarithm of the number required.

This method may be useful to a beginner, but to any one, who understands decimals, I should recommend the following in preference.

In taking the same example, observe that the logarithms of the next lower and next higher numbers have the common figures 372, and a difference of 176 only, between the next three figures.

Now, by the principles of decimals, each (decimal) figure is itself a decimal to the one immediately preceding; the figures  $\cdot 56$ , therefore, in the given numbers, are decimals to the preceding figure 7; the measure, of the difference between 7 and 8, or of an unit of difference, being 176, the figures to be annexed to the next lower logarithm must be less than 176, and must depend upon the proportion, that the given decimal  $\cdot 56$  bears to an unit.

Multiply, therefore, this difference, or measure of the unit (176), by the given decimal  $\cdot 56$ , and you obtain the proportion of the 176, to be added to the next lower logarithm.

$$\begin{array}{rcl} \text{Ex.} & \text{difference of logarithm} & = 176 \\ & \text{decimal of numbers} & = \cdot 56 \\ & & \hline & & 1056 \\ & & 880 \\ & & \hline \end{array}$$

The figures to be added to  $98\cdot 56$

the next lower logarithm  $= 2\cdot 372360$

The required logarithm  $= 2\cdot 372458 = \log. 235\cdot 756$

What are the logarithms of the following numbers?

$$\begin{array}{rcl} \log. & 3421\cdot 56 & = 3\cdot 534228 \\ \log. & 2987\cdot 245 & = 3\cdot 475271 \\ \log. & \cdot 0342172 & = - 2\cdot 534244 \end{array}$$

*To find the number, answering to a given logarithm.*

Seek in the tables for the given logarithm, opposite which, in the left hand column, is the number answering to the given logarithm.

What are the numbers corresponding to the following logarithms?

$$\begin{array}{rcl} & 2\cdot 843855 & = \log. 698\cdot 00 \\ - & 3\cdot 530200 & = \log. \cdot 00339 \\ & 1\cdot 301030 & = \log. 20\cdot 000 \\ & 4\cdot 600973 & = \log. 39900 \\ - & 2\cdot 714338 & = \log. \cdot 05180 \\ & 5\cdot 804412 & = \log. 637400 \\ - & 1\cdot 937518 & = \log. 0\cdot 86600 \end{array}$$

When the given logarithm cannot exactly be found in the tables, seek for the next lower logarithm; opposite which will be found the next lower number corresponding thereto; then say,—

As the difference between the next lower and the next higher logarithm, is to the difference between the next lower and the next higher number, taken as an unit, so is the difference between the next



lower logarithm and the given logarithm, to the difference between the next lower number and the number required, or to the decimal of that unit, to be annexed to the number of the next lower logarithm.

For example, to find the number answering to the logarithm 4.127860, the next lower logarithm is .127753; the difference of this and the next greater is 323; and the number answering to the former is 1342.

Then, as  $.323 : 1 :: .107 : .333$ , which, annexed to the first number, will be 1342333, and, pointing off according to the index, is 13423.33, the required number; or, briefly, the decimal to be annexed, is the decimal of a vulgar fraction, whose *denominator* is the difference between the next lower and next higher logarithms (323), or the logarithmic measure of an unit; and the *numerator* the decimal of that unit, or the difference between the given logarithm and the next lower (107); viz.  $\frac{107}{323} = .333$ .

EXAMPLES.—What are the numbers corresponding to the following logarithms?

$$\begin{aligned} - 3.246418 &= \log. 0017636727 \\ 3.517694 &= \log. 3293.889 \\ - 8.106457 &= \log. 0.00000001277783 \end{aligned}$$

*Multiplication* of numbers, by logarithms, is performed by finding the logarithms of the numbers; adding them, and finding the number answering to their sum. *Division*, by subtracting them.

EXAMPLES.—Divide 56.05 by .075.

$$\begin{aligned} \log. 56.05 &= 1.748576 \\ \log. .075 &= -2.875061 \\ \log. 747.4 &= -2.873515 \end{aligned}$$

$$\text{Multiply 50 by 75} = 3750$$

$$\text{Multiply 48 by 24} = 1152$$

$$\text{Divide 4.167 by 38.56} = 0.108065$$

$$\text{Divide 42.167 by .061089} = 690.255217$$

#### INVOLUTION.

Find the logarithm of the number; multiply it by the index of power required, and find the number answering to the product.

EXAMPLES.—Required the cube of 5.68.

$$\begin{aligned} \log. 5.68 &= 0.754348 \\ \text{index} &= 3 \\ \log. 188.345 &= 2.263044 \end{aligned}$$

What is the square of  $4.678 = 21.883678$

Required the cube of  $321.5 = 33230963.375$

Find the 4th power of  $4.23 = 320.15587041$

EVOLUTION.

Divide the log of the number by the index of the root, and find the number answering to the quotient.

EXAMPLES.—What is the cube root of  $57.08$  ?

$$\log. 57.08 = 1.75684$$

$$\text{div. by } 3 = \log. 385 = 0.58594$$

$$\text{Find the square root of } 5.008 = 2.237856$$

$$\text{What is the 5th root of } 67.40 = 2.3213$$

$$\text{Required the cube root of } 6 = 1.81712092$$

Note.—That, in Involution, if the index of the logarithm of the number be negative (the logarithms of decimals being always affirmative), it will be necessary to multiply them separately.

EXAMPLE.—To find the cube of  $.05$

$$\log. .05 = -2.698970$$

3

$$2.096910$$

$$-2 \times 3 = -6$$

$$\log. .000125 = 4.096910$$

Also, in Evolution, where the index is negative, and is not some multiple of the index of the root, add to each, viz. (the index and the log.), some number that will make the index a multiple required, and divide the two separately, as in Involution. *make it divisible*

EXAMPLE.—Find the 6th root of  $.05$ . *any 12, 18, 24, &c.*

$$\log. .05 = -2.698970$$

$$= -2. + 698970$$

$$\text{Add } -4. + 4$$

$$-6. + 1098970$$

and dividing by 6  $= -1.183161$ , the number answering to which is  $.01525$ .

EXAMPLES TO THE TWO LAST CASES.

$$\text{Required the cube of } .105 = 0.0011576245$$

$$\text{What is the square of } .00534 = .0000285156$$

$$\text{Find the 4th power of } .00062 = .000014776336$$

$$\text{What is the 4th root of } .365 = 777272325$$

$$\text{Required the cube root of } .0003214 = .068498635$$

$$\text{What is the 10th root of } .0016 = 0.525305609$$

*If the given number be a fraction, to find the logarithm.*

**Rule.**—Subtract the logarithm of the denominator from that of the numerator, and the remainder is the logarithm of the fraction required.

**EXAMPLES.**—Required the logarithm of  $\frac{7}{8}$ .

$$\log. 7 = 0.84510$$

$$\log. 8 = 0.90309$$

$$\log. \frac{7}{8} = -1.94201$$

Required the logarithms of the following fractions,  $\frac{1}{3}$ ,  $\frac{1}{4}$ ,  $\frac{1}{16}$ ,  $\frac{1}{49}$ ,

$$\log. \frac{1}{3} = -1.5228787$$

$$\log. \frac{1}{4} = -1.7569620$$

$$\log. \frac{1}{16} = -1.4748500$$

$$\log. \frac{1}{49} = -1.6320232$$

$$\log. \frac{1}{324} = -3.4894550$$

$$\log. \frac{1}{7644} = -4.8045432$$

# MENSURATION OF PLANES.

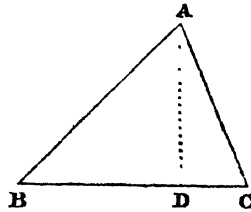
## PROBLEM I.

To find the area of a triangle, when the base and perpendicular height are given.

*Rule.*—Multiply the base by  $\frac{1}{2}$  the height.

**EXAMPLE 1.**—What is the area of a triangular field, whose base is 5 chains 50 links, and height 3 chains 20 links?

$$\begin{array}{r}
 \text{base} = 5.50 \text{ chains} \\
 \frac{1}{2} \text{ height} = 1.60 \text{ chains} \\
 \begin{array}{r}
 33000 \\
 550 \\
 \hline
 10 \overline{) 8.8000} \text{ sq. chains} \\
 \underline{88000} \\
 4
 \end{array} \\
 \text{roods} \quad 3.52 \\
 \underline{40} \\
 \text{perches} \quad 14.080
 \end{array}$$



A. R. P.  
*Answer* 0. 3. ~~14~~ 2

**EXAMPLE 2.**—Required the area of a triangle, whose base is 7 chains 25 links, and perpendicular height 90 links. A. R. P.

*Answer* 0. 1. 12

**EXAMPLE 3.**—How many square yards are contained in a triangular plot of ground, whose base and height measure respectively, 8 chains 50 links, and 5 chains 50 links?

$$\begin{array}{r}
 \text{base} = 8.5 \text{ chains} \\
 \text{height} = 5.5 \text{ chains} \\
 \begin{array}{r}
 425 \\
 425 \\
 \hline
 2 \overline{) 46.75} \\
 10 \overline{) 23.375} \text{ sq. chains} \\
 \underline{23375} \text{ acres}
 \end{array} \\
 \text{multiply by the sq. yards in 1 acre} = 4840 \\
 \hline
 \text{area} = 11313.5 \text{ sq. yards.}
 \end{array}$$

**NOTE.**—[The statute English measure of an acre is ten square chains, (each chain being 66 feet or twenty-two yards,) or 4840 square yards, or 43560 square feet. Areas are generally reckoned in acres, roods, and perches, square measures; 4 square roods being equal to an acre, and 40 square rods to a rood.]

## PROBLEM II.

*To find the area of a triangle, when the three sides are given.*

**Rule.**—Take half the sum of the three sides, subtract each side severally from this quantity; then multiply this and the three remainders together, and take the square root for the area.

**EXAMPLE 1.**—What is the area of a triangle, whose three sides are 30, 40, and 50 chains?

$$\begin{array}{r} 30 \times 40 \times 50 \\ \hline 2 \end{array} = 60 = \text{the half sum.}$$

$$\begin{array}{r} 60 \quad 60 \quad 60 \\ 30 \quad 40 \quad 50 \end{array}$$

$$30 \times 20 \times 10 \times 60 = 360000 \text{ sq. chains.}$$

$$\text{Answer} = \sqrt{360000} = 600 \text{ sq. chains} = 60 \text{ acres.}$$

**EXAMPLE 2.**—Required the area of a triangular field, whose three sides measure respectively 25 chains, 42 chains, and 56 chains?

$$\begin{array}{r} \text{A. R. P.} \\ \text{Answer } \underline{27 \cdot 2} \text{ } \underline{28 \cdot 84} \text{ } 49 \end{array}$$

**NOTE.**—Where the lengths of the sides are not full chains, as in the following example, the solution must be obtained by the general logarithmic expression, viz.:—

$\log. \text{ area} = \frac{1}{2} \log. s \times \frac{1}{2} \log. s-a \times \frac{1}{2} \log. s-b \times \frac{1}{2} \log. s-c$   
 where  $s = \text{the semi-perimeter, and } a, b, c = \text{the sides.}$

**EXAMPLE 3.**—The three sides of a triangular plot of ground are respectively 20 chains 40 links, 25 chains 20 links, and 30 chains 50 links. What is the area?

$$\begin{array}{r} 20 \cdot 40 \text{ chains} \\ 25 \cdot 20 \\ 30 \cdot 50 \\ \hline 2 \mid 76 \cdot 10 \end{array}$$

$$\begin{array}{r} 38 \cdot 05 - 20 \cdot 40 = 17 \cdot 65 = \overline{s-a} \\ 38 \cdot 05 - 25 \cdot 20 = 12 \cdot 85 = \overline{s-b} \\ 38 \cdot 05 - 30 \cdot 50 = 7 \cdot 55 = \overline{s-c} \end{array}$$

$$38 \cdot 05 = s.$$

$$\log. s = 38.05 = 1.5803547$$

$$\log. s-a = 17.65 = 1.2467447$$

$$\log. s-b = 12.85 = 1.1089031$$

$$\log. s-c = 7.55 = 8.8779470$$

$$2 \mid 4.8139495$$

$$2.4069748 = \log. 255.255 = 25.5255$$

$$\underline{4}$$

$$2.1020$$

$$\underline{40}$$

$$\text{Area} = 25. \quad \text{A.} \quad \text{R.} \quad \text{P.} \quad 4 = 4.0800$$

EXAMPLE 4.—Given the sides of a triangle, 24 chains 72 links ; 38 chains 75 links ; and 44 chains 68 links ; to ascertain the area in square yards.

$$\text{Answer } \sqrt{31700} \text{ sq. yards.}$$

### PROBLEM III.

*To find the area of a triangle, having two of its sides, and the included angle given.*

*Rule.*—Multiply half the product of the two given sides by the natural sine of the given angle ; or, where it is desirable to use logarithms, apply the following expression, viz.:—

$$\text{Log. area} = \log. \frac{b}{2} + \log. c + \log. \text{sine } A - 10.$$

EXAMPLE 1.—What is the area of a triangle, whose two sides are 40 and 50 chains, and their contained angle 37 degrees 24 minutes ?

$$\text{Natural sine } A, 37^\circ 24' = .60738$$

$$40 \times 50 \quad \underline{\quad} \quad 1000$$

$$10 \mid 607.38 \text{ sq. chains.}$$

$$60.738 \text{ acres}$$

$$\underline{4}$$

$$2.952$$

$$\underline{40}$$

$$38.080$$

$$\text{A.} \quad \text{R.} \quad \text{P.}$$

$$\text{Answer } 60. \quad 2. \quad 38$$

**EXAMPLE 2.**—What is the area of a triangle, whose sides are 42 chains, 56 links, and 11 chains 28 links, and the included angle 46 degrees 15 minutes?

$$\begin{array}{rcl}
 \log. \frac{AB}{2}, 21 \cdot 28 \text{ chains} & = & 1 \cdot 3279716 \\
 \log. AC, 11 \cdot 28 \text{ chains} & = & 1 \cdot 0522091 \\
 \log. \sin. A, 46^\circ 15' & = & 9 \cdot 8587561 \\
 & & \hline
 & & 12 \cdot 2390368 \\
 \text{acres} & \text{--- radius} & = 10 \\
 \log. 17 \cdot 339 = 173 \cdot 39 \text{ chains} & = & 2 \cdot 2390368 \\
 & & \hline
 & & 4 \\
 & & \hline
 & & 1 \cdot 356 \\
 & & \hline
 & & 40 \\
 & & \hline
 & & 14 \cdot 240
 \end{array}
 \qquad
 \begin{array}{rcl}
 & & \text{A. R. P.} \\
 \text{Area} & = & 17. 1. 14
 \end{array}$$

**EXAMPLE 3.**—Required the area of a triangular field, two of whose sides are 456 feet, and 327 feet, and the included angle 17 degrees 15 minutes?

$$\begin{array}{rcl}
 & & \text{A. R. P.} \\
 & & 0. 2. 1\frac{3}{4}
 \end{array}$$

What is the area of a triangle, whose sides are 11 yards and 24 yards, and the included angle 165 degrees 46 minutes?

$$\begin{array}{rcl}
 & & \text{A. R. P.} \\
 \text{Answer} & = & 1. 0. 1
 \end{array}$$

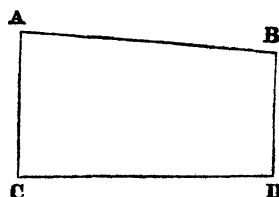
#### PROBLEM IV.

*To find the area of a trapezoid.*

**Rule.**—Multiply half the sum of the two parallel sides by the distance between them.

**EXAMPLE 1.**—In a trapezoid, whose parallel sides are 7 chains 25 links and 8 chains 63 links, the distance being 11 chains 65 links, how many acres are there?

$$\begin{array}{rcl}
 7 \cdot 25 \\
 8 \cdot 63 \\
 2 \mid \hline 15 \cdot 88 & \text{chains} & \text{acres} \\
 7 \cdot 94 \times 11 \cdot 65 = 92 \cdot 50 & = & 9 \cdot 250 \\
 & & \hline
 \text{Area} = 9. 1. 0 & & 1 \cdot 000
 \end{array}$$



**EXAMPLE 2.**—What is the area of a trapezoid, the parallel sides of which are 14 chains 20 links, and 12 chains 35 links, and the distance between them 27 chains 25 links?

A. R. P.

*Answer* 36. 0. 28

**EXAMPLE 3.**—Required the area of a trapezoidal field, whose sides are 40 chains and 27 chains, and distance 15 chains and a half.

A. R. P.

*Answer* 51. 3. 28

### PROBLEM V.

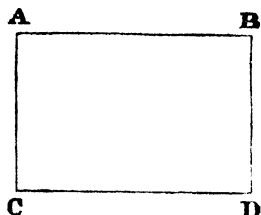
*To find the area of a quadrilateral right-angled figure.*

**Rule.**—Multiply the length by the breadth, the product will be the area.

**EXAMPLE 1.**—What is the area of a rectangular field, whose length is 35 chains 40 links, and breadth 24 chains 36 links?

chs.	chs.	sq. chs.	acres.
35·40	×	24·36	= 862·34 = 86·234
			4
			— 936
			40
			— 37·440

*Answer* A. R. P. 86. 0. 37



**EXAMPLE 2.**—Required the area of a right-angled parallelogram, whose length is 56 chains 24 links, and breadth 35 chains 42 links.

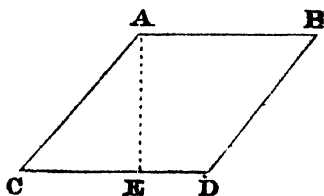
A. R. P.

*Answer* 99. 0. 32

### PROBLEM VI.

*To find the area of a parallelogram, whose angles are not right angles.*

Let ABCD be the parallelogram, whose area is required; let fall the perpendicular AE. The measurement of the parallelogram is AE. CD, and AE is the sine of the angle at C, to radius CA; therefore the area = AC. CD. sine  $\angle C$ .



**Rule.**—Multiply the product of the two adjacent sides by the sine of the angle between them.



**EXAMPLE 1.**—Required the area of a four-sided regular field, whose sides are 20 chains 15 links, and 16 chains 89 links, and the included angle 30 degrees.

$$\sin \angle 30 = \frac{\text{rad}}{2} = \frac{1}{2} = .5$$

$$\text{AC. CD} = 20.15 \times 16.89 = 340.3335 \text{ chains} = 34.03335 \text{ acres}$$

$$\text{Multiplying by sine } \angle 30 = \frac{.5}{17.016675}$$


---


$$\begin{array}{r} 17.016675 \\ 4 \\ \hline .066700 \\ 40 \\ \hline 2.668000 \end{array}$$

A. R. P.  
*Answer* 17. 0. 2

**EXAMPLE 2.**—When the sides are 15 chains 25 links, and 21 chains 18 links, and the included angle 45 degrees, what is the area of the field?

A. R. P.  
*Answer* 22. 3. 14

**EXAMPLE 3.**—With the same sides, but with an angle of 105 degrees, what is the area?

A. R. P.  
*Answer* 31. 0. 31

## PROBLEM VII.

*To find the area of a trapezium.*

**Rule.**—Divide the trapezium into two triangles by the longest diagonal; take this as the common base of the two triangles, and multiply it by half the sum of the two perpendiculars, let fall upon it, from the opposite angles.

**EXAMPLE 1.**—To find the area of a trapezium, whose diagonal is 20 chains, and the two perpendiculars 2 chains 50 links, and 3 chains 40 links?

$$\begin{array}{r} 2.50 \\ 3.40 \\ \hline 2 \mid 5.90 \end{array}$$

$$2.95 \times 20 = 59 \text{ sq. chains} = 5.9 \text{ acres.}$$

A. R. P.  
*Answer* 5. 3. 2

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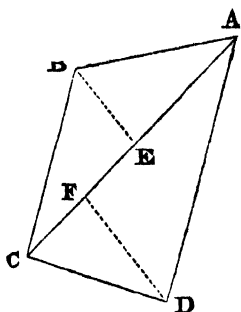
4  
3.6

**EXAMPLE 2.**—How many square feet of paving are there in a court yard, whose diagonal is 2 chains 64 links, and perpendiculars 95 links and 84 links?

*Answer* 10292 square feet.

**EXAMPLE 3.**—In the field ABCD on account of obstructions, the following distances only could be taken, viz., AB, 12 chains; BC, 8 chains; and the distances ~~AC~~, 10 chains; and ~~AD~~, 6 chains 50 links, (on the diagonal which was 15 chains,) required the area?

A. R. P.  
*Answer* 8. 1. 33



**EXAMPLE 4.**—Given the distances AB, 6 chains; and AD, 8 chains. In consequence of obstructions, DB could not be measured; though the perpendicular points E, and F, upon it, were ascertained, and the lengths of the perpendiculars were found to be 3 chains and 4 chains. What is the area?

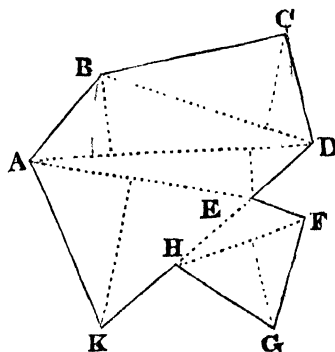
A. R. P.  
*Answer* 3. 3. 38

### PROBLEM VIII.

*To find the area of an irregular polygon.*

**Rule 1.**—Divide the polygon into trapeziums and triangles. Then find the sum of the areas of each, as in the following example:—

Let ACFK be an irregular polygon, whose diagonals BD, AD, AE, HF, and sides AB, BC, CD, &c., are given, viz. :—



the diagonals  $\left\{ \begin{array}{l} BD = 7 \text{ chains} \\ AD = 10\cdot00 \\ AE = 8\cdot00 \\ HF = 6\cdot00 \end{array} \right.$

sides

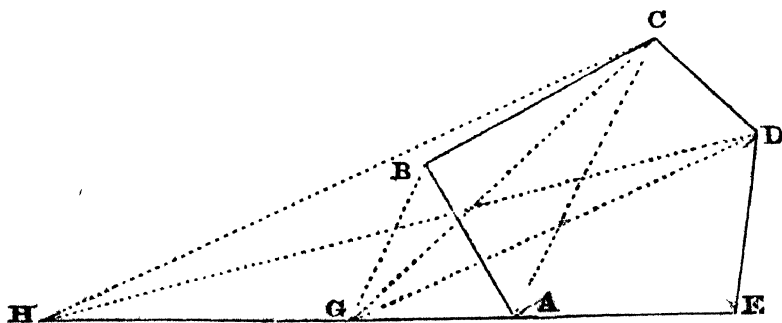
$\begin{array}{l} AB = 4\cdot00 \\ BC = 6\cdot35 \\ CD = 3\cdot00 \\ DE = 2\cdot50 \\ EF = 3\cdot50 \\ FG = 4\cdot00 \\ HG = 4\cdot00 \\ HK = 3\cdot00 \\ HE = 3\cdot50 \\ AK = 6\cdot00 \end{array}$

	chains.
Area of $\triangle ABD$	$= 10.928$
$\triangle BCD$	$= 9.140$
$\triangle ADE$	$= 6.684$
$\triangle EFH$	$= 5.408$
$\triangle FGH$	$= 7.936$
$\triangle AEK$	$= 19.171$
Total area of polygon	$= 59.267$ square chains.
	$5.267$
	$4$
	$.7068$
	$40$
	$28.2720$

**NOTE.**—[The lengths of the sides of the figures have been preferred to those of the perpendiculars, because the lengths of the sides would have to be taken, in practice, for the sake of the offsets, and the sides, together with the diagonals, furnish sufficient data for the polygon; whereas the perpendiculars, though more easily measured, and more readily effective, in determining the areas, would be useless for ascertaining the irregularities of the hedges.]

**Rule 2.**—Construct carefully the given polygon, and reduce it geometrically to an equivalent triangle, as in the following example. Then measure the sides of this triangle, off the scale, by which the polygon was constructed, and find the area. This will be the area of the polygon.

Let  $ABCDE$  be the given polygon; it is required to reduce it to an equivalent triangle.



Join  $AC$ , through  $B$ ; draw  $BG$ , parallel to  $AC$ ; join  $GC$ ; then the triangle  $GAC$  is equal to the triangle  $ABC$ , being on the same base

AC, and between the same parallels. And, therefore, GCDE, the four-sided figure, is equal to the original polygon. Again join GD; and through C, draw CH parallel to DG; and join HD. HDE is the triangle required.

### PROBLEM IX.

*To find the area of any regular polygon.*

Let the given polygon be divided into as many triangles as it has sides, by lines drawn from the centre of the circumscribing circle; then the *area of one of these triangles*, multiplied by the number of sides of the polygon, will give the area required.

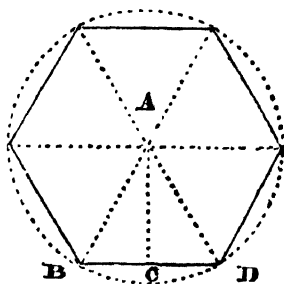
*To find this unit of area.*

Let ABD be one of the triangles of a regular polygon, whose side BD = unity, then as  $BC \times AC =$  the area of the triangle BAD,

and  $BC = \frac{BD}{2} = \frac{1}{2}$ ; and  $AC = \cot$

$\angle BAC$  to radius BC, which is equal to  $\frac{1}{2} \cot. \frac{180^\circ}{n}$ , (because  $BC = \frac{1}{2} BD = \frac{1}{2}$ .)

where  $n =$  number of sides of the polygon; therefore  $\frac{1}{2} \times \frac{1}{2} \cot \frac{180^\circ}{n} = \frac{1}{4} \cot \frac{180^\circ}{n} =$  the required area of the triangle ABD.



Multiply this by  $n$ , the number of sides, and you obtain  $\frac{n}{4} \cot \frac{180^\circ}{n} =$  the area of any sided polygon, whose side  $BD =$  unity. But areas of similar figures are to each other as the squares of their homologous sides. Therefore  $L^2 \cdot \frac{n}{4} \cot \frac{180^\circ}{n} =$  the area of any regular polygon, when  $L =$  the length of its side; hence is deduced the

**1st Rule.**—Square the side of the polygon, and multiply it by one fourth the number of sides, multiplied into the cot. of (180 degrees, divided by the number of sides).

**EXAMPLE 1.**—Let the side BD of the regular hexagon be equal to 20 chains; it is required to find the area.

$$\begin{array}{rcl}
 L^2 \cdot \frac{n}{4} \cot \frac{180^\circ}{n} & = & \text{area.} \\
 \log. & 400 & = 2.602060 \\
 + \log. & 6 & = 0.778151 \\
 + \log. \cot. 30^\circ & = & 10.238561 \\
 & & \hline
 & & 3.618772 \\
 - \log. & 4 & = 0.602060 \\
 \log. 1039.23 \text{ chs.} & = & 3.016712 = \\
 1039.23 \text{ chs.} & = & 103.923 \text{ acres.} \\
 & & \hline
 & & 4 \\
 & & 3.692 \\
 & & \hline
 & & 40 \\
 & & \hline
 & & 27.680
 \end{array}
 \qquad \begin{array}{r}
 \text{A. R. P.} \\
 \text{Answer } 103. \quad 3. \quad 27
 \end{array}$$

**Rule 2.**—Square the side of the polygon, as in the first rule, and multiply it by the tabular area, or multiplier placed, collaterally with it, in the following table.

**EXAMPLE 1.**—Taking the same data as in the preceding, making the side of the hexagon 20 chains, we have

No. of sides.	Names.	Areas, side = 1	
3	triangle	0.4330127	$20^2 = 400$
4	square	1.0000000	tabular area of the
5	pentagon	1.7264774	hexagon = 2.5980762
6	hexagon	2.5980762	multiply by 400
7	heptagon	3.6339124	sq. chs. = 1039.2304800
8	octagon	4.8284271	acres = 103.923
9	nonagon	6.1818242	4
10	decagon	7.6942088	3.692
11	undecagon	9.3656399	40
12	dodecagon	11.1961524	$\text{A. R. P.}$
			$\text{Ans.} = 103. \quad 3. \quad 27 \text{ as before. } 27.680$

**EXAMPLE 2.**—Required by both rules, the area of the regular pentagon, whose side BD is 27 chains.

$\text{A. R. P.}$   
*Answer* 125. 1. 27.

**EXAMPLE 3.**—What is the area of a regular dodecagon, whose side is 3 chains 49 links.

$\text{A. R. P.}$   
*Answer* 136. 1. 19

PROBLEM X.

*To find the circumference of a circle.*

**Rule.**—Multiply the diameter by 3·1416.

Circumferences are to each other as their diameters,  
for  $\pi = 3\cdot1416 =$  the semi-circumference, when radius is unity;  
 $=$  circumference of the circle, when diameter is unity.

**EXAMPLE 1.**—What is the circumference of a circle, whose diameter is 30 chains?

Circumference  $= 30 \pi = 3\cdot2426 \times 30 = 94\cdot2480$  chains.

**EXAMPLE 2.**—Required the circumference of a circle, whose diameter is 17 chains 40 links. *Answer* 54·66 chains.

**EXAMPLE 3.**—What is the diameter of a circle, whose circumference measures 1,000 chains? *Answer* 318 chains 30 links.

PROBLEM XI.

*To find the length of any arc.*

As  $360^\circ$  : to the given degrees of the arc :: so is the whole circumference : to the length of the arc required.

**EXAMPLE 1.**—What is the length of an arc of  $20^\circ$ , of a circle, whose circumference measures 850 chains?

As  $360^\circ : 20^\circ :: 850 \text{ chains} : x$

$$x = \frac{850 \times 20}{360} = 47\cdot22 \quad \text{Ans. } 47\cdot22 \text{ chs.}$$

**EXAMPLE 2.**—Required the length of the quadrant, the circumference measuring 300 chains. *Answer* 75 chains.

**EXAMPLE 3.**—What is the circumference of a circle, when the arc of  $30^\circ$  measures 17 chains 20 links? *Answer* 206·40 chains.

PROBLEM XII.

*To find the area of a circle.*

**Rule.**—Multiply the square of the diameter by ·7854.

*Areas of circles are as the squares of the diameters—*

$$\therefore \text{area of circle.} = \text{rad.}^2 \times \pi$$

$$= (2 \text{ rad.})^2 \frac{\pi}{4} = \text{diam.}^2 \frac{\pi}{4}.$$

**EXAMPLE 1.**—What is the area of a circular field, whose diameter is 18 chains?

$$\begin{array}{r}
 \cdot 7854 \\
 18^2 = \quad 324 \\
 \hline
 31416 \\
 15708 \\
 \hline
 23562 \\
 10 \overline{) 254 \cdot 4696} \\
 \underline{25 \cdot 4469} \phantom{0} \\
 4 \phantom{0} \\
 \underline{1 \cdot 7876} \phantom{0} \\
 40 \\
 \hline
 31 \cdot 5040
 \end{array}
 \quad \begin{array}{l}
 \text{A. R. P.} \\
 \text{Answer } 25. 1. 31.
 \end{array}$$

**EXAMPLE 2.**—The area of a circular plot of ground is required, whose diameter is 27 chains.

$$\begin{array}{l}
 \text{A. R. P.} \\
 \text{Answer } 2. 0. 19.
 \end{array}$$

**EXAMPLE 3.**—What is the area of the circle, that can be described by a rope, measuring  $5\frac{1}{2}$  chains, having one end fixed, as a centre?

$$\begin{array}{l}
 \text{A. R. P.} \\
 \text{Answer } 1. 2. 37.
 \end{array}$$

### PROBLEM XIII.

*To find the area of an ellipse.*

**Rule.**—Multiply the product of the major and minor axes by  $\cdot 7854$ . An ellipse being a mean proportional between a circle described on its minor, and one on its major axis.

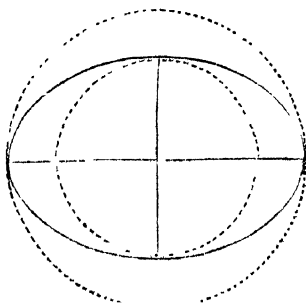
*Let  $b$  = the diameter of the smaller circle; and  $a$ , that of the greater;*

$$\text{then } \begin{cases} \cdot 7854 \times a^2 = \text{area of greater circle,} \\ \cdot 7854 \times b^2 = \text{area of less circle,} \end{cases}$$

*therefore  $\sqrt{\cdot 7854^3 (a^2 b^2)} = \cdot 7854 ab = \text{ellipse.}$*

**EXAMPLE 1.**—Required the area of an ellipse, whose transverse diameter is 12 chains, and conjugate, 8 chains 50 links.

$$\begin{array}{l}
 \text{sq. chains.} \\
 \cdot 7854 \times 12 \times 8 \cdot 50 = 80. 11. \\
 \text{A. R. P.} \\
 \text{Answer } 8. 0. 1.
 \end{array}$$



EXAMPLE 2.—What is the area of the ellipse, whose major and minor axes are, respectively, 22 chains 40 links, and 15 chains 50 links?

A. R. P.  
Answer 27. 1. 3.

EXAMPLE 3.—Required the area of an ellipse, whose transverse diameter is 36 chains 20 links, and conjugate, 24 chains.

A. R. P.  
Answer 68. 0. 37.

# PROBLEM XIV.

To find the area of a parabola.

Rule. — Multiply the base by  $\frac{2}{3}$  of the perpendicular height. The area of a parabola being equal to  $\frac{2}{3}$  of its circumscribing rectangle.

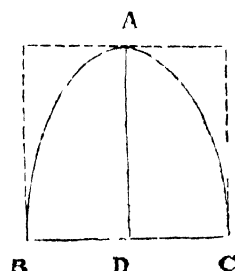
EXAMPLE 1.—Required the area of a parabola, whose base is 22 chains 50 links, and perpendicular height, 22 chains 50 links also.

$$22\cdot50 \times \frac{2}{3} = \frac{45}{3} = 15$$

sq. chains.

$$15 \times 22\cdot50 = 337\cdot50 = \text{area.}$$

A. R. P.  
Answer 33. 3. 0.



EXAMPLE 2.—What is the area of a parabola, whose base is 15 chains 25 links, and perpendicular height, 11 chains 30 links.

A. R. P.  
Answer 11. 1. 38.

EXAMPLE 3.—The base of a parabola is 12·50 chains, and its perpendicular height, is equal to its base. Required the area.

A. R. P.  
Answer 13. 1. 26.



## PLANE TRIGONOMETRY.

## DEFINITIONS.

1. Angles are measured by the arcs of circles, cut off by the two sides, that compose the angles ; *i.e.*, the angle GCB is measured by the arc GB.

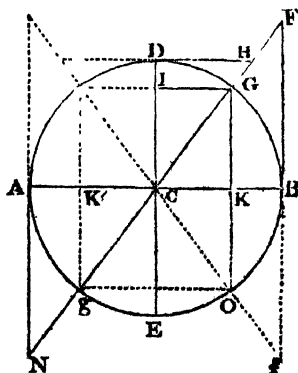
2. The chord of an arc, is a straight line, joining the extremities of an arc, GO being the chord of the arc GBO.

3. The circumference of a circle, is divided by English Mathematicians into 360 equal parts or degrees ; each degree, into 60 minutes ; and each minute, into 60 seconds, which are thus marked :  $4^{\circ}$ ,  $3'$ ,  $15''$  ; 4 degrees, 3 minutes, and 15 seconds. A quarter of a circle, or a quadrant, contains therefore 90 degrees.

4. The complement of an angle, is the difference between that angle and 90 degrees ; therefore, *each of the acute angles of a right-angled triangle is the complement of the other.*

5. The supplement of an angle, is the difference between that angle and 180 degrees ; or, measured on the circumference, the difference between the arc and the semi-circumference. And, as the sum of the three angles of any triangle is equal to two right angles, or 180 degrees, *each angle of any triangle has for its supplement, the sum of the other two.*

6. The sine of an arc, is a perpendicular, drawn from one extremity of the arc, upon the diameter of the circle, which passes through the other extremity, *therefore the sine of any two angles is equal to the sine of the third ;* thus, in the annexed figure, GK, is the sine of the arc BG. Produce GK to O ; GO is bisected ; and the arc GB = the arc BO ; therefore *every sine is equal to half the chord of twice the arc.*



7. The Cosine of an arc, is the sine of its complement, or, is that portion of the diameter, subtended between the sine and the centre, and measured from the centre to the sine. Thus, CK is the cosine of the angle GCB.

8. The *versed sine* of an arc; is that portion of the diameter, subtended between the sine, drawn from one extremity of the arc, and the other extremity, and is equal to (radius—cosine of arc,) being either BK or BK'.

9. The *tangent* of a circle, is a line drawn from any point of the circumference, at right angles to the diameter; the tangent of an arc, therefore, is a line drawn at right angles to the diameter from one extremity, and intersected by

10. The *secant*, which is a line drawn from the centre through the other extremity, or, in a contrary direction; thus BF is the tangent to the arc BG, and CF is the secant of the same arc.

11. The cotangents, cosecants, &c., are but the tangents and secants of the complements of the arc;

Thus DH is the cotangent, and CH the cosecant of the arc BG.

And in the same way GK, BF, and CF are respectively the cosine, cotangent, and cosecant of the co-arc DG; and BK, the co-versed sine of DG.

12. The supplemental arc ADG has the same sine GK, and equal tangents and secants; but the tangents and secants are of the opposite kind, being measured the other way; if the supplemental arc is greater than 90 degrees, they are negative, and those of the arc positive; if less than 90 degrees, the reverse.

## CHAPTER I.

### I. EQUIVALENTS.

The several triangles CKG, CBF, and CDH, are equilateral, and therefore similar; hence result, by proportion, the several values of the sines, cosines &c.

$$1. \quad GK : CG :: CD : CH; \quad \text{or } GK = \frac{CG \cdot CD}{CH}$$

$$\text{i.e. } \text{sine} = \frac{\text{rad.}^2}{\text{cosec.}}; \quad \text{or } \text{cosec.} = \frac{\text{radius}^2}{\text{sine}} = \frac{1}{\text{sine}}, \text{ where } R = 1.$$

$$2. \quad CK : CG :: CB : CF; \quad \text{or } CK = \frac{CG \cdot CB}{CF}.$$

$$\text{i.e. } \text{cosine} = \frac{\text{rad.}^2}{\text{sec.}} = \frac{1}{\text{sec.}}, \text{ where } R = 1:$$

$$\text{and } \text{sec.} = \frac{\text{rad.}^2}{\text{cosine}} = \frac{1}{\text{cosine}}.$$

$$3. CK : KG :: CB : BF ; \quad \text{or } CK = \frac{KG.CB}{BF} ;$$

$$i. e. \cosine = \frac{\text{rad.}}{\tan.} = \frac{\text{sine}}{\tan.} \quad \text{or } \tan. = \frac{\text{sine}}{\cosine} .$$

$$4. KG : GC :: BF : FC ; \quad \text{or } KG = \frac{GC.BF}{FC} ;$$

$$i. e. \text{sine} = \frac{\text{rad. tan.}}{\sec.} = \frac{\tan.}{\sec.} .$$

$$5. GK : KC :: DC : DH ; \quad \text{or } GK = \frac{HC.DC}{DH}$$

$$i. e. \text{sine} = \frac{\cosine \text{ rad.}}{\cotan.} = \frac{\cos.}{\cotan.} ; \quad \text{or } \cot. = \frac{\cos.}{\text{sine}} .$$

$$6. BF : CB :: CD : DH ; \quad \text{or } BF = \frac{CB.CD}{DH}$$

$$\tan. = \frac{\text{rad.}^2}{\cot.} ; = \frac{1}{\cot.} ; \text{ where } R = 1 :$$

Hence we have these general values.

$$1. \text{sine} = \frac{\text{rad.}^2}{\text{cosec.}} = \frac{1}{\text{cosec.}} = \frac{\tan.}{\sec.} = \sqrt{\text{rad.}^2 - \cosine^2} = \sqrt{1 - \cosine^2} .$$

$$2. \cosine = \frac{\text{rad.}^2}{\sec.} = \frac{1}{\sec.} = \frac{\cot.}{\text{cosec.}} = \sqrt{\text{rad.}^2 - \text{sine}^2} = \sqrt{1 - \text{sine}^2} .$$

$$3. \tan. = \frac{\text{rad.}^2}{\cot.} = \frac{1}{\cot.} = \frac{\text{sine}}{\cos.} \times R = \frac{\text{sine}}{\cos.} .$$

$$4. \cotan. = \frac{\text{rad.}^2}{\tan.} = \frac{1}{\tan.} = \frac{\cosine}{\text{sine}} \times R = \frac{\cosine}{\text{sine}} .$$

$$5. \sec. = \frac{\text{rad.}^2}{\cosine} = \frac{1}{\cos.} = \sqrt{\text{rad.}^2 + \tan.^2} = \sqrt{1 + \tan.^2} .$$

$$6. \text{cosec.} = \frac{\text{rad.}^2}{\text{sine}} = \frac{1}{\text{sine}} = \sqrt{\text{rad.}^2 + \cotan.^2} = \sqrt{1 + \cotan.^2} .$$

$$7. \text{and versed sine} = 1 - \cos. :$$

$$8. \text{The } \tan. \text{ of } 45^\circ = \text{chord of } 60^\circ = \text{sine } 90^\circ = \text{radius} :$$

And generally, if, in a right-angled triangle, the *base* be taken as the actual measurement of the radius, the *hypotenuse* and *perpendicular* become the actual lengths of the *secant* and *tangent* (to the same radius); and, if the *hypotenuse* be taken as radius, the *perpendicular* and *base* become the actual lengths of the *sine* and *cosine*, to the angle, at the point, considered as the centre of the circle.

\* II. VALUES OF THE SINES, COSINES, &c.

*In the different quadrants of the circle.*

Let A be any arc in the first quadrant, BG; then  $\pi - A$  or BDG',  $\pi + A$ , or BDAg;  $2\pi - A$ , or BDAgO, will be the arcs in the other quadrants.

In the first quadrant.		In the second quadrant.	
See figure page 36.	GK = sin. A	sin. $\overline{\pi - A}$ = GK =	+ sin. A
	CK = cos. A	cos. $\overline{\pi - A}$ = CK' = -CK =	-cos. A*
	FB = tan. A	tan. $\overline{\pi - A}$ = Bf = -BF =	-tan. A†
	CF = sec. A	sec. $\overline{\pi - A}$ = Cf = -CF =	-sec. A

\* CK' being measured in an opposite direction to CK is *negative*.

† Bf, being measured opposite to BF, is also *negative*; and Cf being measured, not *through* the other extremity (see def. 10), but, away from it, is *negative* also.

The values of the tan.  $\overline{\pi - A}$  and sec.  $\overline{\pi - A}$  may be thus verified:—

$$\tan. \overline{\pi - A} = \frac{\sin. \overline{\pi - A}}{\cos. \overline{\pi - A}} = \frac{\sin. A}{-\cos. A} = -\tan. A.$$

$$\text{and sec. } \overline{\pi - A} = \frac{\text{rad.}^2}{\cos. \overline{\pi - A}} = \frac{1}{-\cos. A} = -\sec. A.$$

In the third quadrant.		In the fourth quadrant.	
sin. $\overline{\pi + A}$ = K'g =	-sin. A	sin. $\overline{2\pi - A}$ = KO = -KG =	-sin. A
cos. $\overline{\pi + A}$ = CK' =	-cos. A	cos. $\overline{2\pi - A}$ = CK =	+cos. A
tan. $\overline{\pi + A}$ = BF =	+tan. A	tan. $\overline{2\pi - A}$ = Bf = -BF =	-tan. A
sec. $\overline{\pi + A}$ = -(CF) =	-sec. A	sec. $\overline{2\pi - A}$ = Cf =	+sec. A

Sec.  $\overline{\pi + A}$  = CF, but *negative*, for it is not drawn through g, the other extremity, but away from it through G.

Sec.  $\overline{2\pi - A}$  is *positive*, being measured *through* the other extremity O.

These values also, may be verified by their equivalents :—

$$\tan. \pi + A = \frac{\sin. \pi + A}{\cos. \pi + A} = \frac{-\sin. A}{-\cos. A} = \tan. A;$$

$$\sec. \pi + A = \frac{1}{\cos. \pi + A} = \frac{1}{\cos. A} = \sec. A.$$

$$\tan. 2\pi - A = \frac{\sin. 2\pi - A}{\cos. 2\pi - A} = \frac{-\sin. A}{\cos. A} = -\tan. A;$$

$$\sec. 2\pi - A = \frac{1}{\cos. (2\pi - A)} = \frac{1}{\cos. A} = \sec. A.$$

### III. NUMERICAL VALUES,

*Of the sines, cosines, &c., of 30°, 45°, and 60°.*

Chord of 60° = radius = 1;

$$\sin. 30^\circ = \frac{\text{chord } 60^\circ}{2} = \frac{1}{2} = .5$$

$$\cos. 30^\circ = \sqrt{1 - \sin.^2 30^\circ} = \sqrt{1 - \frac{1}{4}} = \sqrt{\frac{3}{4}} = \frac{1}{2} \sqrt{3}$$

$$\tan. 30^\circ = \frac{\sin. 30^\circ}{\cos. 30^\circ} = \frac{1}{2} \times \frac{2}{\sqrt{3}} = \frac{\sqrt{3}}{3}$$

$$\sec. 30^\circ = \frac{1}{\cos. 30^\circ} = 1 \times \frac{2}{\sqrt{3}} = \frac{2}{\sqrt{3}} = \frac{2}{3} \sqrt{3}$$

$$\begin{aligned} \sin. 45^\circ &= \cos. 90^\circ - 45^\circ = \cos. 45^\circ \\ \text{rad.} = 1 &= \sqrt{\sin.^2 45^\circ + \cos.^2 45^\circ} = \sqrt{2 \sin.^2 45^\circ} = 2 \sin. 45^\circ \\ \therefore \sin. 45^\circ &= \sqrt{\frac{1}{2}} = \frac{1}{2} \sqrt{2} \end{aligned}$$

$$\cos. 45^\circ = \sin. 45^\circ = \frac{1}{2} \sqrt{2}$$

$$\tan. 45^\circ = \frac{\sin. 45^\circ}{\cos. 45^\circ} = \frac{1}{1} = 1.$$

$$\sec. 45^\circ = \frac{1}{\cos. 45^\circ} = 1 \times \frac{\sqrt{2}}{1} = \sqrt{2}$$

$$\sin. 60^\circ = \cos. 30^\circ = \frac{1}{2} \sqrt{3}$$

$$\cos. 60^\circ = \sin. 30^\circ = \frac{1}{2} = .5$$

$$\tan. 60^\circ = \frac{\sin. 60^\circ}{\cos. 60^\circ} = \frac{1}{2} \sqrt{3} \times \frac{2}{1} = \sqrt{3}$$

$$\sec. 60^\circ = \frac{1}{\cos. 60^\circ} = 1 \times \frac{2}{1} = 2$$

## IV. RELATIVE VALUES

*Of the sines, cosines, &c., to different radii.*

As the actual lengths of the sines, cosines, &c., of different angles, and to different radii, vary in a ratio, compounded of the ratio of their subtending angles and their given radii, so, when the *angles* are constant, they vary as their radii; and when the *radii* are constant, as their angles.

Let  $\sin a = \sin$  of any angle to radius (unity); and  $\sin A = \sin$  of the same angle, to radius (R); then  $\sin a : \sin A :: 1 : R$ ;

$$\therefore \frac{\sin a}{1} = \frac{\sin A}{R}.$$

whence, to transform formulas, calculated to radius unity—to others, to radius R; wherever  $\sin a$ ,  $\tan a$ , &c., occur, their equivalents,  $\frac{\sin A}{R}$ ,  $\frac{\tan A}{R}$ , &c., must be substituted in their stead.

**EXAMPLE 1.**—To find the value of the  $\tan. A$  (to radius R,) in terms of the sine and cosine.

$$\tan. a = \frac{\sin. a}{\cos. a}$$

$$\frac{\tan. A}{R} = \frac{\sin. A}{R} \cdot \frac{R}{\cos. A} = \frac{\sin. A}{\cos. A}.$$

$\therefore \tan. A = \frac{\sin. A}{\cos. A} \times R$ , which also verifies the following proposition, that

$$\text{as } \sin. a = \frac{\sin. A}{R}$$

$$\therefore \sin. a \times R = \sin. A,$$

or, that the actual value of the sine, cosine, &c., of any angle A, whose radius is any length, R, is equal to that of the sine, cosine, &c., of the same angle, *calculated to unity*, multiplied by the given radius, R.

**EXAMPLE 2.**—What is the numerical value of the  $\tan. 45^\circ$  to radius 500?

$$\tan. 45^\circ \text{ to rad. } 1 = 1.$$

$$\frac{\tan. 45^\circ}{\text{to rad. } R} = \tan. A = \tan. a \times R = 1 \times 500 = 500.$$

**EXAMPLE 3.**—What is the value of the  $\sec. 30^\circ$  to radius 300?

$$\sec. a = \frac{1}{\cos. a} = \frac{1}{\cos. 30} = 1 \times 2\sqrt{1} = 2\sqrt{3}$$

$$\frac{\sec. 30^\circ}{\text{to rad. } 300} = \sec. A = \sec. a \times R = 2\sqrt{3} \times 300 = 600\sqrt{3}$$

## CHAP. II.

*and per. are called* **RIGHT-ANGLED TRIANGLES.**

**CASE. I.**—When one acute angle and one side are given.

There will be no difficulty, if the following be borne in mind, viz.:—

1. That, if the *hypotenuse* be made *radius*, each leg will represent the sine of its opposite angle to that radius.

2. And if either the *base* or *perpendicular* be made *radius*, the hypotenuse and other side will represent the secant and tangent of the angle at the centre, and that the sides are proportional to the lines they represent.

Hence (as in the first case) as radius : to the hypotenuse :: sine of either angle : to its opposite side.

And (in the second case) 1st, as radius : is to the *perpendicular* :: so is tangent of the vertical angle : to the base ; and also : so is secant of the vertical angle : the hypotenuse.

And 2nd, as radius : is to the base :: so is tangent of the angle at base : to the perpendicular ; and : : so is secant of the same angle : to the hypotenuse.

**EXAMPLE 1.**—Let the hypotenuse of a right-angled triangle be 3 chains 50 links, and the angle at the base 25 degrees 15 minutes. Required the perpendicular and the base.

As radius	= 10·000000	
to log. hypotenuse 3·50	= 0·544068	} <i>added &amp; after a c</i>
so is log. sine 25° 15'	= 9·629989	
to log. perpendicular 1·49	= 0·174057	
and as radius	= 10·000000	
to log. hypotenuse 3·50	= 0·544068	
so is log. cosine 25° 15'	= 9·956387	
to log. base 3·16½	= 0·500455	

*Ans.* Perpendicular = 1·49 chains

Base = 3·16½ chains.

**Ex. 2.**—Given the vertical angle 35° 54', and the perpendicular 12·20 chains. Required the base and hypotenuse.

*Ans.* Base = 8·83 chs.; Hyp. = 15·06 chs.

**Ex. 3.**—What are the hypotenuse and perpendicular of a right-angled triangle, whose base is 25 chains 80 links, and the vertical angle 89° 27'?

*Ans.* Perp. = 24 links; Hyp. = 25·80 chs.

CASE II.—When two sides are given.

As in all right-angled triangles, the square of the hypotenuse is equal to the sum of the squares of the base and perpendicular, if any two sides are given, the third can be found by one of the following formulas :—

$$\text{Hypotenuse} = \sqrt{\text{perp.}^2 + \text{base}^2}$$

$$\begin{aligned} \text{Perpendicular} &= \sqrt{\text{hyp.}^2 - \text{base}^2} = \sqrt{(H+B).(H-B)} \\ &= \frac{\log. \overline{H+B}}{2} + \frac{\log. \overline{H-B}}{2} \end{aligned}$$

$$\begin{aligned} \text{Base} &= \sqrt{\text{hyp.}^2 - \text{perp.}^2} = \sqrt{(H+P).(H-P)} \\ &= \frac{\log. \overline{H+P}}{2} + \frac{\log. \overline{H-P}}{2} \end{aligned}$$

In the case, however, of the base and perpendicular being given, it is sometimes useful to obtain the hypotenuse *trigonometrically*, thus :

As perp. : rad. :: base : to tang. angle at vertex ;  
then, as sine angle at vertex : base :: radius : hypotenuse.

EXAMPLE 1.—What is the hypotenuse, when the perpendicular is 2·50 chains, and the base 3·90 chains ?

First hyp.  $= \sqrt{2\cdot50^2 + 3\cdot90^2} = \sqrt{21\cdot4600} = 4\cdot63$  chains.  
secondly, by the above formula,

by saying,	as log. perpendicular 2·50	= 0·397940
	is to log. radius	= 10·000000
	so is log. base 3·90	= 0·591065
	to tang. angle at vertex 57·20	= 10·193125

and then,

as log. sine angle at vertex 57·20	= 9·925222
log. base 3·90	= 0·591065
so is radius	= 10·000000
to log. hyp. 4·63	= 0·665843

we obtain the hypotenuse 4·63, the same length as in the preceding.

The angles can be obtained from the first method, by the following proportion, viz. :—

As log. hyp. 4·63 chains	= 0·665581
: log. rad.	= 10·000000
: : log. perpend. 2·50	= 0·397940
: log. sine angle at base 32° 41	= 9·732359
and : log. cos. angle at vertex 57° 19' ;	



**EXAMPLE 2.**—Let AC equal 45 chains, BC 36 chains, and the angle A,  $51^{\circ} 25'$ ; required the base AB.

As log. BC, 36 chains	= 1.556303
to log. sin. $\angle A$ , $51^{\circ} 25'$	= 9.899041
so is log. AC, 45 chains	= 1.653213
	<hr/>
	11.546254
	<hr/>
	1.556303
to log. sin. ABC, $77^{\circ} 43'$	= 9.989951
or to its supplement, $102^{\circ} 17'$ .	

The angle ABC has therefore two values, depending upon whether it is greater or less than a right angle; and also the angle ACB, and its opposite side AB, which may be either AB or AD.

**EXAMPLE 3.**—The two sides of a triangle are AC, 50 chains; CB, 25 chains; and the angle ACB, 16 degrees 45 minutes; what are the other sides and angles? *Answer* Base 27.07 chs.  $\angle B = 147^{\circ} 49'$   $\angle A = 15.26'$ .

**CASE II.**—When the two sides and included angle are given.

**Rule.**—Subtract the given angle from  $180^{\circ}$ , which will give the sum of the other two; then,  $x$  and  $y$  being any two angles,

$$x = \frac{1}{2}(x+y) + \frac{1}{2}(x-y),$$

$$\text{and } y = \frac{1}{2}(x+y) - \frac{1}{2}(x-y).$$

Then, to obtain  $(x-y)$ ; say, as the sum of the two sides : is to their difference :: so is the tan.  $\frac{1}{2}$  sum of the two unknown angles,  $(x+y)$  : to the tan.  $\frac{1}{2}$  difference  $= (x-y)$ .

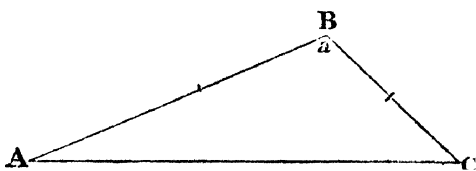
By substituting the values of  $x+y$ , and  $x-y$ , which are now both known, in the above formula, the values of  $x$  and  $y$  will be obtained.

In the triangle ABC, let the two sides, AB and BC (AB being greater than BC) be given, and the angle, at B  $= a^{\circ}$

then  $180^{\circ} - a^{\circ} = \text{sum of angles at A and C}$ ; and  $(x-y) = \text{difference}$ .

$$\angle ACB = \frac{180-a}{2} + \frac{x-y}{2}$$

$$\text{and } \angle BAC = \frac{180-a}{2} - \frac{x-y}{2}$$



It is now reduced to the condition of Case 1st; proceed, therefore, according to the rules there given.

The following formula may sometimes be useful, in at once obtaining the base.

Find first the value of  $(x-y)$  as above, then say—

$$\text{As } \cos. \frac{1}{2}(x-y) : \cos. \frac{1}{2}(x+y) :: AB+BC : AC$$

or,

$$\text{As } \sin. \frac{1}{2}(x-y) : \sin. \frac{1}{2}(x+y) :: AB-BC : AC$$

$$AC = \frac{\cos. \frac{1}{2}(x+y)}{\cos. \frac{1}{2}(x-y)} \cdot (AB+BC).$$

$$\text{or } AC = \frac{\sin. \frac{1}{2}(x+y)}{\sin. \frac{1}{2}(x-y)} (AB-BC)$$

that is,

as $\cos. \frac{1}{2}$ (difference of angles),	or	as $\sin. \frac{1}{2}$ (difference of angles),
: $\cos. \frac{1}{2}$ (sum),		: $\sin. \frac{1}{2}$ (their sum),
: : sum of the two sides,		: : difference of the two sides,
: to the base,		: to the base.

The advantage of using this formula is, that the logarithmic values of the *cosines* of  $(x+y)$  and  $(x-y)$  can be obtained at the same time as those of their tangents, which are wanted, in the first formula, to obtain the value of  $x-y$ ; and the *separate* values of the angles are not required.

**EXAMPLE 1.**—Let the two sides AB, BC of the triangle ABC be 36.40 and 24.40 chs., and the included angle 16 degrees 26 minutes. Required the other side and the remaining angles.

$\begin{array}{r} 180^{\circ}00' \\ 16^{\circ}26' \\ \hline 2 \mid 163^{\circ}34' \\ \hline 81^{\circ}47' = \frac{x+y}{2} \end{array}$	$\begin{array}{r} 36.40 \\ 24.40 \\ \hline 60.80 = AB + BC \\ 12.00 = AB - BC \end{array}$
$\begin{array}{l} \text{As log. } AB + BC, \\ \text{is to log. } AB - BC, \\ \text{so is log. tan. } \frac{x+y}{2}, \end{array}$	$\begin{array}{r} 60^{\circ}80' = 1.783904 \\ 12^{\circ}00' = 1.079181 \\ \hline 11.919616 \\ 53^{\circ}49' = 10.135712 \end{array}$
$\begin{array}{r} 81^{\circ}47' \\ 53^{\circ}49' \\ \hline 135^{\circ}36' = \angle ACB \\ 27^{\circ}58' = \angle BAC \end{array}$	
$\begin{array}{r} \text{Add } \angle ABC = 16^{\circ}26' \\ \hline 180^{\circ}00' \text{ sum of the three angles.} \end{array}$	

The question being now reduced to the first case, the base may be obtained, as follows, by saying:—

$$\begin{array}{rcl}
 \text{As log. sine } \angle \text{ BAC, } 27^{\circ} 58' & = & 9.671134 \\
 \text{is to log. BC, } 24.40 & = & 1.387390 \\
 \text{so is log. sine } \angle \text{ ABC, } 16^{\circ} 26' & = & 9.451632 \\
 & & \underline{10.839022} \\
 & & 9.671134 \\
 \text{to log. AC—} 14.72 & = & \underline{1.167888}
 \end{array}$$

or, otherwise, by using the formula, above referred to, of the cosines of the sum and difference of the unknown angles, and saying:—

$$\begin{array}{rcl}
 \text{As log. cosine } \frac{1}{2} x - y, 53.49 & = & 9.771125 \\
 : \text{ to log. cosine } \frac{1}{2} x + y, 81.47 & = & 9.155083 \\
 :: \text{ is log. (AB + BC), } 60.80 & = & 1.783904 \\
 & & \underline{10.938987} \\
 & & 9.771125 \\
 \text{to log. AC, } 14^{\circ} 72' & = & 1.167862
 \end{array}$$

differing only in the two last places of decimals, which do not agree with the preceding, in consequence of the cos.  $\frac{x-y}{2}$  ( $53^{\circ} 49'$ ), not being the exact value of the logarithm,  $10.135712$ ; there is no difference, however, in their values, as to links, which is sufficiently near for practical purposes.

For finding the exact values of logarithmic sines, cosines, &c., and carrying the whole work out to seconds, see chapter on the "Theodolite and Trigonometrical Calculations."

**EXAMPLE 2.**—Given the two sides 45 chains 44 links, and 40 chains 48 links, of any triangle ABC, and the included angle  $117^{\circ}$  50 minutes; what are the two other angles and the remaining side?

$$\begin{array}{lcl}
 \text{Ans. Base} = 136.91 \text{ chs.} & \angle \text{ at B} = & 32^{\circ} 47' 30'' . 33 \\
 & \angle \text{ at C} = & 29^{\circ} 22' 00'' . 4
 \end{array}$$

**EXAMPLE 3.**—The included angle of a triangle is 45 degrees; the two sides are respectively 70 chains and 50 chains; what will be the length of the base?

$$\text{Ans. } 49.50 \text{ chs.}$$

**CASE III.**—When the three sides only are given.

Let fall a perpendicular from the vertex upon the base, dividing the triangle into two right-angled triangles. We shall then have one side, and the right-angle known in each. And, by finding the lengths of the

segments of the base of the whole triangle, we obtain in each of the smaller right-angled triangles two given sides and an angle. Now, by saying,—

As the whole base, or sum of the segments  
: is to the sum of the two other sides,  
: : so is the difference of those two sides,  
: to the difference of the segments of the base.

And, by adding half the sum to half the difference, we obtain the *greater* segment, which is always adjoining the *greater* side of the given triangle, and, by subtracting half the difference from half the sum, we obtain the *smaller* segment, adjoining the *smaller* side of the whole triangle.

The case having come under the form of two right-angled triangles, the required sides and angles must be worked out by the rules before given.

Let ABC be a triangle, whose three sides AB, BC, and AC, are given.

From B, let fall BD perpendicular to AC. In the two triangles ABD, CBD, AB and BC being known, and the angles at D right angles, we have to find first the two sides AD and DC; say,—

As  $AC : AB + BC :: BC - AB : x$   
when  $x = AD \sim DC$ ,  
and because BC is greater than AB,  
therefore DC is greater than AD;

then, by saying, (*see case of right-angled triangles*),—

as  $AB : \text{rad.} :: AD : \cos. \angle \text{ at } A$

we obtain the angle at A; and, by similar proportions, the remaining angles at C and B.—*The angles ABD, CBD, calculated from the smaller triangles, should be together equal to the supplement of the sum of the angles at A and C.*

**EXAMPLE 1.**—Let ABC be a triangle, having the sides AB, BC, and AC, respectively, equal to 25 chains, 36 chains, and 48 chains. It is required to find the angles.

Let fall the perpendicular BD, which will divide the given triangle into two smaller right-angled triangles.

Then, to find the difference of the segments of the base, say,—

$$\text{As } AC : BC + AB :: BC - AB : (AD - DC)$$

$$\text{or, } 48 : 61 :: 11 : 13.98-$$

and adding half the sum to half the difference, for the greater segment, and subtracting them, for the less, we have,—

$$\frac{1}{2} \text{ of } 48 = 24 ; \text{ and } \frac{1}{2} \text{ of } 13.98 = 6.99.$$

$$\text{where } 24 + 6.99 = 30.99 = \text{greater segment, DC,}$$

$$\text{and } 24 - 6.99 = 17.01 = \text{smaller segment, AD,}$$

It is now reduced to the case of right-angled triangles; next say,—

$$\text{As log. AB, 25,} \quad = 1.39794$$

$$\text{is to log. rad.} \quad = 10.00000$$

$$\text{so is log. AD, 17.01,} \quad = 1.23070$$

$$11.23070$$

$$1.39794$$

$$\text{to log. cos. angle A, } 47^\circ 8', \quad = 9.83276$$

$$\text{And as log. BC, 36,} \quad = 1.55630$$

$$\text{to log. rad.} \quad = 10.00000$$

$$\text{is log. DC, 30.99,} \quad = 1.49122$$

$$11.49122$$

$$1.55630$$

$$\text{to log. cos. angle C, } 30^\circ 35', \quad = 9.93492$$

Adding the angle at A and C, and deducting them from  $180^\circ$ , we obtain the angle at B.

$$47^\circ 8' + 30^\circ 35' = 77^\circ 43'; \text{ and}$$

$$180^\circ - 77^\circ 43' = 102^\circ 17' = \text{angle B.}$$

**EXAMPLE 2.**—Given a triangle, whose sides are 25 chains 16 links, 24 chains 13 links, and 16 chains 17 links, respectively; required the angles.  
*Ans.*  $74^\circ 20'$ ;  $67^\circ 25'$ ;  $38^\circ 15'$ .

**EXAMPLE 3.**—A triangle, whose sides are respectively, 40, 50, and 60 chains, being given, it is required to find the angles.

$$\text{Ans. } 82^\circ 49'; 41^\circ 25'; 55^\circ 46'.$$

**EXAMPLE 4.**—What are the angles of that triangle, whose sides are 498 chains 16 links, 464 chains, and 40 chains, respectively?

$$\text{Ans. } 125^\circ 29'; 52^\circ 11'; 2^\circ 20'.$$

2ND METHOD.—When the nature of the question merely requires the determining of one of the angles, the following formula may be used, viz.:—

$$\cos. \angle C = \frac{AC^2 + BC^2 - AB^2}{2 AC CB} \text{ or, (vide fig. p. 49) making } AB=c$$

$BC = a, \text{ and } AC = b, \text{ corresponding to the angles to which they}$

$\text{are opposite. } \cos. C = \frac{a^2 + b^2 - c^2}{2 ab}$

For  $AB^2 = AC^2 + BC^2 \pm 2 AC. CD$  (see Theor. 7, page 13.)

$$\therefore \pm CD = \frac{AC^2 + BC^2 - AB^2}{2 AC}$$

now  $BC : CD :: \text{rad.} : \cos. C$

$$\therefore \pm \frac{CD}{BC} = \pm \frac{\cos C}{R} \text{ and } \pm CD = BC \cos. C \text{ (when radius} = 1)$$

$$\text{(by substituting) } BC \cos. C = \frac{AC^2 + BC^2 - AB^2}{2 AC}$$

$$\therefore \cos. C = \frac{AC^2 + BC^2 - AB^2}{2 AC. CB} = \frac{a^2 + b^2 - c^2}{2 ab};$$

Where  $\cos. C$  is that of an angle, either greater or less than a right angle, whose value is expressed in terms of its sides;  $a$  and  $b$  being the sides that include it, and  $c$  the side opposite to it.

This formula, however, can only be used in its *present* form, when the sides are *small*, and when they can be calculated arithmetically, as in the case of the 1st Example, in the *first method*, the result of which we will now proceed to verify, by the present formula.

The given sides are 25, 36, and 48 chains.

Substituting these lengths in the above formula, we have,—

$$25^2 = 625; 36^2 = 1296; \text{ and } 48^2 = 2304$$

$$\text{then } \cos. C = \frac{625 + 2304 - 1226}{2 (1200)} = \frac{1633}{2400}$$

$$\frac{1633}{2400} = 0.6804166 = \text{natural } \cos. C$$

$$\text{natural } \cos. = \cos. \text{ to rad. } (1) = \cos. a.$$

$$\log. \cos. = \log. \text{ cosine to rad. } 10^{10} = \cos. A.$$

$$\cos. A = \cos. a \times R \text{ (see page 41)}$$

$$\therefore \log. \cos. A = \log. \cos. a + \log. 10^{10} (10)$$

by substituting natural cos.  $a = 0.6804166$

$$\log. \cos. a = -1.832781$$

$$+ \log. R = 10$$

$$47^\circ 8' = \log. \cos. A = 9.832781, \text{ the same result as before.}$$

To find the angle  $C$  in the same triangle.

$$\cos. C = \frac{AC^2 + BC^2 - AB^2}{2 AC. BC}$$

$$\text{or } \cos. C = \frac{2304 + 1296 - 625}{2 (1728)} = \frac{2975}{3456} = .85999$$

$$\log. 85999 = -1.93492$$

$$+ \log. R = 10$$

$$30^\circ 35' = \log. \cos. C = 9.93492$$

To find the angle at  $B$ .

$$\cos. B = \frac{AB^2 + BC^2 - AC^2}{2 AB. BC}$$

$$\text{or } \cos. B = \frac{625 + 1296 - 2304}{2 (900)} = \frac{-383}{1800} = -.21278$$

$$\log. .21278 = -1.32793$$

$$+ \log. \text{rad.} = 10$$

$$\log. -\cos. 77^\circ 43' = 9.32793$$

but this is the supplemental angle, being the value of  $-\cos. B$ ;

therefore  $180^\circ - 77^\circ 43' = 102^\circ 17' = \cos. \text{ angle } B$ .

It will be seen that these angles are the same as those which were obtained by the first method, where the same triangle was used.

WHEN THE SIDES, HOWEVER, ARE LARGE, it is necessary to convert this formula into a more convenient expression, for logarithmic calculation. viz.—

$$\text{Because, } \cos. C = \frac{a^2 + b^2 - c^2}{2 ab} \text{ (to rad. 1)}$$

$$\frac{\cos. C}{R} = \frac{a^2 + b^2 - c^2}{2 ab} \text{ to rad. } R \text{ (see page 51)}$$

$$\therefore \cos. C = R \frac{(a^2 + b^2 - c^2)}{2 ab}$$

and versed sine = rad.  $-\cos.$

$$\text{vers. sin. } C = R - R \frac{(a^2 + b^2 - c^2)}{2 ab} = R \left(1 - \frac{a^2 + b^2 - c^2}{2 ab}\right);$$

$$= R \frac{(2 ab - a^2 - b^2 + c^2)}{2 ab}$$

but  $\sin \frac{1}{2} C = \frac{1}{2} \text{ rad (vers. sine } C)$

$$\therefore \sin \frac{1}{2} C = \frac{\text{rad}^2 (2 ab - a^2 - b^2 + c^2)}{4 ab}$$

$$= \text{rad.}^2 \frac{(c^2 - (a-b)^2)}{4ab}$$

$$= \text{rad.}^2 \frac{(c+a-b) \cdot (c-a+b)}{4ab}$$

$$\text{sine } \frac{1}{2} C = \text{rad.} \sqrt{\frac{(s-a) \cdot (s-b)}{ab}}$$

where  $s$  = half the perimeter—

and for the  $\cos. \frac{1}{2} C$ , it may be proved in the same way, that

$$\cos. \frac{C}{2} = \text{rad.} \sqrt{\frac{(s-c)s}{ab}}$$

*Assuming the same data as before, for the purpose of verifying the former results, and substituting their values, respectively, in the present formula of the values of half the sines and cosines, we obtain,—*

$$\text{where } s = \frac{1}{2} \text{ perimeter} = \frac{25 + 36 + 48}{2} = 54.50$$

$$\text{then sine } \frac{1}{2} A = \text{rad.} \sqrt{\frac{(54.50-25) \cdot (54.50-48)}{25 \times 48}}$$

log. 29.50	=	1.46982
log. 6.50	=	0.81291
—log. 25	=	1.39794
—log. 48	=	1.68124
		2.28273
		—3.07918
		<hr/>
		2   —1.20355
		—1.60178
+ log. rad.	=	10.00000
log. sine $\frac{1}{2} A = 23^\circ 34'$	=	9.60178
		<hr/>
		2

angle  $A = 47^\circ 8'$ , the same as before.

By the 2nd formula—that of  $\cos. \frac{1}{2} A = \text{rad.} \sqrt{\frac{(s-a)s}{bc}}$ , where  $A$  is the given angle required, and  $a$  is the side opposite to it.

log. 18.50	=	1.26717
log. 54.50	=	1.73640
		3.00357
—log. 25	=	1.39794
—log. 48	=	1.68124
		—3.07918
		<hr/>
		2   —1.92439
		—1.96219
		10.00000
log. cos. $\frac{1}{2} A = 23^\circ 34'$	=	9.96219
		<hr/>
		2

angle  $A = 47^\circ 8'$  as before.



A few Examples, without answers, are subjoined, which the student is required to calculate and verify, by the various methods above.

**EXAMPLE 1.**—Given the three sides 40 chains, 90 chains and 60 chains, to find the opposite angles.

**EXAMPLE 2.**—The three sides of a triangle are 24·50 chains, 31·60 chains, and 31·96 chains, what are the angles?

**EXAMPLE 3.**—The sides of a triangular field are, respectively, 20 chains, 3·50 chains, and 45·20 chains. What are the opposite angles?

As a knowledge of some of the general relative values of sines, cosines, and tangents, in terms of each other, may be found exceedingly useful in practice to the Surveyor, some of the most useful ones are subjoined.

*The most important formula, the basis, in fact, of the whole, is—*

$$\text{sine } (a + b) = \text{sine } a \cdot \cos. b + \cos. a \cdot \text{sine } b$$

$$\text{sine } (a - b) = \text{sine } a \cdot \cos. b - \cos. a \cdot \text{sine } b$$

$$\cos. (a + b) = \cos. a \cdot \cos. b - \text{sine } a \cdot \text{sine } b$$

$$\cos. (a - b) = \cos. a \cdot \cos. b + \text{sine } a \cdot \text{sine } b$$

These are the values of the sines and cosines of the sum and difference of two arcs, in terms of the simple arcs.

By *adding* and *subtracting*, we obtain,—

$$\text{sine } (a + b) + \text{sine } (a - b) = 2 \text{sine } a \cos. b$$

$$\text{sine } (a + b) - \text{sine } (a - b) = 2 \cos. a \text{sine } b$$

$$\cos. (a + b) + \cos. (a - b) = 2 \cos. a \cos. b$$

$$\cos. (a - b) - \cos. (a + b) = 2 \text{sine } a \text{sine } b$$

which are the values of the sum and difference of the sines and cosines of the sum, and difference of the arcs.

Again, by making  $a + b = m$

$$\text{and } a - b = p.$$

we have—

$$\text{sine } m + \text{sine } p = 2 \text{sine } \frac{1}{2}(m + p) \cos. \frac{1}{2}(m - p)$$

$$\text{sine } m - \text{sine } p = 2 \cos. \frac{1}{2}(m + p) \text{sine } \frac{1}{2}(m - p)$$

$$\cos. m + \cos. p = 2 \cos. \frac{1}{2}(m + p) \cos. \frac{1}{2}(m - p)$$

$$\cos. p - \cos. m = 2 \text{sine } \frac{1}{2}(m + p) \text{sine } \frac{1}{2}(m - p)$$

but  $m$  and  $p$  are *any angles whatever*, so that

$$\text{sine } a + \text{sine } b = 2 \text{sine } \frac{1}{2}(a + b) \cos. \frac{1}{2}(a - b)$$

$$\text{sine } a - \text{sine } b = 2 \cos. \frac{1}{2}(a + b) \text{sine } \frac{1}{2}(a - b)$$

$$\cos. a + \cos. b = 2 \cos. \frac{1}{2}(a + b) \cos. \frac{1}{2}(a - b)$$

$$\cos. b - \cos. a = 2 \text{sine } \frac{1}{2}(a + b) \text{sine } \frac{1}{2}(a - b)$$

This formula is useful in facilitating the transformation of the *product* of the sines of the sums and differences of angles into simple angles.

Now, by dividing the last class of formula, the one by the other, we have,—

$$\begin{aligned}\frac{\sin a + \sin b}{\sin a - \sin b} &= \frac{\sin \frac{1}{2}(a+b) \cos \frac{1}{2}(a-b)}{\cos \frac{1}{2}(a+b) \sin \frac{1}{2}(a-b)} = \frac{\tan \frac{1}{2}(a+b)}{\tan \frac{1}{2}(a-b)} \\ \frac{\sin a + \sin b}{\cos a + \cos b} &= \tan \frac{1}{2}(a+b) \\ \frac{\sin a + \sin b}{\cos b - \cos a} &= \cot \frac{1}{2}(a-b) \\ \frac{\sin a - \sin b}{\cos a + \cos b} &= \tan \frac{1}{2}(a-b) \\ \frac{\sin a - \sin b}{\cos b - \cos a} &= \cot \frac{1}{2}(a+b) \\ \frac{\cos a + \cos b}{\cos b - \cos a} &= \frac{\cot \frac{1}{2}(a-b)}{\tan \frac{1}{2}(a-b)}\end{aligned}$$

And by making  $b = 0$ , we obtain the value of the  $\tan \frac{1}{2}a$ ;  $\cot \frac{1}{2}a$ ,  $\tan^2 \frac{1}{2}a$ ,  $\cot^2 \frac{1}{2}a$ .

Again,—

$$\text{As } \tan. \frac{a+b}{2} = \frac{\sin(a+b)}{\cos(a+b)} = \frac{\sin a \cos \frac{b}{2} + \cos a \sin \frac{b}{2}}{\cos a \cos \frac{b}{2} - \sin a \sin \frac{b}{2}}$$

Dividing by  $\cos a \cos \frac{b}{2}$ , we have,—

$$\frac{\frac{\sin a}{\cos \frac{b}{2}} + \frac{\sin b}{\cos b}}{1 - \frac{\sin a \cos \frac{b}{2}}{\cos a \cos \frac{b}{2}}} = \frac{\tan a + \tan \frac{b}{2}}{1 - \tan a \tan \frac{b}{2}}$$

$$\text{so also, } \tan. (a-b) = \frac{\tan a - \tan \frac{b}{2}}{1 + \tan a \tan \frac{b}{2}}$$

Make  $b = a$  (in the first value)

$$\text{and } \tan. 2a = \frac{2 \tan a}{1 - \tan^2 a} \text{ \&c., \&c.}$$

[NOTE.—For a fuller investigation of this subject, see Hall's Trigonometry, which I would strongly recommend to every practical man, who might have leisure to follow it up.]

# LAND SURVEYING.

## Part the First.

### CHAP. I.

#### ON THE CHAIN.

As every measurement, whether of work, solidity, or superficial extent, must be measured by some unit, to which it can bear constant relation, it has been concluded in the case of land measurement, to make that unit an acre; the acre, or arpent, is the generally recognised unit of land measurement; it varies, however, considerably in different Counties.\*

THE STATUTE ACRE in England consists of ten square chains, or one chain front by 10 chains deep, or of its equivalent rectangle  $ax$ , where  $a$  can be any number, and  $x = \frac{10}{a}$  square chains; each chain containing 22 yards, or 4 poles, or 66 feet, or 100 links, and each link 792 inches.

The acre, therefore, is equal to 10 square chains

or (4) <sup>s</sup> poles	×	10	=	160	square poles.
or (22) <sup>s</sup> yds.	×	10	=	4840	square yards.
or (66) <sup>s</sup> feet	×	10	=	43560	square feet.
or (100) <sup>s</sup> links	×	10	=	100,000	square links.

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\* For the different values of the acre in different Counties of England, see the subject, at the end of the book. / / /

The chain, in common use, is called Gunter's chain, from its inventor, and is divided into ten equal parts, distinguished by a piece of brass, with notches; the brass at the first division, from either end, having one notch; at the second division, two notches; at the third, three; at the fourth, four; and at 50 links, or the middle of the chain, there is a round piece of brass.

The object of marking these divisions from either end of the chain, is to enable the surveyor to measure *either way* from each end; but it is productive sometimes of serious mistakes, as in consequence of 60 links being marked the same as 40, when the eye is not accustomed to distinguish the distance between them, the one is frequently mistaken for the other; and, in measuring with the chain, no mistake is so common as this to a young beginner; and I would particularly charge him to be careful, that in taking off a distance of, what he may suppose to be, 2 chains 45 links, he is not mistaking for it 2 chains 65 links; there is less likelihood of mistaking 30 links for 70, though a stupid inattentive assistant will sometimes even make that mistake.

Each of the above brass divisions of the chain is again sub-divided into other ten parts, or links. If therefore a chain = unity;

$$\text{each brass division} = \frac{1}{10}, \text{ or } 0.1 \text{ chains;}$$

$$\text{and each link} = \frac{1}{100}, \text{ or } 0.01 \text{ chains.}$$

The advantages of this arrangement is, that, in measuring a line, it matters not whether the distance be termed 7.32 chs. (7 chains 32 links,) or 732 links; and, as 10 square chains make one acre, or 10 times 100 links squared, it is only necessary to multiply together the length and breadth, given in links, of a piece of ground, whose area is required,

and set off four decimal places, when the integers will be square chains; or set off five decimal places, and the integers will be the required acres.

**NOTE.**—[In doing the chain up, after the day's work, begin always at the middle, folding two links up at once. When done up in this way, by taking hold of the two handles, and throwing the other part from you, you are safe from any entanglement of the chain, when you want to use it.]

*To measure a straight line with a chain.*

In measuring with the chain, it is requisite to have ten small arrows, or pins of strong iron wire, about a foot and a half long, and pointed at one end, to stick into the ground.

A piece of red cloth should also be tied to the ring of each arrow, so that they may be more easily perceived in the midst of grass or underwood.

Having determined upon the starting point and the direction of the line, it is always requisite to have the line carefully ranged, and generally one or two poles or flags set between the two end flags; these flags should consist of a straight pole about an inch or  $\frac{3}{4}$  inch diameter, pointed and shoed at the bottom, and having a white and red flag, about nine inches square at the top, they are generally about six feet high, though in the taking of long base lines, they are occasionally made of from 10 to 15 feet.

On a large survey, a Surveyor should be at least furnished with three of the latter and half a dozen of the former.

There is a great deal of mechanical nicety in setting these poles, and preserving a correct line throughout. To obtain this,—

Set up a flag at your starting point, and one at the termination of your intended line, and, having selected the distances between, where you are desirous of setting up flags, direct your assistants to hold the flags carefully upright and move as you direct them, one way or the other.

It is frequently the practice for persons, who are directing the ranges, to stand immediately behind the starting flag, and, by first looking on one side and then on the other, to range the intervening flags with the distant one : this is by no means a correct method ; it is, in all cases, whether of directing the fixing of a flag by another between two fixed points, or the continuing of those points onwards oneself, an act of necessity, to stand some 10 or 20 paces off from the nearest flag, and, by contracting its visual breadth, to secure the correct position of the flag that is to be put down, by at once covering the three.

Having thus obtained a practical method of measuring along a base line, and not deviating at the fixing of every chain's length to the right or the left, we have but to secure a correct and equal extension of the chain at every length, to obtain the true length of the line required. Let one assistant take the lead, who is called the *leader*, having the ten pins and the chain handle in his left hand, and proceed towards the mark at the end of the line ; while the other assistant, called the *driver*, holding the other end of the chain also in his left hand, keeps it close against the mark or starting point ; when the leader has come to the end of the chain, let him turn round with his face towards the driver, and, taking one of the arrows in his right hand, let him pull the chain tightly, keeping it on the ground, and fix the arrow he held in his right hand in the ground.

As the chain is seldom straight at first, when the leader faces the driver, the driver must always keep *his* end to the ground, close to the mark ; but the leader, previously to finally setting the arrow, should raise the chain with both hands, shake it to the very end, and before he puts his arrow down, see that it is perfectly straight.

The leader must also observe, as he is putting down the chain, to look towards the driver, to see if he is himself in the right direction, or whether the driver is noting him to the right or to the left ; when he has succeeded in hitting the right direction, (which he knows by the driver calling out "*down*," ) and, obtaining the proper measurement of the chain, by carefully complying with the directions above, and has put his first arrow down, he must *return* the cry of "*down*" to the driver, and, taking the chain up in his right hand, must proceed onwards until the driver, coming to the first arrow, cries out "*stop*," when the same process is repeated. The leader having a second time returned the cry of "*down*," the driver picks up the arrow, carrying it in his left hand, taking care, as long as he has two or more flags to cover before him, to direct the leader ; when (the leader), however, has past all

but the last flag, the proper position of the arrows depends upon himself, which he must carefully do, by covering the backward flags.

When the leader has put down the ten pins, he calls out "*tally*," and the driver, dropping his end of the chain, comes up to the place, picks up the last arrow put down, counts over the whole of the arrows, and, putting his foot to the mark, returns them again to the leader, who finding they are the right number (10), proceeds with the chaining as before; the second "*tally*" is then called; then the third; each cry, to prevent mistakes, being repeated by the driver. I would here observe, that many errors have arisen from the omission of these apparently trifling superfluities, and much valuable time lost. It is this superfluity of care which distinguishes the man of practice from the novice—the latter *thinks* it so simple that he cannot make a mistake; the former *knows* that, from that very simplicity, error is more likely to ensue.

In this manner they continue, till the whole line is measured. Should it not be a complete chain to the end, it is usual for the leader to put his end of the chain down to the mark, and for the driver to read off the distance; and this is certainly the correct method, except there are offsets to be taken between the last full chain and the end of the line, when the driver had better put his end down, and the leader pull up, as the position of the offsets is measured from the driver's end.

It is advisable, at the end of each tally, to leave a mark in the ground, and note it in the field book, especially when the line is long, as the latter prevents mistakes, and the former makes the correction of it more easy.

It need only be added that the leader having put down his arrow, should, on proceeding, walk so as to keep the flags, that are before him, always covered, and, on turning round, should so arrange himself as to cover the starting point (as near as he can) with the last arrow put down; this will be saving the driver some considerable trouble in keeping him in the proper direction, and will, at the same time, give the leader himself a facility in keeping the course, when a hill, or other local obstruction, may hide the forward flags from the driver's view.

*For practice*—I would recommend some line to be selected of about half a mile, which should be carefully measured two or three times, until not more than an error of a link should be detected, in two or three consecutive trials.

NOTE.—[It is requisite, to ensure accuracy, that the chain should be frequently measured. If found incorrect, take care that the error be divided among the whole, so that *each* 10 links be of equal length; in each 10 links, it is better to cut off a little from several rings, than take a whole ring away.]

Professional Surveyors measure their chains regularly every morning, when they are on service.

## CHAP. II.

### THE OFFSET STAFF,

*(For the Chain.)*

Is a narrow slip of wood about  $1\frac{1}{2}$  inch wide by 1 inch thick, and generally 10 links long, divided into links; it is made of deal or some light wood, and should be furnished at one end with a small notch or hook, to put the chain through the hedges, and be numbered on both sides, from different ends. It is used for the purpose of measuring short distances, called *offsets*,\* from the line to the hedges, &c.

As these offsets must be measured at right-angles to the chain, the Surveyor should stand on the opposite side of the chain to the hedge, or object to be measured to, and walking along the chain, looking at either end, mark where a perpendicular from the given object would fall upon the chain. These are but approximations, but practice will soon make them as practically accurate as is necessary.

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\* These offsets should never exceed one chain.



## CHAP. III.

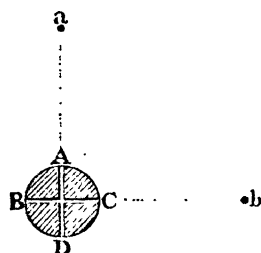
## ON THE CROSS STAFF.

THE cross staff is about 5 feet 6 inches high, pointed, and strongly shoed, to drive into the ground, and having at the top a circular piece of brass, or hard wood, so fixed as to be taken on and off.

The piece of brass is furnished with four brass pins, set at right-angles to each, and the wood head is divided into four divisions, by grooves, also at right angles, cut about half an inch down into the wood, which is about 2 or  $2\frac{1}{2}$  inches thick. This head is sometimes made with a spring, to move up and down the pole of the staff, and to remain firm at any height.

It is always requisite to test the accuracy of these grooves, which can be done in the following way:—

Let ABCD be a cross staff; and AD, BC, its sights or grooves; place a flag, *a*, in the direction of the grove AD, and another at *b*, in that of BC; half reverse the cross, so that DA point to *b*, and CB to *a*. Then, if the flags are still in the direction of the grooves, the instrument is correct.



The principal use of the cross staff is in the measurement, by perpendiculars, of straight-sided fields; and in the setting off of short perpendiculars from a base line; in the laying out of streets and building lots: and generally, in determining, by means of a perpendicular therefrom, the position of a house, mile-post, pond, or any conspicuous object, whose approximate situation alone is required, either to fill up or elucidate the plan, and whose distance from the measured line might exceed the limit of an offset.

It is especially useful where no great accuracy is required, when, in the computation of the areas, it is not requisite to take into account the irregularities

of the hedges, as, *in surveying a field*, it saves the necessity of measuring the whole way round, and makes a diagonal, with two perpendiculars from each of the opposite corners, all the admeasurements required.

In the measurement of straight-sided fields, when the perpendiculars are taken from the diagonal to the opposite corners, considerable trouble is sometimes experienced in determining the position of the perpendiculars on the diagonal; and two or three trials are often required, when accuracy is wished for. To remedy this, a little portable instrument was invented by a friend, which can be carried in the hand, and which at once determines the required position of the perpendicular. It is a small oblong piece of wood, of about two inches thick, with two pins placed vertically, having a small mirror, at an angle of  $45^\circ$ , which being held in the line of the diagonal, and turned towards the flag, which is set in the corner of the field, gives you, when your hand is at the required position, the reflection of the flag in the same line with the pins.

By holding this in your hand, keeping the pins in the direction of the diagonal, and walking along it till you perceive the reflected flag immediately in the line of the pins, the true position of the perpendicular can be correctly determined.

It is a simple but very useful instrument.

## CHAP IV.

### THE FIELD BOOK.

THE field book should be of a convenient size for the pocket, having the left page ruled with a central column, and the right page left blank for remarks. The central column should be headed "*Chains*," on either side "*Offsets*," and the right page "*Remarks*."

The central column is intended for all actual lines measured, and, by commencing *from the bottom of the page*, the page becomes a smaller representation of the reality with the line measured from you, and the

offsets, at their respective distances on that line, taken at so many links to the right or to the left, as are actually on the ground. In keeping the field book, it *first* should ever be remembered, that the central column is virtually but one line, representing the chain, the space *within* the column being merely required for the several distances on the chain, whence the offsets are taken; and, *secondly*, that all offsets, read either way, outward *from* the central column, in the same way as they are measured outwards *from* the chain.

To preserve uniformity, as it is more natural to measure from left to right, the place measured *from*, is put on the left of the central column, at the bottom of the line, and the station measured *to*, is put at the top, to the *right*; the points of commencement and termination of the line can thus be immediately seen.

The book should be interleaved with blotting paper, and the entries, if possible, should be made in ink. The pages should also be numbered, before beginning for facility of reference.

If the direction of the line is determined by an angle taken by the theodolite, or the bearing of the line be given by the circumferentor, the angle of the former or the bearing of the latter is placed in the central column, immediately above the starting point, according to the examples below:—

When the line crosses a road, or hedge, &c., make corresponding lines in the field book, as, in the first example of Theodolite Surveying, at the several distances on the line: of 2 links; 1 chain 10 links; 1 chain 26 links; 11·20 chains; 11·40 chains; 13·21 chains; always considering the column as but a line.

In taking "offsets" to corners of fences, houses, &c., mark the relative position of the corner, as to the chain line (see distance 6·79 on line 6·90; and 0·10 on line 7·31, in the example of "CHAIN

SURVEYING") and generally be careful to make the field book, as much as possible, a *fac simile* of the ground itself, with each post, hedge, house, &c., placed on the book, as to the central column, considered always as a line, in the same position as they stand to the chain on the ground. And think not that any time is gained to the Surveyor by hurrying over the notes in the field. *A little care in the field will save much trouble in the office.*

Stations are generally expressed in the field book by the following character  $\Delta$ , which, in the plan, is represented by a circle in pencil, drawn round the station point, which should be always that of a needle.

I would never recommend the use of letters for stations in Chain Surveying. In the first place, they are soon exhausted; in the second, they in no way assist the memory. The BASE LINE, perhaps, had better, when referred to, be termed the *base line* AB, in contradistinction to the secondary lines, which are required in surveys of some extent, and are virtually base lines to their own portions of survey, and may be lettered CB, EF, &c., &c.

In all other cases, distinguish the lines by their lengths, and the points upon them, by the distance of those points from the zero end of the lines;—thus, in the *first* example of the method of keeping the field book for chain surveying, "*from 609 on 609, to 0 on 609,*" the line begins at 609 on 609, and runs to 0 on 690—that is, the measured line is a line, connecting the end of the line 609 with the commencement of the line 690.

And again, in the *second* example, "*from 685 on 731, to 574 on 635,*" the point started from, is that point upon the line 731, which is 6 chains 85 links from the zero end of the line, and its termination, a point 574 on another line 635.

In *theodolite* surveying, it is better perhaps to use letters (see example), as the stations are but few, and mostly come within the exception above referred to, letters, in this case, as being usually applied to trigonometrical stations, may be, therefore, more characteristic and distinctive.

Surveyors sometimes take the bearing of the base line at the commencement of a survey, and enter it at once in the field book; this enables them to plan the estate in reference to the meridian line. It must be remembered, that this is but the magnetic bearing, and must be entered as such.

The correction for the variation\* must be subsequently determined, and the true bearing of the line inserted in the book.

### EXAMPLES OF FIELD NOTES.

(To be read upwards.)

<i>Chain Surveying.</i>			<i>Theodolite Surveying.</i>		
6.90 to 574 on 635			20.47 $\Delta$		
— D —	6.79	— X	path —	13.21	— X
10 + 1	700		hedge —	11.40	— X
10 + 5	500		D —	11.20	— X
10 + 4	400			11.11	$\Delta$ <sup>D</sup>
10 + 3	300			2.00	41 + 12
10 + 3	200		H —	1.26	— X
10 + 2	100		D. of Rd. —	1.10	— X
<sup>D</sup>			to G.P. 33	0.80	
10 + 0	0.50		Road —	0.02	— X
<sup>D</sup>		<sup>D</sup>	between 0		and 20.47
7	0.00	3	on 1077	123° 23'	on 20.47
from 685 on 731			at 1077 on 1077		
EXAMPLE II.			EXAMPLE II.		
Maiden {— 7.31 — X	} Lane				
top of bank {— 6.95 — X					
10 + 20	6.70	on 690	between $\Delta$ C	82° 55'	and $\Delta$ A
10 + 30	6.00		at $\Delta$ B	72° 24'	and $\Delta$ E
10 + 36	5.00			68° 28'	and $\Delta$ D
10 + 70	1.00				
✓ 90	0.10		between $\Delta$ D	100° 20'	and $\Delta$ B
from 609 on 690			at $\Delta$ C	14° 53'	and $\Delta$ A
EXAMPLE I.					
			between $\Delta$ E	180° 0'	and $\Delta$ C
			at $\Delta$ D	168° 49'	and $\Delta$ B
				89° 42'	and $\Delta$ A
			from $\Delta$ A	9.13 chs.	to $\Delta$ B
			between $\Delta$ B	151° 59'	and $\Delta$ E
			at $\Delta$ A on	86° 27'	and $\Delta$ D
			base line AB	11° 32'	and $\Delta$ C
			EXAMPLE I.		

of this subject, see "chapter on the

*Surveying by the Circumferentor.*

	17.84	to 0 on 755
	6.00	25 to fence
	250	30 to fence
	S.63°45'W.	
pond {	15.41	0
	60+10	12.00
	50+12	20 to fence
		10.00
		15 to fence
	2.00	10 to fence
	N.6°15'E.	
	8.82	4
	1.00	28
	S.75'E.	
from corner of field near road	7.55	6 to fence
	3.50	16 to fence
	0.00	3
	S.49°30'E.	

As it is indispensable, for the proper keeping of the field book, that the common general and local divisions of property should be understood, I have subjoined a few remarks on that subject.

## THE BOUNDARIES OF PROPERTY.

The usual boundary of the field, where the ditch is between it and the hedge, is the brow of the ditch, or that edge of the ditch which is furthest from the hedge. This, however, is not always the case, as sometimes it is the stem of the quickset, or the roots of the hedge; depending upon local custom.

The common allowance from the quick root, for the brow of the ditch, varies, in different places; being  $4\frac{1}{2}$  links, 6, 8, and sometimes as many as 9 links.

When between fields, it is ordinarily 5 links; when the ditch separates two contiguous properties, 6; and adjoining waste lands, moors, commons, roads, &c., generally 7 links. A wall is generally the division line between the properties, on which side soever the ditch may be placed. When there is a boarded fence between two fields, the fence belongs to the occupant of that side where it is clap-boarded, as the nails are considered to be driven home.

The *centre* of a stream running between two properties, is usually the boundary line.

In most parishes in England, a parish ditch is the boundary; the course of this ditch having been altered by time and circumstances, will account for the apparent freaks of the division of properties, which is so striking in the map; running across fields, dividing a pond, and putting one end of a street in one parish, and the other end in another.

It is particularly important in taking the notes of any survey, whenever any of the measured lines cross, or come near any of the division lines of the adjoining property, that these division lines should be noted in the field book. It at once localises the estate surveyed, and is oftentimes of great use to the Surveyor afterwards.

NOTE.—[It were needless to observe, that, whether the contents of the ditch are, or are not, to be taken into account, depends upon the object of the measurements. No one would think of *including* them, in the measurement of a crop of wheat or barley, or of *omitting* them, when the area of each field is required, for the determining of the area of the whole.]

## CHAP. V.

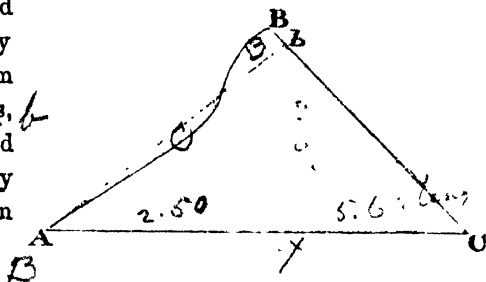
### SURVEY OF A SINGLE FIELD.

#### *Triangular.*

HAVING given a description of the chain, offset-staff, &c., together with the method of ranging correctly, and the best plan of keeping the field book, we will now proceed to the several plans adopted in actual survey, according to the circumstances.

Let ABC be a triangular field to be surveyed.—The first thing to be ascertained, is, whether a plan is required, or whether the area alone is sufficient.

If only the area, observe whether the sides AB and BC are sufficiently straight and regular to warrant any point *b* to be taken, from which straight lines, drawn to A and C, would approximate sufficiently to the area of the given triangle ABC.

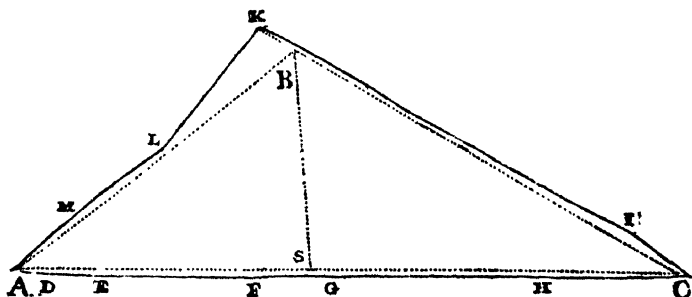


FIELD BOOK.

OFFSET.	CHAIN.	OFFSET.
perp. from 260 on 560	3.15	to <i>b</i>
	5.60	to C.
from $\Delta$ A	2.50	

Having carefully selected the point *b*, it will only be requisite, in measuring along AC, to observe where a right angle from (*b*) would fall upon AC, note in the field book 250; leave a mark in the ground, and measure to C = 5.60. Draw a line in your book, and measure from 2.50 to *b* = 3.15; then  $\frac{1}{2}(3.15)(5.60) = 8.82$  square chains = area of triangle ABC.

If a plan be required, or the hedges AB and BC be so irregular as to require offsets being taken, it will be requisite, for the sake of the offsets alone, to measure round the whole field along the three sides. Let the field now assume the form in the accompanying diagram:—



select any points A, B, C, commanding the longest and nearest lines to the several hedges of the field, and place flags at those points A, B, and C.



FIELD NOTES. $\Delta B$	
$\Delta S$ From 5.70 on 12.73.	4.25 to 8.50 on 9.27
$\times$	6.80 0 + D (to 0 on 12.73)
	5.26 8 being $\Delta A$
$\Delta$ from 8.50 on 9.27.	3.20 4
ditch—	9.27 — crosses 4 + D
$\Delta$	8.50 3
	3.20 3
	1.20 14
from 12.73 on 12.73.	0.00 0 + D
D—	12.73 — $\times$ 0 + D to $\Delta C$
	10.18 4 + D $\times$
	6.15 6
$S \Delta$	5.70 4
	4.82 15
	1.60 5
	.56 4 + D
from $\Delta A$	.00 0 + D

First observe, whether the point A be exactly at the ditch; if it is, put in the centre line 0, and in the right hand offset line 0; proceed till you come opposite 56 links, the first corner or bend D; take the offset (4) to D; this line, connected with A, gives the actual position of the ditch. Had there been no remark at 0, on the line AC, there would have been nothing to show where the ditch went to from the point D; proceed along the line AC, marking down the several lengths, as marked above, with their respective offsets, and selecting some point S, 5.70, to be noted as a station, whence to measure afterwards, a check or *tye*-line to B; note it  $\Delta$  (a station) in the left hand column, as the *tye*-line will be to the left. Having completed the measurement of the line, put *where to* in the right offset column at the top, and draw *one* line over the whole; all *loose* lines, that is, lines not connecting certain points, have *one* line only drawn over them; *fixed* lines have *two*. Measure next from C to B, write from 12.73 on 12.73 on the left offset column in your field book, and observe, as before, what the position of the ditch is; it will, of course, be 0, as before, as the station was at the end of the previous line, and the line was measured up to the edge of the ditch, which is denoted by the ditch crossing,—thus; ditch—(12.73)— $\times$



and mark BA—the direction you are going in—whether to the right or the left, CB being the direction you have just come: BA the way you turn (to the left); proceed, therefore, measuring along CB, and marking the offsets, till you come to B, which enter in your field book 8·50  $\Delta$ , proceed with the measurement to the hedge, making 9·27; put down in your centre column 9·27, offset 2 to right, and draw a line, thus:—

hedge—(9·27)— $\times$  i.e. hedge crosses.

From B measure to A, and as B is 8·50, write in your book, in the left hand column, from “ $\Delta$  8·50 on line 9·27,” and proceed as before to A, which is a point 0, or zero, on line 12·73, mark this on the right hand column; and, as this completes the triangle, draw *two* lines above it, instead of one.

Then, from point 5·70 on line 12·73, measure carefully the tie-line to  $\Delta$  B, being 8·50 on 9·27, as the whole check of the accuracy of the measurements of the two sides AB, BC depends upon the correctness of SB; and draw *two* lines over it in the field book, writing *check* line across it.

NOTE.—[The reader's attention had here better be directed to the object, had in view, in the measurement of this line BS; it is not for the sake of determining the correctness of the point S, but that of the lines AB and CB. If either of those lines be made longer than it really is, the point B, on the line BS, correctly measured, would be found in the plotting to fall within the triangle; if shorter, it would fall without; but the position of the point S might be materially altered, without affecting the correctness of the length of BS. Yet the reason of the inutility of the line BS, as a check upon S, becomes the cause of its efficiency, in verifying AB and BC, as, whether S should be 5·90 or 5·50, when it is called 5·70, is not of much moment, as the *length* of BS would scarcely be affected by it.]

It is sometimes necessary and always desirable, that the field notes should be sufficiently distinct, as to show at once, when referred to, on which side of the ditch of a field the hedge is. The black line on the plan is the *ditch* line—the hedge line is seldom noted; to obtain the ditch line, when the hedge is between the measured or chain line, and the brow of the ditch, the offsets would have to be measured through the hedge to the brow of the ditch. To do this at every offset, would be too troublesome; the plan, therefore, generally adopted, is, at the first offset, to measure up to the roots of the hedge (say 12 links), and then from there, to the brow of the ditch ( $+ \frac{D}{8}$ ), and assume this ( $+ \frac{D}{8}$ ), as the average distance, to be added to the hedge offset, throughout the line.

This plan furnishes us at once with a simple method of determining the position of the hedge, thus :—the offset  $(12 + D)$  shows, that the hedge is *beyond* the field; 12 being *up to* the brow of the ditch, denoted by  $(+ D)$ —the distance from there to the roots of the hedge not being wanted, as not belonging to the field under measurement. And the offset  $(12 + \overset{D}{8})$  shows the hedge to be within the field, as the offset, or distance of the chain from the boundary or brow of the ditch, is not 12 links, but  $12 + 8$  or 20; the distances being taken separately for the reasons above.

By adopting this plan, the Surveyor can at once define the position of the hedge. When several fields are measured, and the lines cross from one field to another, it is useful to mark, both where the hedges and the ditches cross.

### *To find the areas.*

In finding the *area* of the field, there are two or three practical methods indifferently adopted.

#### THE FIRST METHOD.

That of dividing the field into a triangle (whose sides are the measured lines,) and into trapeziums, (whose heights are the measured offsets,) as being the most correct, though certainly the most tedious and troublesome, shall be the first described.

NOTE.—[The example is worked out fully for the use of the student, who may choose out of the three methods for himself.]

EXAMPLE 1.—To find the area of the previous figure from the field notes.

1. To find the area of the triangle ABC.

AB = 6·80	Area = $\sqrt{S(S-a)(S-b)(S-c)}$			
BC = 8·50		14·01	14·01	14·01
AC = 12·73		6·80	8·50	12·73
2   28 08		7·21	5·51	1·28

$14\cdot01 = S = \text{semi-perimeter.}$

log. 14.01	=	1.146438	10   26.69
log. 7.21	=	0.857935	A. 2.669
log. 5.51	=	0.741152	4
log. 1.28	=	0.107210	2.676
	2	2.832735	40
log. 26.69 chs.	=	1.426368	A. R. P. 27.040
Area of triangle = 2. 2. 27.			

TO FIND THE AREAS OF THE TRAPEZIUMS  
On line AC.

Rule. - Multiply the lengths, by half the sum of the heights, for each area.

Lengths	Perpend. Heights.	.56	1.040	3.22	.880	.45	.403
		.02	.045	.10	.105	.06	.05
		.0112	5200	.3220	4400	.0270	.2015
56	0		4160		8800		
104	4		.046800		.092400		2.55
322	5	.0112		10   .7519			.02
88	15	.3220		.07519			.0510
45	15	.0468		4			
403	6	.0270		.30076			
255	6	.0924		40			
	4	.2015		12.03040			
	4	.0510					
	0	.7519 chains.				A. R. P.	
						Areas = 0. 0. 12.	

On line BC.

Lengths.	Perpend. Heights.	1.20	5.30	2.000	7.70
		.07	.03	.085	.035
		.0840	.1590	10000	3850
1.20	0			16000	2310
200	14	.0840	10   4399	.170000	.026950
530	3	.1590	.04399		
.77	3	.1700	4		
	4	.0269	.17596		
	3	4.399 sq. chs.	40		
			7.03840		
					A. R. P.
					Areas = 0. 0. 7.

On line BD.

Lengths.	Perpend. Heights.	8.200	2.06	1.54	10   1.4812
		.405	.06	.04	.14812
		16000	.1236	.0816	4
3.20	17	128000			59248
2.06	4	1.296000			40
1.54	4				23.69920
	8				
	6				

$$\begin{array}{r}
 \cdot 1236 \\
 \cdot 0816 \\
 \hline
 1 \cdot 2960 \\
 \hline
 1 \cdot 4812 \text{ sq. chs.}
 \end{array}
 \qquad
 \begin{array}{r}
 \text{A. R. P.} \\
 \text{Areas} = 0. 0. 23.
 \end{array}$$

$$\begin{array}{r}
 \text{A. R. P.} \\
 \text{Area of triangle} = 2. 2. 27 \\
 \text{Trapeziums on AC} = 0. 0. 12 \cdot 03 \\
 \qquad \text{on BC} = 0. 0. 7 \cdot 03 \\
 \qquad \text{on BD} = 0. 0. 23 \cdot 69 \\
 \hline
 \text{Area of field} = 2. 3. 29 \cdot 75 \text{ acres.}
 \end{array}$$

**EXAMPLE 2.**—Take, for practice, the field notes of the Fields Nos. 1 and 2, Plan No. 1, at pages 78 and 80, and calculate the areas, checking them by the following method.

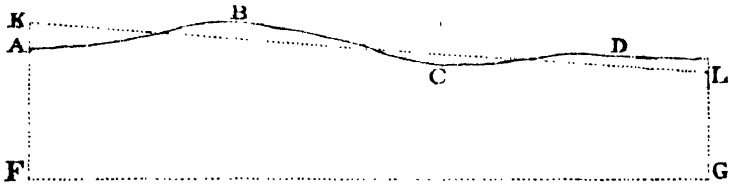
**NOTE.**—[Though this method is seldom requisite in small surveys, yet it is important that the reader should be acquainted with the best means of ensuring accuracy in cases of importance.]

#### THE SECOND METHOD.

A common method in practice, in computing the areas of irregular-sided fields, is to have a piece of transparent horn—and by *giving* and *taking*, as it is termed—to draw a straight-sided polygon, equal to the given irregular one, and, by means of the compasses and scales, to measure the lengths of the new lines, and from these lengths to calculate the area. This is a sufficiently close approximation in skilful hands; though I would counsel young beginners not to attempt it at first, until, by computing the area, by means of triangles and offsets, on several occasions, they have had opportunities of testing their own correctness.

**EXAMPLE.**—Let ABCDE be the irregular outline of a hedge or ditch, by placing along it a straight-sided piece of transparent horn; the position of the line KL can be determined, such that the area KLG F shall be

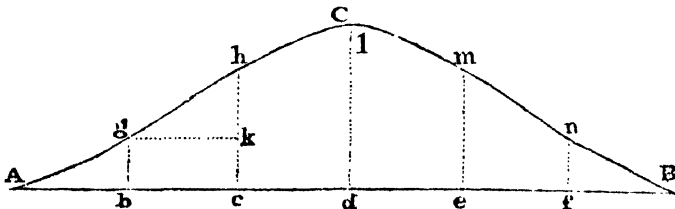
equal to the area of ABCDEFG; so that the pieces, taken in at A and C, shall be equivalent to the pieces given up at B and D.



NOTE.—[I would recommend the beginner to find by this method the areas of the preceding figure, and of the fields in the several figures in the work, comparing them with the calculated areas.]

As it is sometimes desirable to calculate in the field, without plotting, the area of an irregular figure, which has been surveyed by the circumferentor; and, as the space included within the lines of survey can always be calculated separately, without reference to the areas between these subsidiary lines and the natural boundaries, I subjoin an useful practical method of determining this area, when the offsets are supposed to be taken at regular distances.

To approximate to the areas of offsets, let AB be a straight line, near the curvilinear hedge ACB, and let it be required to determine the area, included between the line AB and the hedge.



In measuring from A to B, take at every chain's length, Ab, bc, cd, &c., the offsets bg, ch, dl, em, &c.; then the whole area ABC, shall

be equal to the sum of  $gb + ch + dl + em + fn$ , multiplied into their common distance of one chain, or 66 feet.

For the area of  $Abg = \frac{1}{2} Ab. bg = \frac{d}{2}(bg)$ , where  $d$  = the distance.

$$gbch = bc \left( \frac{bg + ch}{2} \right) = d \left( \frac{bg + ch}{2} \right) = \frac{d}{2}(bg) + \frac{d}{2}ch$$

$$hcdl = cd \left( \frac{ch + dl}{2} \right) = d \left( \frac{ch + dl}{2} \right) = \frac{d}{2}(ch) + \frac{d}{2}dl$$

$$ldem = de \left( \frac{dl + em}{2} \right) = d \left( \frac{dl + em}{2} \right) = \frac{d}{2}(dl) + \frac{d}{2}(em)$$

$$mcf n = ef \left( \frac{em + fn}{2} \right) = d \left( \frac{em + fn}{2} \right) = \frac{d}{2}(em) + \frac{d}{2}(fn)$$

$$nfB = \frac{fB}{2}(fn) = d \left( \frac{fn}{2} \right) = \frac{d}{2}(fn).$$

$$\therefore \text{Area} = d(bg + ch + dl + em + fn).$$

EXAMPLE 1.—Let  $d$  = one chain, and  $bg$ ,  $ch$ , &c., respectively 10, 15, 17, 14, 9 links, what will be the area of the figure?

10	·65 sq. chs. = ·065 acres.
15	4
17	·260
14	40
9	Poles 10·400
·65	<u>          </u>
1	<u>          </u>
·65	

square chains.

EXAMPLE 2.—Given the several offsets 15, 25, 40, 10, 60, 30, 25, 8, 18, 9, 8, 4, 6, 0, taken at one chain's distance. Required the area.

A. R. P.

Ans. 0. 1. 0.

#### TO MEASURE A FOUR-SIDED FIELD.

This must, in all cases, be divided into two triangles. If the boundaries are irregular, each hedge must be taken by means of a subsidiary line and offsets, and a diagonal line, drawn from the two opposite corners, that are most remote from each other.

If the boundaries are *regular*, the diagonal line alone need be measured, together with the lengths of the perpendiculars to each of the other corners.

*To compute the Area.*

Find the area of each of the triangles that compose the trapezium, separately, adopting one or the other of the different methods of computation before enumerated, and their sum is the area required.

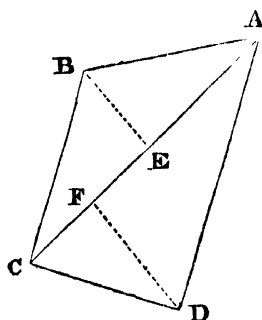
Let ABCD be the given trapezium, and first let it be straight-sided; measure AC and BE, and FD

then the area will be equal to  $\frac{BE+FD}{2}$

$\times AC$ ; for it is equal to  $AC \cdot \frac{BE}{2}$

$+ AC \cdot \frac{FD}{2} = AC \cdot (\frac{BE}{2} + \frac{FD}{2})$

$= AC \cdot (\frac{BE + FD}{2})$



Now let the sides be *irregular*.—It will be necessary to measure all round for the sake of the offsets; one other measurement ties the whole in, and you have two triangles, ACB and BDC, upon a common base, CB. To prevent errors to young beginners, I would recommend them, in all cases, for some time, taking the tie-line from the base to the vertical angles, till they get into practice; thus, in the last case, in measuring from C to A, when nearly opposite (as well as the eye can tell) to D, take any station F; and, when opposite to B, any station E; after completing the diagonal CA, measure BE, FD; this will at once prove the accuracy of the measurements in the field of the several lines AB, BC, CD, AD, and will, at the same time, guard against any error in the plotting at home.

The following field notes, accompanied with a plan (being Field No. 1, in Plan No. 1,) of a four-sided field, near Maiden Lane, surveyed in the above manner, are given as an example of the method of keeping the notes in the field, and of plotting the work at home.



It were advisable that the student should plot every one of the examples of field notes *himself*, for practice, and compare them with the plans given.

FIELD No. 1, PLATE I.

From 230 on 873 check line	3·67	to 480 on 485			
From 490 on 873 check line	4·71	to 574 on 635	70   40	D—	7·24 — ×
	8·73	to 619 on 624		8	7·19
	4·90	Δ		7	600
	2·30			5	500
from 34 on 485				6	400
				6	300
	7·16	to 34 on 485	to gate post	10	2·79
10+24	7·00		D+4		2·00
10+21	600		D+5		1·00
10+19	500				
to gate post 22	392				
10+11	300				
10+ 7	200		from 490 on 485		
10+ 7	1·00				
to top of bank 10+8			D —	485	— ×
from 574 on 635	0·00		+ 13	480	Δ
			+ 28	400	
			+ 32	300	
Maiden { D—	635	— ×	D		
top of bank { D—	597	— ×	10 + 27	200	
	587	— ×	D		
4	500		10 + 28	1·00	
4	400		D		
2	300		10 + 10	0·34	Δ
D+2	200		10 + 15	0·00	
D+5	100		D —		— ×
to 2d gate post 12	0·08		Maiden		— Lane.
from 719 on 724					

Commencing on top of bank by the road side, at the south-east corner of the field.

## THE PLOTTING OF THE ABOVE NOTES.

*First lay off the measured lines, independently of the offsets.*

Draw any line AB, indefinitely in pencil, as your guiding line, so placed, as to throw the plan into a favourable position on the paper. Now, for the length of this line, by looking at the field notes you





will find that the whole line is 4·85 ; but there is a  $\Delta$  at 24, and another at 480 ; 480—24, or 4·56, is the length of this subsidiary line, forming one of the lines of triangulation ; write in pencil 456 against this line.

The next line runs to the right, from 480 on 485, (which 480 from 0 is equivalent to 456 from 24,) and is 724 long ; but the station is at 7·19 ; draw any line, making the supposed angle with AB, and mark 7·19 against it.

The third line also turns to the right, and though continued to 635, across Maiden Lane, has its station point at 5·74 ; mark this line also, in its supposed position.

The Fourth line does not say to the right or to the left, because it runs from a station 574 on 635, the end of the last line, to a previous station, 24 on 485, which was the starting point ; this line is 716 long ; mark this distance against it.

The next measured line is from 24 on 485, or the starting point, and runs to another previous station, 619 on 624 ; its total length is 873, and there are two stations upon it, 230 and 490 ; draw this line in its proper position, and mark off the stations, 230 and 490.

The next two lines are check-lines, the one (471) from 490 to 574 on 635 ; the other 3·67 from 230, to 480 on 485 ; mark these also.

Having placed roughly these several lines, with their given lengths, in their supposed position, proceed to plot them off correctly by triangles, marking in every case, on the plan, the direction the line was measured in on the ground, (see plan).

AB, of course, is the base of the whole. Upon AB lay off a triangle, whose other two sides are 7·19 and 8·73, laying off upon 8·73 the stations 2·30 and 4·900. Then, to verify the correctness of the work thus far, measure the distance of the station 2·30 from the point B ; this, if the work be correct, should be 3·67, the length of the dark line.

Next, upon the line 873, lay off another triangle, whose sides are 574 and 7·16 ; the distance of the previous station 490, from the vertex of this triangle, should, if the work be right, be found 4·71. The whole field is now plotted.

#### METHOD OF SURVEYING AND PLOTTING TWO FIELDS TOGETHER.

The field in the previous example having been already surveyed, the adjoining field was added to it,

which is to be plotted from the accompanying field notes.

## FIELD No. 2, PLATE II.

			Maiden	D —	7.31	— ×	} Lane.
				D —	6.95	— ×	
from 590 on 868	5.35	to 685 on 731	top of bank	—	6.85	— ×	
check line			10 + 20		6.70		
			10 + 36		500		Δ
	8.68	to 609 on 609	D				
from 574 on 635	500	Δ	10 + 70		100		
	690	to 574 on 635	..... 90		0.10		
			from 609 on 609				
— D —	6.79	— ×	.....	D 15	609	Δ	
10 + 1	600			6 + 4	5.82		
10 + 5	500			D + 4	500		
10 + 4	400			D + 10	400		
10 + 3	300			D + 16	300		
10 + 2	100			D + 22	200		
				D + 27	100		
to top of bank 10	0.50	0 to D	to gate post 40		0.50		
to top of bank 7	0.00	3 to D in field	35				
from 685 on 731			pond	+ 21	.32		
			pond	+ 20	.12		
			from 719 on 724				produced.

Having the previous notes of Field No. 1, the following notes were taken for the survey of the adjoining Field No. 2.

Produce the line 7.19 to 609 further, which is a station—then, turning to the right, agreeably to the field book, mark off the distance 685, which is the station point in the next line 731; from this point, 685, the distance to a known corner in the first field, (being 574 on 635), is 690.

Now, because 574 on 635 is a known fixed point, and the line 7.19 of the last survey, is a fixed line, its production is also fixed, and the end 609 is a fixed point; the line joining 5.74 on 635, and this point is, therefore, a fixed line without being measured; upon this base, therefore, describe a triangle, whose other two sides are 685 and 690, and their intersection is also a fixed point. By measuring the diagonal line 869, its distance, determined in position by joining two fixed point, is checked by its measured distance.

Again, by taking the distance 500, upon this diagonal, and measuring the check-line 535, this line, measured from a point in a fixed line, to the intersection of two other lines, becomes a check upon the vertex of the triangle.

Having found the plotted check-lines agree with the measured distances, draw the subsidiary lines carefully in *red ink*, and proceed to lay off the offsets.

### TO LAY OFF THE OFFSETS.

#### *Take the first Field.*

The offsets, on the line AB, are all to the left, mark off, therefore, the several offsets in their proper places, observing, in the case of

D

the second offset, that  $10+10$  is ten links up to the hedge, which, with the ditch, is ten links wide. This determines the position of the

D

hedge to be *within* the field; without the 10, or, had it been  $(D+10)$ , the hedge would be *beyond* the boundary of the field. At 4'85, the cross ditch of the field intersects the line.

In the next line the offsets are still to the left, and are marked  $(D+5)$ , showing that, in this case, the hedge is without the field. At 7'19, there is an offset of 8, to where the edge of the side ditch intersects the side of a pond; at 7'24 the cross ditch of the field intersects the line as before. The width of the pond 40 links, and its length 70 links, are marked.

In the next line, the offsets are still to the left, and the  $(D+5)$  shows, that the hedge, in this case, is beyond the field. As this line, if produced, will cross Maiden Lane, it is produced across for the purpose of determining its width and position. In every case, in addition to the mere measurement of the field, it is advisable to annex such collateral localities as roads, turnpikes, ponds, &c., as may determine the relative position of the field. At 0'08 there is an offset of 8 links (on the left) to the second gate post. In the following line 716, where the offsets are "still to the left" at 392, there is an offset of 22 to "gate post" only. Observe—that it is usual to take to the *first* gate post, and allowing the average width of gate to be 15 or 16 links, to determine the position of the gate by the direction of the line. It is difficult to fix the position without some fixed rule, such as the above, of always taking the *first* point of the gate, as you come to it, on the line, whence the offset is taken.


The first offset on the line 716 is  $(10+8)$ , always on the left. This 8 is up to the bank, bounding the lane, and 10 is the distance to its top. This 10 is repeated throughout, the average width having been taken. The other three lines have no offset to them.


### OFFSETS OF SECOND FIELD.

The first line is 609, and the first offset is at  $\cdot 12$  (still on the left), being (pond+20), or, 20 up to the edge of the pond+20 across; the second offset at  $\cdot 32$ , is  $(35+21)$ , that is, 21 up to the pond +35 across the pond, which, by the accompanying diagram, ends there; the distance  $\cdot 32$  being taken, in order to show that there was the end of the pond.

The next offset is at 50, that is, 40 to first gate post; at  $1\cdot 00$ , the offset is  $(D+27)$ , that is, 27 to the brow of the ditch, which being beyond the boundary of the field, the width of that and the hedge is not required; the other offsets are regular to the distance  $5\cdot 82$ , which

has an offset of  $(6+4)$ , that is, 4 up to the hedge, and 6 through the hedge, which is now within the field, to the brow of the ditch; the previous offset was  $(D+4)$ , that is 4 to the brow of the ditch (the hedge being then beyond the field), the ditch therefore *changes* at

this point, denoted in the diagram by . There is here also a cross

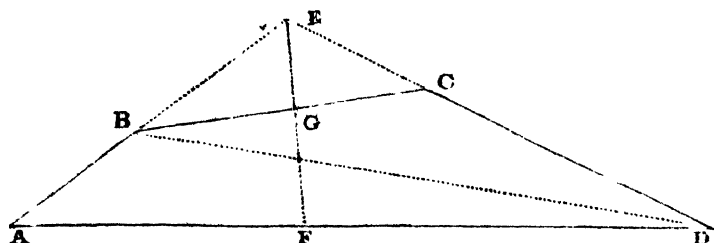
hedge, on the left, which the diagram  shows too.

In the next line, at  $\cdot 10$ , there is an offset, taken to the corner of the field,  $\cdot 90$  links; and the relative position of the sides, to the offset line, is expressed by the diagram in the notes, which should ever be, as much as possible, a ground plan of the locality.

In line 690, the first offset taken is at the station point or  $0\cdot 00$ , 7 to the left, and 3 to the right: showing, that this point is between the top of the bank by the road, and the *field* ditch at the bottom; at 50 links, the line touches the ditch, having 0 offset to it.

There are no offsets to the two following lines.

There is a form, however, of a four-sided field, which it would be dangerous to measure in the manner before referred to.



Let ABCD be the field. To measure BD as a diagonal, and to let fall perpendiculars thereon (supposing the field to be *straight sided*), would be troublesome, and might be productive of error; and, to divide the field into two triangles, with BD for their base, subtended, in the one case, by so obtuse an angle as C, and, on the other, by so acute an angle as A, would be equally hazardous. One good tye line, from C to BD, might be taken; but a tye line from A, except upon the production of DB, which might be impracticable, would be useless.

It would be better, according to the circumstances of the locality,—

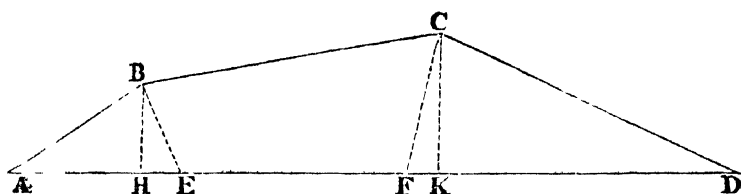
1st.—Either to produce DC and AB to their intersection at E; and, taking all the requisite measurements, for the computation of the triangle AEC, take, at the same time, those for the area of BEC, and subtract the smaller from the greater, for the required area.

EXAMPLE—Let AD be the base measured, and take as a station, any point F, opposite to E; from A measure along AB, towards E; at the distance AB, in the field book, mark “*fence of field crosses*,” (and also “*out of field*”) and continue the measurement to E. From



E measure towards D, marking (out of field) till you come to C, where, at the distance EC, say fence of field crosses, and proceed to F. From the station at F, towards E, measure to CG, (where fence crosses,) and continue to E; measure CB.

2nd.—If the hedges be so high or thick, that you cannot range beyond—or, if there be any other local objection, I would prefer the following method:—



In measuring from A to D, take a point E, about the same distance from A as B is, and mark it a  $\Delta$ ; and at F, take any other point, making FD about equal to CD, marking it in the field book as  $\Delta$ , and proceed to D; measure the other three sides, which must be done, for the sake of the offsets. Then, from E measure to B; and, from F to C, the points B and C become the vertices of two triangles, ABE, and FCD, whose bases, AE and FD, being given, as well as the lengths of their respective sides, the vertices B and C are given, and therefore the line BC, between them, without being measured, is given also; by measuring the actual distance between B and C, BC becomes a check-line. Other check-lines BH, and CK, to the several triangles upon the base AD, can be taken or not, as the importance of the survey may or may not render it necessary.

#### THE LINEAR MEASUREMENT OF AN ANGLE BY THE CHAIN.

It must have been observed by the student, that this method of chain surveying is but a practical application of Euclid's problem, (see Problem x, page 5,) "for constructing an angle, equal to a given angle,"

viz., by completing the triangle, and making a second triangle equal to the first.

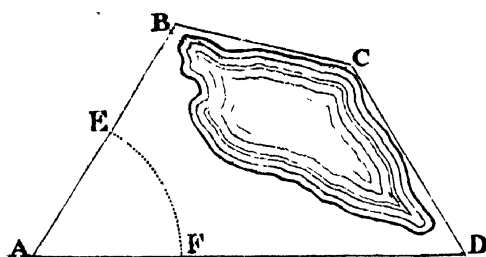
In *chain surveying*, an angle is not measured by the number of degrees and minutes it contains, but by the length of the base that connects its sides, either completing the field (if it be of a triangular shape), or, as a diagonal, forming the base of another triangle, or of a new system of triangles.

The length and position of this connecting side, or linear measurement of the angle, should not be taken indifferently, but should be so selected, as to connect sides, as nearly equal as possible to each other, and to the line that connects them, forming an equilateral triangle, where possible, or an isosceles.

Again, when practicable, the largest triangle should be taken, though it is not actually necessary, as is shown in the following example.

*Field*

Let ABCD, be the given angle.



It is not actually necessary, to make AB, AD, the two sides of a triangle, whose base, BD, shall determine the measure of the angle at A. Any points

on AB, and AD, may be taken at pleasure, as E, and F. The distance EF, will determine the angle at A, and the position of the sides AE, AF. These lines are but parts of the straight lines AB, AD; and, therefore, AB, AD, are determined; and the points B, and D, upon them.

There might be many local obstructions to measuring from B to D, and AD might be disproportionately long to AB. By making AF

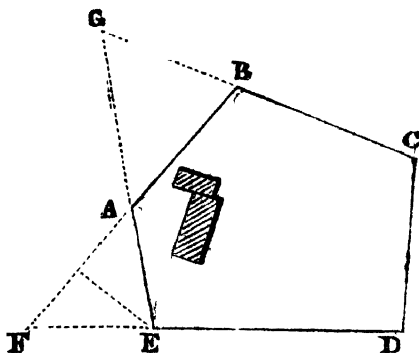
as nearly equal as possible to  $AE$ , you obtain a far more favourable measure of the angle at  $A$  than  $BD$  might be.

THE MEASUREMENT OF AN ANGLE, EXTERNALLY, BY MEANS OF THE OPPOSITE, OR SUPPLEMENTAL, ANGLE.

It sometimes happens, that an angle cannot be measured internally, from local obstructions, such as trees or buildings intervening between the sides that include it. In this case the following plan should be adopted.

Let  $ABCDE$  be any enclosure, such as a wood, river, &c., whose angles cannot be measured internally; or, a farmstead, which, from the position of the buildings, must be measured from without.

Assume the side  $AE$ , as the base line, and, for the sake of the offsets, measure all round the enclosure, taking the lengths



of the sides,  $ED$ ,  $DC$ ,  $CB$ ,  $BA$ . The nature of the obstructions prevents the measuring of the side  $AD$ , in the triangle  $ADE$ , which would give the angle  $AED$ ; and, also, the length of  $BE$ , in the triangle  $ABE$ , to obtain the angle  $BAE$ . Neither of these angles, therefore, can be obtained in the usual way.

Though the angle  $BAE$ , however, cannot be *directly* determined, there are three different angles, from which it can be deduced, viz.:—its own opposite angle,  $GAF$ , or either of the supplemental angles,  $FAE$ ,  $BAG$ . Some one or other of these may be free from local obstructions.

First, let the supplemental angle,  $GAB$ , be free; produce the range  $EA$  to  $G$ , where  $CB$ , also produced, intersects it; measure  $AG$ ,  $AB$ , and  $GB$ ;  $AGB$  is a triangle upon the base  $AG$ ;  $AE$  is determined in position, and, therefore, its production  $AG$ , is so also; and  $GB$ , and  $AB$ , being measured,  $B$  becomes determined; and the line  $GB$  and its *Production*  $BC$ , and therefore  $C$ ; but  $E$  is determined; an imaginary

line, EC, connecting them, becomes so also. This being the base of the triangle CDE, whose two sides, CD and DE, are measured, D becomes determined, and therefore the whole figure is determined, by measuring the supplemental angle GAB, making, in all cases, its sides, BG and AG, the *production* of the sides, CB and EA respectively.

For the purpose of *obtaining the offsets*, it would be necessary to measure every side; there are, therefore, but two, not subsidiary, viz., the supplementary sides, AG, BG, which have to be measured, *purely as such*.

The same plan can be adopted, with any number of angles, modified according to circumstances, and having always regard to the using of as few supplementary lines as a due attention to accuracy will permit.

## CHAP. VI.

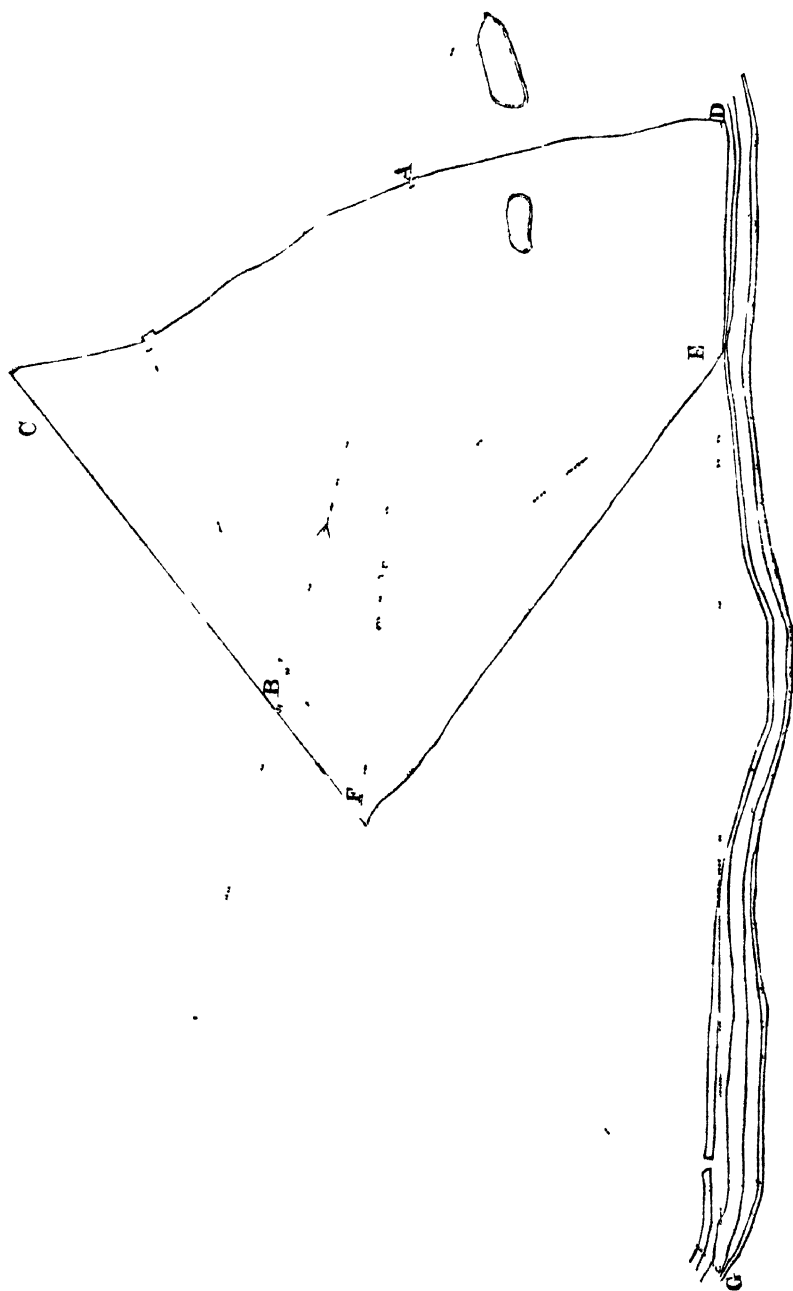
### SURVEY OF FIELDS OF MORE THAN FOUR SIDES.

MANY-SIDED fields are of so many various forms, that it is impossible to give any specific rule for all.

The chief point in this, as in most cases, to be secured, is, that as the *whole circumference* of the field *must* be measured for the *sake of the offsets*, a selection should be made of the fewest and the most favourable supplementary or diagonal lines, with the best points for check-lines, to the corners.

The following field notes, of a five-sided field near Highgate, are given as an example, which the student is recommended to plot for himself, and compare with the accompanying plan.

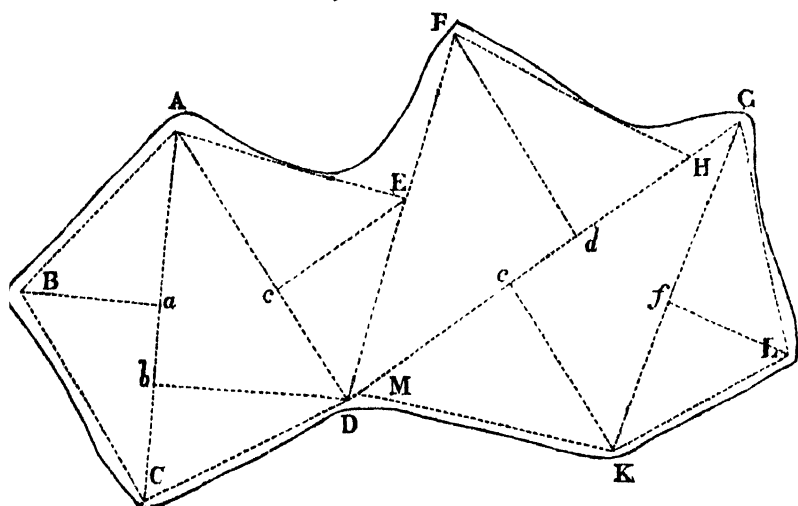




I have subjoined, for the practice of the student, a few examples of the best methods that might, under different circumstances, be advantageously adopted.

### EXAMPLE 1.

In the survey of many-sided fields, it must be remembered, that the whole field can always be divided into as many triangles (less two) as the figure has sides. Thus in the annexed diagram,



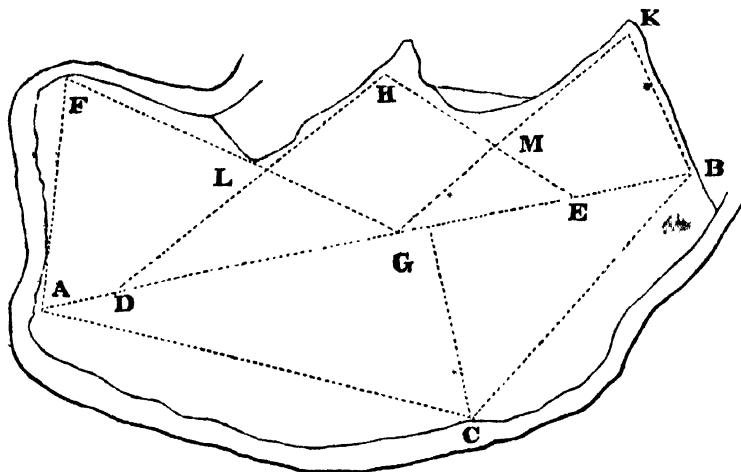
the whole figure  $ABCKG$ , by drawing the diagonals,  $AC$ ,  $AD$ ,  $FD$ ,  $DG$ , and  $GK$ , is divided into triangles.

I would not, in most cases, recommend this method, for it is violating one of the first principles of surveying, viz., that of working from *whole* to *part*.

Instead of confining the whole field within the limits and error of one triangle, this method not only depends upon a *number* of triangles and the errors of each, but the error of any one of the triangles is not confined to itself, but is carried through the whole. Thus the triangle  $GKL$ , (being correct in itself,) depends upon the correctness of the triangle  $GKM$ ; the two triangles,  $GKL$  and  $GKM$ , upon the triangle  $DFH$ ;  $DFH$  upon  $AED$ ; the triangle  $AED$  upon  $ADC$ ; and  $ADC$ , and all the preceding, upon the correctness of the one triangle  $ABC$ .

### EXAMPLE 2.

In the survey of the Field, in the accompanying diagram, a different arrangement has been adopted.



One base line, AB, has been taken through the centre of the field, and the various triangles have been based upon it, determining the several points required: each point in this arrangement depending solely upon the accuracy of the admeasurement of the triangle to which it belongs.

For example, take the point C, by the side of the river—this point is the vertex of the triangle ACB, whose base is the base line AB, and whose accuracy is secured by the check-line GC. The points F, H, K, also, are the vertices of the triangles, AFG, DHE, GKB, whose bases are respectively AG, DE, GB, portions severally of the base line AB. The distances DL, ME, not being required for the determining the vertex H, become check-lines upon all three.

The base line AB, and the check lines GC, LG, ME, are the only supplementary lines required.

### EXAMPLE 3.

The next example is that of a very irregular field (selected for practice sake from the centre of a survey),



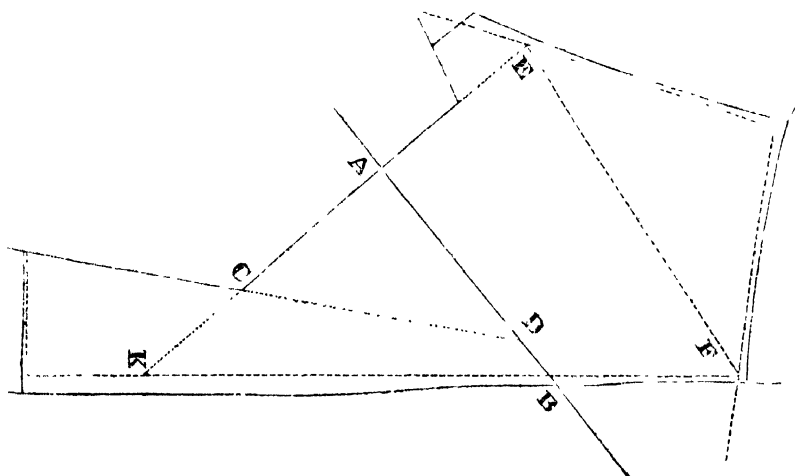


measurements  $GH$ ,  $HC$ , determine the position of  $H$ . All these lines are required for the offsets; by measuring, along the footpath, the line  $HP$ , you have a check upon the whole of this latter part of the work.

All the supplementary lines, required for the survey of the whole field, are  $Pb$ ,  $aN$ ,  $Eq$ ,  $aB$ , and  $PH$ .

#### EXAMPLE 4.

This example is that of a field under similar circumstances, of a "base line running through it," but the general position of the field is different, and a different arrangement required.



The same principle is still kept in view, of making the sides of the triangles serve the purpose of taking the offsets.

In this case there is but one supplementary line  $CD$ , and two check-lines,  $BF$ , and  $CK$ , or lines of verification to the whole.

#### EXAMPLE 5.

This example is that of a long narrow slip, crossed at one corner by the base line.

The best method of surveying it, if circumstances will allow, is given in the accompanying diagram.

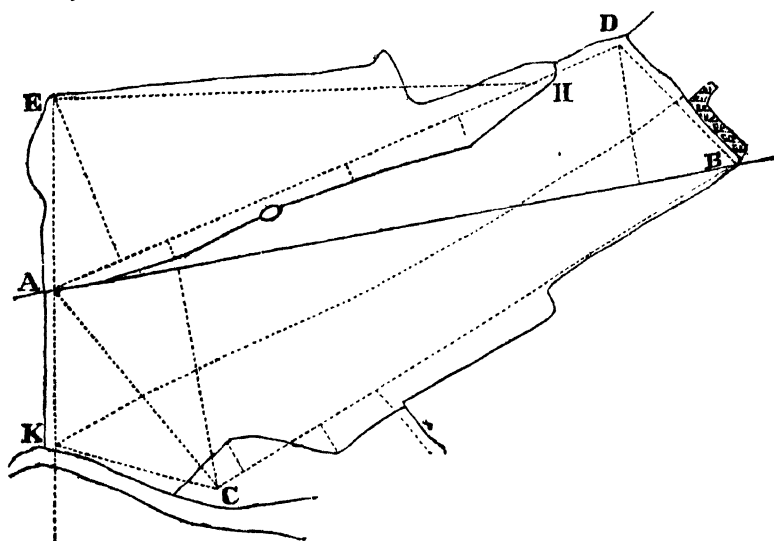


that are merely used in determining the triangles, and do not also serve for taking the offsets.

I subjoin a few examples of this kind, with field notes (of the two last), plans, and descriptions of the mode adopted in making the survey, and the reasons for selecting and arranging the several lines.

**EXAMPLE 1.**

In this case, there are two contiguous fields irregularly bounded.



Let the base AB (which may be taken arbitrarily) pass in a diagonal direction through the corners of the larger field.

By basing upon AB, the triangles ACB, ADB, you obtain the position of the three hedges of the larger field, one of which is common to the adjoining field.

By basing upon AH, the triangle AEH, the whole of the second field is measured, and the side EA is a fixed line; its production to K is also fixed; and, as C is fixed, being the vertex of the triangle ACB, the line KC, without measurement, is determined; when measured, *which it must be for the sake of the offsets to the river*, its measurement is a base of verification to the whole.



The methods above given, the student must bear in mind, have been supposed *free* from those *local obstructions* which will often materially change the arrangement. He must keep in view the difference he will find in laying down a method of survey, *when all the plan is before him, and he has a bird's eye view of the whole, which is supposed to be perfectly flat and free from every obstruction*; and, when, ignorant of the locality, unable to comprehend its content at a glance, he has also to contend with the local difficulties of hills, vallies, woods, rivers, houses, &c.; in fact, of all that constitutes, in all cases, the difference between theory and practice.

NOTE.—[The various methods of overcoming these difficulties will be found fully explained in Part the Second.]

## CHAP. VIII.

### SURVEY OF A PORTION OF A PARISH.

EXAMPLE.—This example is that of a larger portion of country, comprising some 80 or 100 fields, 6 or 8 roads, a village, river, canal, &c., and a line of rail road running through a corner of it, towards the village. It is an actual survey of a portion of the county of Derby, in the parish of Willington.

A base line AB has been selected, contiguous to an intended line of railway, and running through the whole of the survey, intersecting the road HC, at C; the road GK at D; and the road MO at E, which distances are carefully marked in the field book. Where the base line crosses the hedges also, at the most favourable place for running cross lines along the hedges, stakes must be put in, and the points carefully noted, taking offsets *en route* to any hedges or corners of hedges, fences, or other objects, that may be within an offset distance.

The method actually adopted would be, in order to avoid any needlessly going over the same ground twice, to commence at A, measure AN, NVH, then HC, CV, and the fields within the block VCH, and NC.

From H, the next line measured would be HG, observing carefully where the best stations could be taken for the cross-hedges, on the same side of them, as the stations were selected in the base line.

Then measure GD and HD, observing, in measuring HD, to have the range of the line carefully defined, where the several hedges cross: so as accurately to define the several points in the line CD, where the cross hedge lines, from AB to HG, intersect. By this plan, all these cross-lines are check-lines.

Produce GD, to K, in the same straight line; measure KC, taking notice as before, where the cross hedges come, and on their proper side, and *complete* the block HGKC.

Then measure CL and LK.

Now return to C, and produce HC to F, where it intersects the base line; marking the several points P, Y, and X, upon it, and the several cross hedges.

Then from D produce HD to M; and from M, measure a line in range with EX, which produce to O; join OB; then complete the block GXMD, and the triangular piece KDM.

There now remains but the part adjoining the village.

From P, measure PRS, and join ST; then produce VR to *b*, and join *ba*; the lines *ba*, *aT*, TS, SR, *Rb*, will tie the whole of the houses in.

This must always be the plan adopted in the survey of a village, or farmhouse, or homestead; to confine







all the areas within one triangle, whose three sides should severally pass through the principal points of the place.

Having given the method, adopted in practice, for saving time in the survey of the plan, we will proceed to explain the nature and use of the several main lines.

The line ME, of the triangle DME, is the measure of the angle MDE; but CH, in the triangle CHD, is the measure of the opposite and equal angle CDH. Therefore the measured and the determined distance, agreeing or disagreeing, of either the side CH or ME, is a proof of the correctness or incorrectness of the angle at the vertex D.

Produce ME, the *fixed* line, to O; any points upon this production are also *fixed*. The point X, which is in a range with the road HP, is fixed; and H, being a fixed point, the lengths of HX is determined.

Its measurement becomes a line of verification to the opposite angle HMX, or (MDE, being supposed correct,) of the supplemental angle DEM: which is the angle that this new line MD makes with the base line. *The line OB, if the nature of the ground will permit its being measured, measures the opposite angle BED, and is another check upon its correctness.*

Again, MX is the measure of the angle MHX; and GD is also the measure of the same angle.

The actual distance of GD, compared with its computed or determined distance, is a check upon the correctness of the length of MX.

Having determined the correctness of these triangles, there can be no error of any moment in the *filling in*.

In fact, all the lines used for the measurement of the offsets to the cross hedges, are only so many additional check-lines to the triangles, or measures of the angles at their vertices.

EK being *determined* by the previous measurements, its measured distance is a check upon the angle CDK, and, therefore, upon the direction of the line KC, relative to the base line AB.

The correctness of the triangle, CLK, is secured by the common check-line to its vertex Lc.

The triangle AHC, having in AC a portion of the base line, depends upon the correctness of the measured distances AN and NC.

A being thus a fixed point, as well as H, measure the line NH, and as H has been previously assumed correct, HN is a measure of

the angle NCH, which is the supplemental angle to the two known angles HCD, ACN; the length VC is a check upon the distances CN and CH.

Now returning to the other parts of the survey, the line HZ, produced to the base line at F, is an additional verification *of the whole of the triangulation*.

To ensure a correct survey of the village, observe that the line MOa passes close to one side of it.

From P, drawing PRS through R, and joining ST, you have *known lines close to the village, on another side*; producing YR to a point *b*, such that a line *ba* shall pass close to the third side of the village, you surround the whole with a fixed triangle. All errors must be confined *within* this limit; and all lines, for the measurement of the streets or lanes, carried through to either of the sides of this triangle, are, as in the case of the cross hedge-lines, in the first part of the survey, virtually but so many corroborative checks of its accuracy.

## CHAP. IX.

### SURVEY OF FIELDS NEAR MAIDEN LANE.

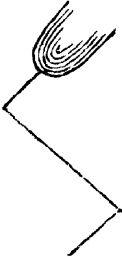
PLATE No. 3 is a plan of the point of junction of Maiden Lane, with the Junction Holloway Road, and of six or seven fields in one of the corners.





The extent of the survey was settled in the first instance, and the whole of the ground carefully examined, and the arrangement of the lines, as much as circumstances might afterwards allow, predetermined.

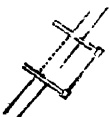



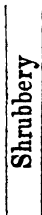

The plan does not take in the whole of the field notes, as the roads surveyed extended much further than the page would admit of being introduced; the notes are, however, retained, as the student will not, of course, be under similar restrictions.

## FIELD NOTES OF SURVEY.

6.34 to 16.61 on 1728	paling—	6.09 —X 13
6.33 15+p+21+15	D—	5.95 X
6.17 9+p+6+15		5.90 2+18 ✓
D		5.83 Δ
5.95 6+p+9+16		5.80 1+d
D		Δ 6.44
5.70 2+p+24+15	path—	5.39 —X
D		5.00 2+d
5.44 30+15		4.48 to 487 on 10.67
p+p—		4.47 0+d
p+5 4.90 28 to end of fence	pond—	0.61 —X
4.51 12+waterfall		13 0.60 33
p+10 4.38 8+fence+d	pond—	0.31 —X
4.23 6+20	pond	
D—		12+4 0.30
D		from 578
13 4.16 20		on 578
D—		
D—		579 to off. 23 at 8.20 on
D		4 570 19.55 (page 107)
3 3.90		8 500
p+11(D—		+2 1.55
3.70 —+		D+6 0.00
326 20+15	from 0.13	
p+5 297 20+20	on 734	
2.32 20+15		
path+10 200 30+15		7.34 to off. 21 at 483 on 968
1.44 25+15		700 2+d
1.23 33+20		4.52 8 to G. P.
D		4.36 10 to G. P.
0.94 28+15 to paling		1.56 17 to 1031 on 1728
path+9 0.70		Δ 0.13
from 544		0.12 18+d
on 609	from 578	
	on 578	
3.54 to 583 on 609		
327 3		to off. 98 at 908 on
300 4		5.78 1728 (page 105)
2.62 10 to paling		5.75 29+
2.50 10+ fence +8		5.60 26+pond
2.39 58+26 to G.P.		5.43 32+pond (35)
209 88 to G.P.		5.25 40 to G. P.
205 60		300 18
1.93 23+18		2.50 14
1.68 4+p+22+fence +		2.10 10
path—		0.72 4
1.65 —+ +path		0.55 0
1.00 9+10+3 to paling		0.25 6
from 860		0.13 12
on 1067	from 616	
	on 1306	

	13.06	to offset 14 at 147
10+30	13.02	on 1955
D		
10+76	6.83	
	6.16	△
D		
10+73	6.02	4+D
D—	6.02	—X
	0.08	8+D
from 1063 on 1067		
 pond		
46+p+0		
	10.67	
	10.63	△
15	10.50	
stile—	10.46	—X 0 to post
8	10.00	6
2	9.36	
P		P
3	9.00	8
path+path—	8.74	—X
to paling 10+6	8.61	
△	8.60	
D—	4.88	—X
	487	△
from 1443 on 1728		
	9.68	to offset 26 at 16 ch.
D+7+44+0	9.35	on 19.55
haystack+2	905	
46	900	
30	800	
7	600	
↑ to fence 21	4.83	
D+12	479	
16	400	
18	300	
5	200	
0	1.46	
D+3	1.00	
from 17.28 on 17.28		
page 105		

from 765 on 11.60 page 104		to offset 27 at 710 * on 12.30 (p. 104)
	5.56	18+13 
	5.41	32
	4.00	26
	3.00	22
	2.70	8+oval
	2.40	
from 765 on 11.60 page 104	1.00	13+hedge
	Δ	4.06 to 100 on 1160
		4.05 11+p+15 
		3.72 27
		3.26 7+ 
		2.95 3+p+26
		1.18 24
path— from 574 on 577	0.93	—X
	↑	
	fence—	5.77 —X
	Δ	5.74
		5.67 <sup>D</sup> 3+16 
		on fence
p—		<sup>D</sup>
	5.58	—X 22+16
	5.00	2+18
	(30) <sup>D</sup> —	4.50 4+16
	<sup>D</sup>	
	30 <sup>D</sup> —	4.12 10+12—X
		4.00 12+14
p—	3.00	9+16
	2.56	30+15
	2.00	41+14
	1.00	50+12
	0.95	342+0
	0.80	21+18
	0.57	—X
p—		<sup>P</sup>
	0.30	11+10+8+12 <sup>D</sup>
<sup>D</sup> —	0.29	—X

from 765 on 1160	0.20	stile	D
		16+12+13+15	
	11.60	to 1728 on 1728	
	16+3	11.56	14+p
	10+6	11.00	17+p
	12+7	10.50	15+p
	D		
	12+10	10.00	8+2 path
	D		
	12+13+p	9.53	- X
	10+11+2+7	9.00	
	12+23+p+26	8.00	
	10+23+p+26	7.90	
	D		
	to post 20+18+p	7.70	
	+25	765	Δ
	D	737	- X
		720	27+D
	37	600	
	+path+35	400	
	+path+21	200	
	Δ	1.00	
	to post stile 16	0.76	
	stile	0.69	- X
	16+6	0.65	
	R		
	to G. P. 21+11	0.36	
	to G. P. 41	0.17	
	from 1230 on 12.30		
	lamp	Δ	post
	12.30		
	76	12.24	
		11.76	30+22
		11.70	4 to pump
		10.92	12+20
		10.50	16+13
		9.32	20+9
	H+9+69	7.77	
		7.21	25+9
		7.10	27
			 


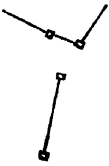
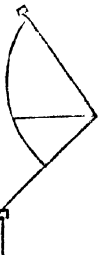
	5.00	<sup>R</sup> 5+30+ <sup>H</sup>
4+15+10+34	3.00	<sup>R</sup> 10+28+ <sup>H</sup>
<sup>R</sup> 11+10+25	1.00	<sup>R</sup> 17+20+ <sup>H</sup> 10
46	0.58	
11+14	0.51	
34+12	0.14	
from 1026 on 1026 along High Road		to Kentish Town
to 2 G. P. 0	10.26	
*to turnpike G. P. 7	10.09	*offset 50 at 19.15 on 19.15
to fence 15	9.82	
to fence 10	9.78	
hedge—	9.73	—X
	9.64	18 to G. P.
	9.58	15   18
\ fence—	9.49	—X
	9.46	2
	9.34	20
from 1728 on 1728		
	Δ	0
	17.28	0 to 2 brick pier
	17.14	11 to pier of small bridge
path—	17.00	—X
	16.93	8 to further post
	16.86	0+stile
to tree 0	10.32	
n—	10.31	—X
	9.08	90 to G. P.
to tree 0	8.39	
n—	8.35	—X
from offsets 14 at 147 on 1955		
	7.53	to 2166 on 2624
	300	28 to 2624
from 300 on 23.46		




Holloway & Highgate Road X		
	23.46	0 to sign post
	23.32	5 to G.P.
	22.00	5 to garden gate
R	21.00	10 to G.P.
10+44+3	20.00	2+8+2
Rd. —	19.00	—X
	17.90	19+14+28
	17.64	19+15+28
	17.44	20 to paling
—15+40	16.66	
	P	
	16.37	7+5+8+2
	R	P
	14.60	12+6—8+4 to G.P.
to G.P. 20+30	13.95	
24+25	12.00	
	10.00	27+4+8+12
		CHURCH LANE
	9.50	26+4+10
25+15	8.92	
	5.34	40+4+12 to G. P.
15+4	4.42	40+4+11 to G. P.
Road—	3.70	—X
	3.00	△
	R	P
	1.00	7+44+4+15
from 2624 on 2624		

NOTE.—[The two following lines not in the engraving.]

	12.58	to 159 on 159
	12.00	20
△	10.16	40
	6.50	76+D
	2.00	23+57
fence—	.70	—+
to fence 4+hedge—	0.90	—X
from 2166 on 2624		90
	△	
	26.24	R
15+0	25.55	40+5+13
R—	25.54	—X
R		R
20+16	24.00	20+5+14
R		R
18+35	22.52	3+6+15
Road—	22.35	—X
to G. P. 20+36	22.34	[from gate.
△	21.66	0 to 9th lamp-post

—  4+20+36+6 to G. P. 20+40+5	21.46	14 to —	on fence
	21.27		
	20.62		
	20.00	8+5	
	19.09	16 to G.P.	
20+35+4	18.93	2+13 to G. P.	
R			
15+30+10	18.00	2+11	
Road—	16.20	—X	
12+30	15.00	3+12+5	
14+20	13.00	13+4+13	
14+10	10.00	24+5+13	
10+9	9.00	28+4+12	
12+7	8.00	30+6+13	
R		R	
20+6	7.00	30+6+13	
16+11	6.00	26+5+12	
R		R P	
15+15	5.00	20+5+10	
R		R P	
17+20	4.00	19+4+10	
R		R P	
D+10+26	3.00	10+10+10+D	
Road—	2.32	—X	
R		P	
D+13+30+3	2.00	10+13+D	
R			
D+28+30+10	1.57		
path—	1.56	—X	
—  gate fence—	.82	—X	12 to stile
from 19.55 on 19.55		Along the Junction Hol-	
		loway Road.	
from 217 on 19.55	3.73	to 638 on 663	
	Δ		
	19.55	0	
	19.26	25	
	19.15	50	
post by hedge	47	19.07	
	19.00	40+20	
	18.72	37	
	18.52	18	

	to G. P.	30	18.42	
	to post	15+20	17.84	10+10 to stone
		17+15	16.00	26 to G. P.
			15.86	15+10 to G. P.
		11+15	15.47	△ 24 to fence
		7+13	14.00	20+6
			12.24	20+8 to G. P.
			12.08	20+6 to G. P.
			8.20	10+13 to ↑ on fence
		3+28	600	
		3+25	500	10+3
		20	3.00	10+3
			2.17	△
		2+25	2.00	10+2
		<sup>R</sup>		<sup>R</sup>
		D+3+27	1.70	10+2+D
			1.47	14 to ↑ on fence
			1.13	22 to fence
		D+4+18	1.00	16+6+D (20)
		— 25	0.00	
from 159 on 159				
from 159 on 159			2.14	to 5.74 on 6.63
		D+0	1.59	<sup>R</sup> △ 30+5
		<sup>D</sup> 6+12	1.53	
		to G.P. 10	1.31	
		to G.P. 12	1.15	<sup>R</sup>
		5+8	1.00	23+5
from 638 on 663				
check-line.				
from 116 on 330			0.32	to 518 on 544
			3.30	to 215 on 663
from 397 on 544			1.16	△
hedge —			6.63	— X
		<sup>R</sup>		
		3+28	639	0+7
		△ road	6.38	— X
		<sup>R</sup>		<sup>R</sup>
		16	600	10+6
		△	574	
		5	500	26+8
		D+0	460	26+8+D

3+8	4.00	24+10
4+10	3.00	20+8
4+16	2.60	15+10
△	2.15	
R		R
D+10+20	2.00	8+7+D
road—	1.00	—X
R		
a+33+0+3	0.55	6+D
┌		
from 518 on 544		
hedge of road	5.44	—X
	5.34	7 to G. P.
△	5.18	15 to G. P.
R		
5+30+6	5.00	20+D
R		
D+8+30	4.61	
road	4.60	—X
3+18	4.00	12+30
△	3.97	
3+16	3.90	
8+15	3.40	20+30
	3.00	15+25
to G.P. 17+18	2.96	
to G.P. 20+20	2.80	
30+20	2.00	15+20
	1.01	△
20+13	1.00	18+10
R		R
D+25+10	.800	20+15
		R
D+10+0	.00	33+20+D
from 23.41		being old sign post.
on 2341		in previous survey.

NOTE.—[The commencement of this line will not, from want of room, be found inserted in the plan.]

For the assistance of the student, we will briefly go over the different selections of the lines, and the arrangement of the notes, making such special comments upon any local obstructions that may have occurred, as may enable the student to meet the same under similar circumstances.

The first line commences at a point 23·41, or 23'41, which is the end of an old line; this point is a sign post, selected as of certain reference and known position. This position is defined at starting, being at 0·00, close by the road side—that is, having the whole of the road (33) on the right side, and 0 (of the road) on the left; the 10 links on the left, and the 20 links on the right, are the width of the grass, common in country lanes, between the road and the hedge.

The station, 101, is a point required for the purpose of connecting this present new line with the line above referred to (23'41). This is, of course, not wanted for our present purpose. At 280, 20+20 to G. P. (or gate-post), at 29 18+17 to G. P. Reference is seldom made to a gateway *twice*. It is usual to mark the position of the *first* post of the gate you come to.

The station 397 is wanted for the purpose of measuring the bend in the road, the remaining distance being taken as one of the sides of the triangle. The distance 215, on the next line, being taken as the second side, and the base of the triangle being the distance 3·30, which will be found immediately after the next line, 663, viz., from 397 on 544 (330) to 215 on 663. There is a *check*-line to this, for the sake of accuracy, from 116 or 330 (·32) to 518 on 544. This gives the direction (relatively) of the line 6·63. At the end of this, the lane again bends to the left, for a distance of 1·59 chains. The position of this line is also fixed by the measured base of 2·14 chains; from the end of 159, to a point 571, on the former line 663. The road now turns quickly to the right, and runs straight for nearly a quarter of a mile, to the turnpike (19 chains 55 links). This sharp turn in the road is measured by the subsequent base line of 3 chains 73 links, from 217 on the new long line, to the station 638 on the second line 6·63.

It is upon this long line, 19 chains 55 links, that all the triangulation is based, and the relative positions of the several roads in the plan determined. The small triangles, used for the windings of the

lane, which came first in the field notes, should be plotted last in the plan. The surveying of a winding road, by means of the chain, is very inaccurate, and should, where possible, be avoided. It, however, seldom happens, that when extreme accuracy in the delineation of a road is required, that this mode of measurement becomes compulsory. One average line, through the whole of its windings, can mostly be taken, when desirable; and when an approximation, as to position (which is frequently the case), is all that is wanted, the present method is sufficiently correct.

In the taking of the offsets upon this base line 19·55, great care is requisite in selecting the most suitable. It often happens, in practice, that stations for the side triangles cannot be taken at the base line itself, but an offset points near to it, such as a post in a fence, a gate post, the corners of a fence, or any other fixed point that can at any time be referred to. For instance, the first station here selected is that of a broad arrow on the fence (offset 14 at 147), for the purpose of measuring the cross hedge.

The next is that at an offset of 23 at 8·20 chains. The third station is that of the offset 24, at 15 chains 47 links.

At 16·00, then, is the common offset of 26 to gate post; at 17·84, 35 on left to a post at the corner of the field; at 1842, 30 to first left hand gate post; at 18·52, 18 links to a post in the corner of the field to the right; at 1872, 37, on the right, to where a small garden enclosure commences; at 19·00, 40 + 20, *i.e.*, 40 up to the enclosure + 20 to the corner; at 19·07 to stile post on left, by side of hedge; at 19·15, 50 to first right hand turnpike gate post; at 19·26, 25 links to second gate post on right; at 19 55, to the further corner of turnpike.

From this corner you turn to the left, on the high junction Holloway Road, for a distance of 26 chains 24 links.

The *direction* of this line is obtained by measuring the line 12 chains 58 links, from the station 21·66, on the line 2624, to the end of the line 159.

The number of the offsets on this line will depend altogether upon the degree of nicety required. At 1·57 the offsets on the left hand are

R

10 + 30 + 21 and D, that is 10 up to the road, which is 30 links wide, beyond which there is a grass plot of 28, up to the ditch. At 2·00, the line is on the right 10 links from the path, and on the left 3 links from the road. At 16·20 the road X, that is, the line

crosses the road, the position of the road in the previous or subsequent offset point will define the side of the road; for instance, at the previous offset, 15 chains, the line is only 3 links from the road; at the subsequent offset there is a space of 10 links between the line and the road on the left. The side of the road, therefore, crossed at 16·20, must be the right side of the road, and you cross out of it. The station selected at 21·66 is a lamp post, being the ninth from the gate. At 22·38, the line crosses the road again, and, as at the next distance you have road on each side of the line, it now crosses *into* the road. At 25·54, the line crosses a side of the road again, and, as in the next station 25·55, the whole of the road is on the right, there being no portion of it (o) on the left, the line crosses the right side, *out* of the road. At 26·24 the line ends. The road, now taking a new direction, another line of 23 chains 46 links is taken, making an angle with the preceding, which angle is measured by the line 7·53 chains, drawn from 309 on 23·46 to 21·61, on 56·24.

There are no particular observations needed on this line; at 9 chains 50 links there is a lane, called Church Lane, to the right; the position of its corners, and their distances from the line, are carefully defined. At 16·37, and at 16·66, on either side, the hedge ceases, and the wood fence begins. Between 17·64 and 17·90 there is a house on the right, 62 links off the line; at 23·46 the station is a sign-post, at the corner, where the Junction Road comes out into the high Holloway Road.

This completes the Holloway side of Maiden Lane. We now return to the Kentish Town side, to the survey of the fields.

From offset 14, at 147 on 1955, turning to the right, a line is measured to the second brick pier of a culvert, being a distance of 17·28; on this there is an offset to a post in the corner of the hedge of the first field, viz., at 9 09, 90 to G.P.

From 17·28 another line is again measured, of 10 chains 26 links, to second gate post of turnpike, which is a known point of offset upon the line 19·55.

This completes the first triangle, which passes obliquely through three or four fields

Again, from this point 10·26, turning to the right, along the same Junction Holloway Road, but in a *contrary* direction, measure 12 chains 30 links, to a lamp post.

From the end of this line measure 11·60 to 17·28 on 17·28, to the second pier of culvert, above referred to. In this line we meet

with a path, running through the field, the position of which it is necessary to define.

Instead of saying, at 200 distances, 20 links to path, &c., I would recommend a plan (which I have adopted in this instance) of  $21 + \text{path}$ , or, as at 7·70, of  $25 + p + 18 + 20$ : the intersection of  $p$ , shows the position of it, without interfering with the system and simplicity of the arrangement.

Upon this line 11·60, there is a third triangle based, for the purpose of obtaining the position of the boundaries of the field notes; the sides of this triangle are respectively 5 chains 74 links, and 4 chains 6 links.

On this line, 577, the first offsets taken are at 30 links, being 16 links after the stile, which is 12 links wide, and 13 links from the ditch, which is 15 links across.

At 4·12, ditch crosses—and, from being on the right side of the line, runs 30 links to the left, forming, as the following offset at 4·50 shows, a small pond, whose length on the line is 31 links, and width 30 links to the left. The offsets, now, go on regularly to the end.

In the next line, 406, which brings us back again, close upon the high road, at the distance of 1·18, there is an offset of 24 to the paling; at 2·95 ( $3 + p + 20$ ), that is, 3 to the path, which is 20 links off from the corner of a small enclosure; at 3·26, the offset of 7 links is taken to the widest part of the oval; at 3·72, you come to the end of it, which is 27 links off from the line; the position of the main fence is fixed by the two offsets, 20 and 27, and the beginning and end of the enclosure by their distances on the line; the size of the oval is known by the distance 7. At 4·05 there is an offset of 11 to the path + 15 to the corner of a garden, on the other side of the field.

The offsets are now carried up to where the triangulation was left off. Let us now, therefore, proceed with the triangulation.

The next line we come to is the line 556, which, turning to the right, runs from a known point, 765 on 1160, alongside of a thick quickset ornamental hedge, to another known point, viz., an offset 27 at 710 on 12·30; this line, therefore, acts as a *check-line*. The first offset, at 100, shows the position of the hedge; the second distance, 246, the commencement of an oval enclosure; the next, 270, shows that, at its widest part, it touches the line; at 3·00 the oval ends, and is 22 links off the line.



At 541, another half-oval begins, which, at its widest part, at the end of the line, is 18 links off, its conjugate axis being 13 links ; this point, which is 31 links from the present line, is the offset point 27, on the old line 12·30. This line completes the Field No. 6.

The next line runs from the brick pier at the corner of No. 5, along the boundary hedge, between Field No. 5 and Fields Nos. 1 and 4. It connects the point 17·28 at the pier, with another known point, at the other end of the field near Maiden Lane, being offset 26 at 16 chains, on the line 19·55. This line finishes Field No. 5.

The following line, 10·46, is a loose line, being one of the two sides of another triangle, based upon a previous line 17·28, and selected for the purpose of surveying fields Nos. 2, 3, and 4. It runs to a stile post at the corner of field No. 3.

The other side of the triangle is the line 13·06, which extends from the same stile post to a known point, viz., the offset 14 at 147, on the line 19·55, being the corner, on Maiden Lane, of field No. 2.

On the line 10·67, there are two stations taken, 487 and 8·60 ; the object of them will be explained presently. The offsets on this line are as usual, except at the end, where they differ somewhat ; for example, at 10·46, the line touches the stile post on the line, denoted by (0+post), and stile crosses at 10·50—there is an offset at 15 links, to the angle of the adjoining property. At 10·07 there is an offset to another angle in the fence.

On the line 13·06, the chief thing to be observed, is, that at 6·02 the line crosses the ditch between fields Nos. 2 and 3, and that the corner of field No. 2 is (10+73) links, to the left of the line, where it crosses, while the corner of field No. 3, is 4 links to the right.

The offsets of  $73+10$ ,  $67+10$ , denote, that on this side of the field the hedge is *within* the boundary of the field, as was explained above. There is but one  $\Delta$  on this line, viz., 6·16.

The next line measured, 578, is from this  $\Delta$ , 6·16, to a previous known point, viz., offset 90 at 908 on 1728. This line is a check upon the line 10·67, and at the same time it gives the position of the boundary between fields Nos. 3 and 2.

From 578, along the ditch, between 4 and 1, the line 734 is measured to a known offset, viz., 21 at 483 on 968. This line is a check upon the measured lengths of fields Nos. 1 and 2, as to their





sides adjoining Maiden Lane. At 156 the line is 17 links distant, from a previous point 1031, on a fixed line 1728. There are, therefore, *two lines* drawn half way across to the central, to show that it is not a *loose* line, and the *half way* denoting that the line, in its fixed direction, is produced further.

Next, from a point on the last line (0·13), a line is measured along the boundary, between 1 and 2, to a known offset on Maiden Lane, being offset 23 at 8·20 on 19·55.

Then, from the end of 578, a line is taken between fields 3 and 4 to a previously known point (487 on 1067) and produced to 609 (denoted by the two lines half way across).

Upon this line there is a  $\Delta$  (544), selected for the purpose of obtaining the winding of the old flat ditch, that runs at the bottom of the field No. 4, and another  $\Delta$  at 5·83, for the corresponding boundary of No. 3.

The next line, 354, is a line connecting a point, 860 on 1067, with this last  $\Delta$  5·83, having offsets to the fence on the right.

And the last line is a line measured from the first of the  $\Delta$ s on the line 609, (1544), to a known point on the line 1728.

The offsets, on this and the preceding line, are many and somewhat complicated, but certainly intelligible (with a little trouble) to the student, who has made himself master of the preceding explanations.

*The Areas of the Fields in the Survey, Plate III, are respectively as follows :—*

	A.	R.	P.
No. 1 contains	4.	3.	0.
2 . .	4.	2.	25.
3 . .	3.	0.	32.
4 . .	3.	1.	28.
5 . .	5.	3.	0.
6 . .	2.	3.	8.
7 . .	11.	2.	28.
Total area	<u>35.</u>	<u>2.</u>	<u>1.</u>

## CHAP. X.

REDUCTION OF CUSTOMARY TO STATUTE MEASURE,  
*and vice versa.*

THE statute length of the perch is 16 and a half feet, but it is different in various counties of England.

In Devonshire and Somersetshire, the customary perch, that is, the local measure of the perch, is less—being but 15 feet.

In Cornwall, it is more, 18 feet; while in Lancashire, it increases to 21; and in Staffordshire and Cheshire it is as much as 24 feet.

This is a *lineal* difference. There is, also, in some counties of England, a *superficial* difference in the measure of an acre; an acre, in Wiltshire, containing only 120 square statute perches, instead of 160.

The Wiltshire customary acre is, therefore, one quarter less than the statute acre, and the rood one quarter less than the statute rood.

As property is frequently bought and sold by the customary measure of the county wherein it lies, the Surveyor is often called upon to reduce it from one to the other.

## DIFFERENT VALUES OF THE ACRE.

The number of (statute) square yards in an acre will, of course, vary with the length of the customary perch of the county.—(An acre consisting of ten square chains or of 160 square perches.)

In the statute acre, a square perch is 272·25 square feet, and the acre, therefore, is equal to

$$\begin{aligned} 272\cdot25 \times 160 &= 43560 \text{ square feet,} \\ &= 4880 \text{ square yards.} \end{aligned}$$

In the acre of Devonshire or Somersetshire, as the square perch contains  $15 \times 15$  square feet, or 225 square feet,

$$\begin{aligned} \text{the number of sq. feet} &= 225 \times 160 = 36000 \\ \text{and of yards} &= 4000 \end{aligned}$$

In Cornwall, where the perch is 18 feet,

$$\begin{aligned} 18 \times 18 &= 324 \times 160 \text{ feet} = 51840 \text{ sq. feet,} \\ &\text{or } 5760 \text{ sq. yards.} \end{aligned}$$

The Lancashire perch is 21 feet long; the square perch, therefore, must contain  $21 \times 21 = 441$  square feet, which will make the acre to contain 70,560 square feet, or 7840 square yards.

The customary acre in Cheshire and Staffordshire is the largest of the whole, each perch being 24 feet; the acre will consist of  $24 \times 24 \times 160$  square feet, which is equal to 92160 square feet, or 10240 square yards; while the Wiltshire acre consists only of  $\frac{3}{4}$  the statute acre, or 3630 square yards.

By dividing the square feet, in a customary perch, by that in the statute perch, you will obtain the measure, the customary is of the statute acre, taken as an unit; and, by multiplying statute square links by this number, you can at once bring the statute acre into the corresponding customary form:—

$\frac{225}{272 \cdot 25} = \cdot 826447$ ; the measure of the Devonshire acre in terms of the statute acre.

$\frac{324}{272 \cdot 25} = 1 \cdot 19$ , that of Cornwall;  $\frac{441}{272 \cdot 25} = 1 \cdot 62$ , of Lancashire.\*

$\frac{576}{272 \cdot 25} = 2 \cdot 1157$ , that of Cheshire.

$\frac{120}{160} = \frac{3}{4} = 0 \cdot 75$ , that of the Wiltshire acre.

#### DIFFERENT LENGTHS OF THE CHAIN.

Because the area of any acre  $A = 10x^2$ , when  $x$  equals a chain's length in feet; therefore  $x = \sqrt{\frac{A}{10}}$  in sq. feet, where  $A$  is the area in sq. feet of the given acre, whether statute or customary:—

The Devonshire acre contains 36000 square feet;

$$\text{the chain} = x = \sqrt{\frac{36000}{10}} = \sqrt{3600} = 60 \text{ feet.}$$

The Statute acre contains 43560 sq. feet.

$$x = \sqrt{4356} = 66 \text{ feet.}$$

The Cornwall acre contains 51840 sq. feet.

$$x = \sqrt{5184} = 72 \text{ feet.}$$

The Lancashire acre contains 70560 sq. feet.

$$x = \sqrt{7056} = 84 \text{ feet.}$$

The Cheshire and Staffordshire acre = 92160 sq. feet.

$$x = \sqrt{9216} = 96 \text{ feet.}$$

\* This is the same as the Irish perch.

As the chain is divided into 100 links, whatever may be the length of the link, and, as in every case, an acre equals 10 square chains, the area of an estate in any county can be *at once* found in the customary measure of that county, by using the proper length of chain, without first calculating it, by statute measure, and then having the trouble to bring it back again to customary.

If a chain be divided into 100 links, or  $a$  in feet  $= 100$  links ; then each link  $= \frac{a}{100}$  feet  $=$  the decimal of the number of feet.

	Feet.	Inches.
The statute link, therefore,	$= \cdot66$	or 7·29 in length.
The Devonshire link	$= \cdot60$	or 7·20.
The Cornwall link	$= \cdot72$	or 8·64.
The Lancashire link	$= \cdot84$	or 10·08.
The Staffordshire link	$= \cdot96$	or 11·52.

*To reduce Statute Measure to Customary, or one Customary to another.*

**Rule 1.**—Bring the acres, roods, &c., in every case, to square perches; multiply these by the number of square feet in the given perch to bring them into square feet (a foot being the common unit of measurement of both statute and customary measure), and divide by the number of square feet in the required perch; this will bring it into perches; raise these perches to roods and acres and the result is the area in acres, roods, and perches, of the customary measure required.

Or **Rule 2.**—Multiply the given area in acres by the measure, the statute perch is of the customary perch, and the product will be in customary acres, which reduce to their proper value.

Or **Rule 3.**—Bring the given area into acres and decimals, and divide by the measure the customary is of the statute acre (considered as an unit.)

**EXAMPLE 1.** (*by the 1st rule.*)—Reduce 25 acres, 2 roods, 26 perches, statute measure, to the customary measure (Derbyshire) of 15 feet to a perch.

A.	R.	P.
25.	2.	24.
	4	
102		
40		
4096 square perches.		

The square feet in a statute perch =  $272\cdot25 \times 4096 = 1115136$  square feet.

$$15 \times 15 = \frac{1115166}{225} = 4956 \text{ customary perches.}$$

$$40 \overline{) 4956}$$

$$4 \overline{) 123} - 36$$

30. 3. 36. Derbyshire measure.

EXAMPLE 2. (*by the 2nd rule.*)—How many Cornwall customary acres are there in 45 acres, 2 roods, and 27 perches, statute measure?

$\frac{272\cdot25}{324} = \cdot84 =$  the measure, the statute perch is of the Cornwall perch, considered as an unit.

$$40 \overline{) 27\cdot00}$$

$$4 \overline{) 2\cdot675}$$

$$45\cdot66875 \times \cdot84 = 38\cdot36175$$

$$\begin{array}{r} 4 \\ 1\cdot44700 \end{array}$$

$$\begin{array}{r} 40 \\ 17\cdot88000 \end{array}$$

A. R. P.  
Ans. 38. 1. 17.

EXAMPLE 3. (*by the 3d rule.*)—How many acres, in the county of Stafford, are required to make a farm of 100 statute acres?

The Staffordshire measure of an acre is  $2\cdot1157$ .

$$100\cdot000 \text{ acres} = 47\cdot426 \text{ Staffordshire acres.}$$

$$2\cdot1157$$

$$\begin{array}{r} 4 \\ 1\cdot704 \end{array}$$

$$\begin{array}{r} 40 \\ 28\cdot160 \end{array}$$

A. R. P.  
Ans. 47. 1. 28.

*To bring Customary into Statute Measure.*

Reverse each of the preceding rules.

EXAMPLE 1.—How many statute acres are there in 28 acres, 3 roods, and 15 perches, of Devonshire measure?

$$4 \overline{) 15\cdot000}$$

$$4 \overline{) 3\cdot375}$$

28·84375 Devonshire acres.

The measure of the Devonshire acre =  $\cdot826447 \times 28\cdot84375 = 23\cdot8378$

$$\begin{array}{r} 4 \\ 3\cdot3512 \end{array}$$

$$\begin{array}{r} 40 \\ 14\cdot0480 \end{array}$$

A. R. P.  
Ans. 23. 3. 14.



EXAMPLE 2.—A farm in Cornwall consists of 100 acres, of local measurement, how many statute acres are there?

$$\begin{array}{r} 100 \text{ acres.} \\ \text{Cornwall acre measure} = 1.19 \\ \text{Ans. } 119 \text{ acres of statute measure.} \end{array}$$

EXAMPLE 3.—In 30 acres, 3 roods, 36 perches, Derbyshire measure, how many statute acres?

$$\begin{array}{r} \text{A. R. P.} \\ \text{Ans. } 25. \quad 2. \quad 16. \end{array}$$

EXAMPLE 4.—A gentleman, wishing to purchase a farm in Lancashire, which is 486 acres and 2 roods, of the statute measure, is desirous of knowing how many acres of customary measure there are in it?

$$\text{Ans. } 300 \text{ acres.}$$

EXAMPLE 5.—In Cheshire, there is a farm of 240 acres 2 roods, of statute measure; a recent purchaser wishes, by the purchase of a portion of the adjoining land, to increase his farm to 350 customary acres: what will it cost him to do so, at the rate of £40 per statute acre?

$$\text{Ans. } £20,000.$$

EXAMPLE 6.—A nobleman wishing to farm 400 acres of customary measure, in the county of Wiltshire, is desirous of knowing what it will cost him, at the rate of £30 per statute acre?

$$\text{Ans. } £9,000.$$

#### MISCELLANEOUS EXAMPLES.

How many acres of Lancashire measure are there in 250 statute acres?

$$\begin{array}{r} \text{A. R. P.} \\ \text{Ans. } 154. \quad 1. \quad 11. \end{array}$$

How many statute acres will make up a farm of 300. 2. 30. acres, of Wiltshire customary measure?

$$\begin{array}{r} \text{A. R. P.} \\ \text{Ans. } 225. \quad 2. \quad 2. \end{array}$$

The base of a triangular field measures 6 chains 25 links, of a Cheshire chain, and the perpendicular, 4.84. How many statute acres are there in it?

$$\begin{array}{r} \text{A. R. P.} \\ \text{Ans. } 3. \quad 0. \quad 32. \end{array}$$

A rectangular field measures, by a Lancashire chain, 12 chains 45 links, and its perpendicular breadth, 8.20 links. How many acres would a farm of the same size contain in the county of Devonshire?

*Scotch Measure.*

The acre in Scotland consists, as in England, of 10 square chains, (each chain divided into 100 links,) and is reckoned in acres, roods, and *falls*, which are equivalent to the English perches; 40 falls making one rood, and 4 roods one acre. The Scotch chain, however, is 8 feet longer than the English, being 74 feet instead of 66.

The acre being 10 sq. chains =  $10 \times 74^2 = 54760$  sq. feet.

The chain or 100 links = 74 feet, therefore 1 link equals  $\cdot 74$  feet = 8·88 inches.

As 10 sq. chains = 160 square perches

$$\frac{54760 \text{ feet}}{160} = \text{one square perch}$$

therefore one square perch or fall = 342·25 sq. feet.

As the Scotch fall = 342·25 sq. feet,

and the English perch = 272·25 sq. feet,

the excess of the Scotch fall = 70 feet sq. feet.

And the measure of the Scotch acre, in terms of the English

$$\text{statute acre} = \frac{342 \cdot 25}{272 \cdot 25} = 1 \cdot 2571.$$

*To bring English Statute Measure into Scotch.*

**Rule 1.**—Reduce the given area into English perches, and then into square feet, by multiplying by **272·25**, the number of square feet in an English statute perch; divide this product by the number of square feet (342·25) there are in a Scotch fall, and you obtain the area in terms of Scotch falls, which bring back to their proper quantities in roods and acres.

**Rule 2.**—Bring the given area into decimals of an acre; divide this by 1·2571, which is the measure of a Scotch acre, when the English acre is unity, and you obtain the decimals of a Scotch

acre, which multiply by 4 and by 40, to bring to their proper quantities.

A. R. P.

EXAMPLE 1.—Reduce 32. 3. 25. English statute measure, into Scotch measure.

By the 1st Rule,

$$\begin{array}{r} 32. 3. 25 \\ 4 \quad \cdot \\ \hline 131 \\ 40 \\ \hline 5265 \text{ square perches} \end{array}$$

$$272 \cdot 25 \times 5265 = 1433396 \text{ square feet in the given area}$$

$$\text{sq. ft. in a fall} = \frac{1433396}{342 \cdot 25} = 4188 \text{ sq. falls}$$

$$\begin{array}{r} 40 \overline{) 4188} \\ 4 \overline{) 104 \cdot 28} \\ \hline 26 \cdot 0 \cdot 28 \end{array}$$

A. R. P.  
Ans. 26. 0. 28.

By the 2nd Rule,

$$\begin{array}{r} 40 \overline{) 25 \cdot 0000} \\ 4 \overline{) 3 \cdot 6550} \\ \hline 32 \cdot 93625 \text{ acres} \\ \hline 1 \cdot 2571 \end{array} = 26 \cdot 175 \frac{4}{704} \frac{40}{}$$

A. R. P.  
Ans. 26. 0. 28. the same as before 25·360

EXAMPLE 2.—Bring 20. 3. 39 acres, English, into Scotch measure.

How many Scotch acres are there in 100 English acres, and by how much does the Scotch exceed the English acre?

THE IRISH MEASURE is the same as the Lancashire; the chain is 84 feet long, and the acres are reckoned in acres, roods, and square perches, as in England.

The perch is 21 feet instead of  $16\frac{1}{2}$  feet; the square perch, therefore, contains 441 square feet, and the acre 70,360 square feet or 7840 square yards.

$$\frac{441}{272 \cdot 25} = 1 \cdot 62 \text{ the measure of an Irish acre.}$$

As 85 feet equals 1 chain, 1 link equals  $\cdot 84$  feet. = 10·08 inches.

The Irish chain being equal to 84 feet

and the English only to 66

In each Irish chain there is an excess of  $\frac{18}{66}$  feet, multiply by  
number of chains in a mile 80

$$3 \overline{) 1440}$$

The Irish exceeds the English mile by 480 yards

English mile contains 1760 yards

Irish mile = 2240 yards

$$2240 \div 160 = \frac{14}{91} \text{ or the Irish is to the English mile, as 14 is to 11.}$$

The same rules must be adopted in this case as in that of English and Scotch measure, in the reduction of statute English to customary, and customary to statute.

EXAMPLE 1.—How many English statute acres are there in 25. 3. 19. Irish acres ?

EXAMPLE 2.—How many Irish acres must be taken to make up a farm of 100 statute acres ?

EXAMPLE 3.—A Gentleman has an estate in a rectangular form,  $2\frac{1}{2}$  miles Irish one way, and  $\frac{3}{4}$  the other. How many English acres are there in it, and what would be the periphery in Scotch miles of the same sized estate in Scotland ?

EXAMPLE 4.—There is an Irish farm of 120 acres, and a Scotch one of 150 acres, the former worth £10 per acre, the latter 15£. For what quantity of land, at £25 per English acre in the county of Cheshire, could they be exchanged without loss ?

#### GAD MEASURE.

In some places in the Country, measurements are taken by the gad pole, a staff of 8, 9, or 10 feet, indifferently divided; either into feet or into tenths.

The square gad is the space inclosed within the length and breadth of a gad.

Acres are reckoned in square gads and square feet, or in square gads and decimals.

The 8-foot gad square is 64 square feet; the 9-foot gad, 81 feet; and the 10-foot, 100.

To ascertain, in gad, the areas of lands, measured by this method, bring the length and breadth into feet or tenths, multiply these together, the product is either square feet or square tenths, or, using

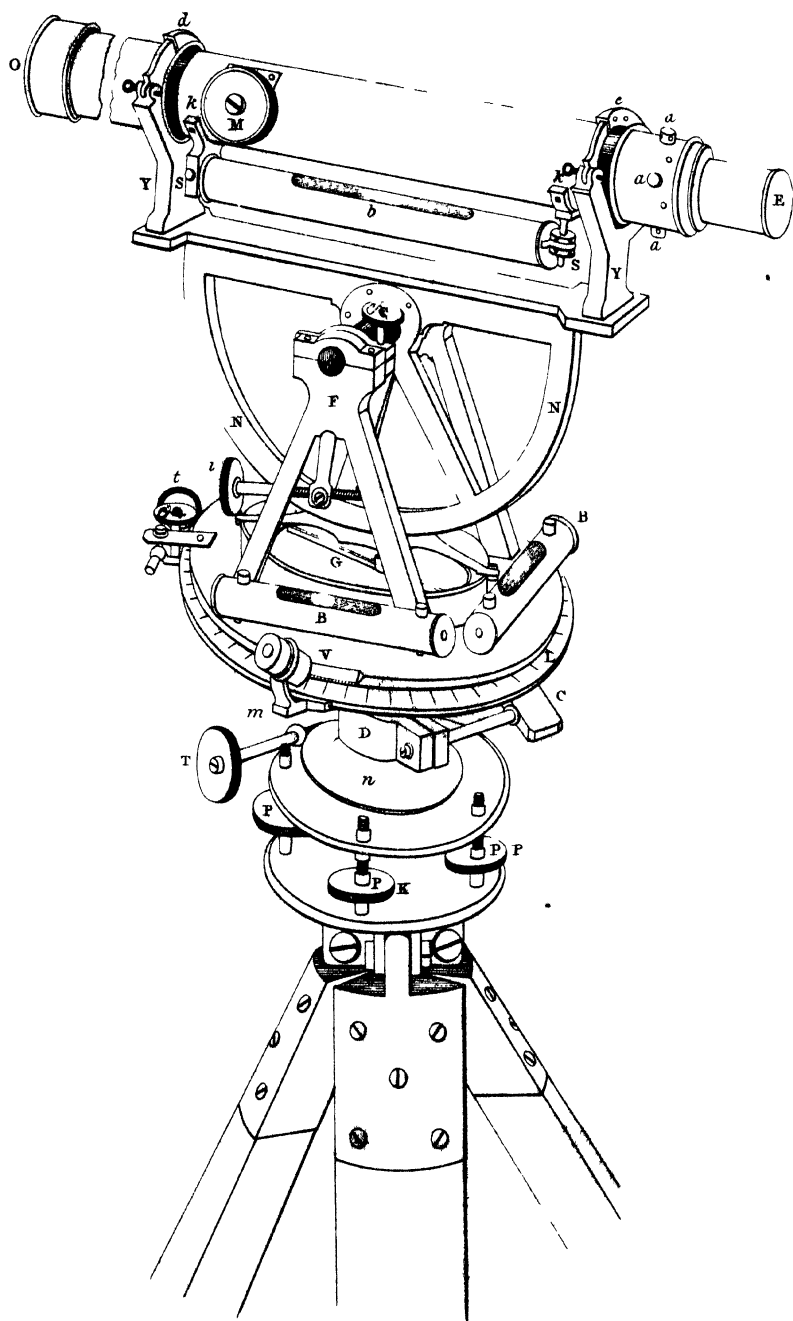
the 10 as decimals, square gads; the latter is complete, the former is to be divided by the number of square feet in the given gad pole.

*To bring areas in gads, and feet of customary measure, into the statute measure of acres, roods, and perches.*

~ Bring the areas into square customary feet, divide this quantity by the square feet in a statute perch, 272·55, and the quotient is the number of statute perches in the given area, which divide by 40 and by 4, as usual, to bring it into roods and acres.

EXAMPLE 1.—A triangular field measures 28,(10 feet) gads and 5 feet, in its longest side, and in its perpendicular, 19 gads and 7 feet—What is the area in square gads, and also in statute measure?

EXAMPLE 2.—How many square gads (9 feet) are there in a rectangular field, whose length is 10·25 Irish chains, and breadth 6·29.





# LAND SURVEYING.

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## Part the Second.

### THE THEODOLITE.

*Its Description, &c.*

THE Theodolite is the most useful instrument, that has been invented, for taking horizontal and vertical angles, as, by the nature of its construction, it is not necessary, that, in the former, the objects should be in the same horizontal plane; or, in the latter, in the same vertical plane.

This instrument stands upon three legs, and consists of three divisions, and has three motions.

1st.—*The absolute horizontal motion* of the whole instrument moving upon its axis, with clamp-screw (C) to fix it, and tangent-screw (T) for fine adjustment.

2nd.—*The relative motion* (as to the lower) of the upper of the two horizontal circles, to which the vernier (V) is attached, with its clamp-screw (c), and adjusting-screw (t).

These two motions are for taking horizontal angles.

3rd.—The relative motion of the vertical circle, which has also, as well as the other two, its clamp-screw (c), and fine adjusting-screw (i).



*Detail of the first motion, which must be perfectly horizontal.*

The lower (K) of the two parallel *Plates* is screwed tightly down to the legs of the instrument. The axis of the whole instrument passes right through to this plate. The centering, at the other end, is fixed to the upper of the two parallel *circles*—the upper one, called the *vernier circle*, from having the vernier attached to it; the lower, the graduated or horizontal limb (L), having its whole circumference graduated into degrees and half degrees. The lower circle has a distinct motion from the upper, working by means of a collar attached to it, upon the centering of the upper. Upon this is again fixed the collar (D) of the large clamp-screw (of the first division), which is attached as well as the tangent-screw to the upper of the parallel plates, which, connected with a ball, works in the socket of the lower plate, and has a double relative motion.

The upper of the parallel *plates* is made *instrumentally* parallel to the lower graduated circle; and the upper circle, (when in correct adjustment), is also parallel to them both.

Upon the upper of these circles are two small spirit levels, B, ~~B~~, at right angles to each other.

By means of two pairs of conjugate screws, (PPP), which alter the relative position of the plates, the upper one can be always made level, as will be immediately seen, by the two bubbles, in the levels, being in the centre of the tube; and, once set level, the instrument, when in adjustment, will be level in any position.

By tightening the clamp-collar, and using the tangent-screw, the finest adjustment can be obtained.

*The second motion.*—Unclamp the horizontal circles, and the upper will move independently of the lower, or body of the instrument, with which it is connected. This motion of the upper circle, or vernier plate, as well as that of the two (or virtually the motion of the lower), will be perfectly level, if the instrument be correct and in adjustment; and the bubbles, now, as then, will be, in every position, in the centre of the tubes. This is, however, the weakest part of the instrument. The upper circle is very thin, and having an unequal and constant pressure exerted in one place, the bubbles, which, when the two circles are clamped together, are in perfect adjustment, are often very far from being so, when the upper plate is no longer held down by the clamp-screw (*c*).

The clamp (*c*), and tangent-screw (*t*), are placed along-side these circles, and have the same office as those of the first motion.

*The third motion.*—In this, which is a vertical motion, a graduated circle (*N*) is made to move instrumentally at right angles, with the horizontal motion plane of the instrument.

This circle moves upon its axis (*A*), which, passing through the common centre of the instrument, is supported by two shoulders or supports (*FFP*), at right angles with the vernier plate, to which this axis is made parallel.

This, like the horizontal circle, is graduated to half degrees, and, like that, by means of a vernier (*v*), supported by the compass box, is capable of being read off to minutes.

Attached to this vertical arc, above it, is placed a telescope, supported on two *Ys* or arms, in the form

of the letter Y. These Ys are virtually tangents to the tube of the telescope, which is generally furnished with bell-metal collars, ground truly cylindrical, to rest upon them. These are kept in by clips (*de*), fastened by a pin.

A long spirit-level (*b*) is fixed beneath, to the telescope, parallel to it. The upper part of the inside of the tube is ground truly circular, so that when it is nearly filled with alcohol, or any other spirit, and its axis in a level position, the fluid may recede from the centre, which will then be its upper part.

There is also a compass box (*G*), with a magnetic needle. This box is generally placed over the two circular plates, and under the vertical arc. It is occasionally used to find the bearing of any line, though it is not correct enough for that purpose. It is really only calculated for ascertaining, *roughly*, the general position, as to the north and south points of the estate.

### *The Working of the Theodolite.*

Adjust, first, the parallel screws (*P P*), so as to have, as nearly equal lengths of the worm, as possible, above the upper plate.

Extend the three legs, approaching or extending each, until the bubbles in the two levels (*B B*) are nearly central, and the plummet, suspended from a hook under the body of the instrument, hangs freely above the centre of the station. The better plan is to move only one leg, which is, of itself, capable of a double motion.

Press the legs firmly in the ground, unclamp the

*whole* instrument by means of the large clamp-screw (C), observing to keep the other motions clamped.

It must now be remembered, that the two levels, on the horizontal plate, are conjugate, *i. e.*, at right angles; and, that the opposite screws, also, are conjugate, each pair of them.

Set one level in a plane, vertically parallel to a plane, passing through any one of the screws; and the other level and the other pair of screws, will be also in parallel vertical planes, conjugate to the former.

If both the bubbles of the levels, thus placed, are not in the centre, take the bubble that is not level, and, loosening one of the corresponding conjugate screws, tighten the other, until the bubble be accurately adjusted. Then loosen and tighten the other pair in the same way, till the same result be obtained. This will probably throw the first out; repeat the process to each, until both bubbles are level.

Having these plates (K k) level, clamp the whole instrument, and, unclamping the parallel circles, set the broad arrow of the vernier, which is in the upper plate, to  $360^\circ$ , or zero, of the larger circle, and clamp it. This must be done by the magnifying or reading glass (*m*), attached to the horizontal circles.

This large circle is divided into 360, and then again, subdivided into half degrees, which are numbered from left to right, and, by means of the vernier, read off to minutes.

Again, unclamp the large clamp-screw, and turn the whole instrument towards the left of the stations, between which, you are desirous of taking the angle, until you can cut the object as accurately with the intersection of the cross wires of the telescope, as can

be done by the hand. Clamp the screw (C), and slowly turning the milled-head tangent-screw (T), you can obtain all possible degree of accuracy.

Now, as the zero points of both upper and lower circles are together in the present position of the telescope, and as the lower circle is graduated from left to right, by separating the upper circle, and turning it round, till the centre of the cross wires of the telescope, which is attached to it, cut exactly the centre of the object at the second station, you obtain the angle between the two, determined by the position of the vernier and the length of the arc of the circle it has described. This can at once be read off from the plate by the broad arrow of the vernier, which will stand exactly above the number of degrees and minutes of the angle, measured between the two given objects.

When the cross wires, therefore, nearly cover the object, clamp the plates, and use the tangent-screw (*t*); and, with the magnifying glass (*m*), read off the angle, by means of the vernier. (For a description of the vernier, see page 137.)

The angle, thus read off, should always, if great accuracy be required, be read off by each of the two verniers. The common 5-inch theodolite is furnished with two, and the larger instruments have three, equidistant from each other, so that the *mean* of the readings, taken at different points on the circumference, should correct the errors of eccentricity or graduation.

In extensive surveys, other securities are adopted against these errors.

Instead of fixing the broad arrow of the vernier at the zero point of the horizontal limb, at starting,

the telescope is directed to the first station, with the broad arrow indifferently placed upon the lower plate, and its position carefully read off by the several verniers, and the *mean* taken. The difference between this *mean*, and that of the reading of the second station, is the measure of the required angle.

In incorrect graduation, this is perhaps the best check that can be used.

As an additional check, these angles are often repeated; that is—the angle is not taken again, by separating the upper plate and bringing the vernier back to zero, and then taking it a second time—but, without detaching the two plates after the last observation, turning the whole instrument bodily round to the first station, and, then unclamping the vernier plate, and turning it round to the second station.

The difference between this and the first reading, before starting, will be double the mean angle. Keep the two plates still together, and turn the whole round, repeating the process as before.

The difference between this third reading, and the reading at starting, will be three times the angle required.

It is requisite that the verniers should be separately marked, as A, B, C.

#### TO TAKE A VERTICAL ANGLE, OR AN ANGLE OF ELEVATION OR DEPRESSION.

First set the whole instrument level, as was explained before, by means of the bubbles on the vernier plate. Then bring the bubble of the telescope level (*b*) to the centre of the tube, observing whether, at the

same time, the zero point of the circular arc coincides with the zero of the vernier. This must be carefully examined by the microscope.

The instrument being thus perfectly level, when the zero point of the circle and the broad arrow are together, raise or depress the telescope, till you distinctly cut the required object with the horizontal wire, or the common intersection of the three wires. The changed relative position of the broad arrow, will give the required angle, which will be an angle of depression, if the broad arrow be found between the zero of the plate and the object-glass of the telescope, and of elevation, if beyond them.

It will be requisite, for particular accuracy, to invert the telescope in its Ys, and read off the same angle from the other end ; half the difference of these two will be the angle of error, of the vernier.

## ADJUSTMENTS.

### THE TELESCOPE.

The accuracy of this instrument, in its application to the purpose of taking angles, depends altogether upon the correctness of the line of collimation. The optical axis of the telescope, which is an imaginary line, joining the centre of the object-glass and eye-glass, should pass through the point of intersection of the cross wires.

These wires are attached to a broad flange of an inner tube, within the tube of the telescope, near the eye-piece, with which it is connected by two pairs of conjugate capstan-headed screws (*a a a*), so as to admit of a double relative motion.

I.—TO ASCERTAIN, WHETHER THE LINE OF COLLIMATION  
IS IN ADJUSTMENT.

Place the telescope within the Ys, and, having found some point clearly defined, which is cut by the intersection of the cross wires, turn the telescope round on its axis, and observe, whether, during its whole revolution, the centre of the wires remains the same, always covering the same point. If it does, it is in adjustment; if not, turn the telescope round on its axis, and correct for half the error, by means of the small capstan-headed screws (*a a a*), loosening one and tightening the other.

II.—WHETHER THE AXIS OF THE LEVEL IS PARALLEL  
TO THE AXIS OF THE TELESCOPE.

Place the telescope on the Ys, and unclamping the large clamp-screw, set the telescope over one pair of conjugate screws, and loosen and tighten them till the level, attached to the telescope, is made perfectly level; or it may be made perfectly level by the vertical tangent-screw.

Then reverse the telescope in the Ys, if the level remains the same, this also is in adjustment; if not, correct for one half the error by the capstan-headed adjusting-screw, at the end of the level (*S*); and the other half by the vertical tangent-screw.

There is also a side adjustment required.

The level may not always be immediately under the telescope, but a little to the right or to the left; this must not affect the position of the bubbles, or a lateral adjustment, similar to the vertical one, is indispensable, by means of the capstan-headed screw, at the other end of the level (*S*).



## III.—HORIZONTAL.

*To mak the axis of the bubble on the vernier plate, parallel to that plate.*

Let one bubble be over one pair of the circular plate screws, then the other bubble will be over the conjugate pair; make both bubbles level, turn them half round the circumference, and if the bubbles deviate from the centre, correct one half the error, by the small milled-headed screws above the levels: and the other half error, by the circular plate screws; repeat this, till the bubbles are level, in every position, throughout a whole revolution of the circumference.

## IV.—HORIZONTAL.

*Whether, after having duly corrected for the third adjustment, or made the "axis of the bubbles, on the vernier plate, parallel to that plate," the bubbles will remain perfectly level, during a whole revolution of the instrument upon the common axis.*

Clamp the two circular plates, and unclamp the large clamp-screw; set the bubbles perfectly level, as before; when, immediately over each pair of conjugate-screws, reverse them; if they continue level, they are in adjustment; if not, the two circular plates are moving upon different axis, and are not parallel to each other. This imperfection can only be well remedied by an instrument maker.

## V.—VERTICAL.

*Whether the vertical arc moves in a truly vertical plane.*

Set the vertical plate, or upper horizontal circle, perfectly level.

Direct the telescope to some well-defined angle of a building ; or, should there be no building convenient, suspend a string, with a plummet attached, from the top of a high pole, and, taking care that the intersection of the wires exactly cut the string, near the plummet, raise the vertical arc, observing whether the cross wires, throughout the whole of the vertical motion of the telescope, cover the vertical string ; if it does, this also is in adjustment.

As it is seldom found that two objects, whose horizontal angle is required, are exactly in the same horizontal plane, this adjustment becomes a very important one, and requires great care. Considerable error has resulted from neglect of it.

#### VI.—VERTICAL.

*Whether the vertical vernier is in adjustment, or perfectly central.*

Direct the telescope to some point of elevation, and note the angle. Reverse the telescope in its Ys, and raising the telescope to the same object, read off the same angle ; if these angles are the same, the vernier is in adjustment ; if not, correct the vernier for the error, by means of the small screw, fastening the vernier to the vertical plate, which can be loosened, and half the difference of these two angles will be the angle of error ; or, which is better, add this angle of error to every angle of elevation, when you use the end that reads off the smaller angle, and subtract the same, from that of depression, under the same circumstances.

When you read with the larger angle, subtract this angle of error from the angle of elevation, and add it for the angle of depression.

## PARALLAX.

Is an error occasioned by the focus of the eye-glass not being at once, with the focus of the object-glass, in the field of the cross wires.

The existence of parallax is determined by moving the eye about, when looking through the telescope, observing whether the cross wires change their position, and are flitting and undefined.

To correct this error, first adjust the eye-glass, by means of the moveable eye-glass tube, till you can perceive the cross wire clearly defined, and sharply marked against any white object.

Then, by moving the milled-head screw (M), at the object-end of the telescope, until you obtain the proper focus, according to the distance of the object, you are enabled at once to see clearly the object, and the intersection of the wires, clearly and sharply defined before it.

The existence of parallax is very inconvenient, and where disregarded, has frequently been productive of serious error. It will not always be found sufficient to set the eye-glass first, and the object-glass afterwards. The setting of the object-glass, by introducing more distinct rays of light, will affect the focus of the eye-glass, and produce parallax or indistinctness of the wires, when there was none before. The eye-piece must, in this case, be adjusted again.

Generally, when once set for the day, there is no occasion for altering the *eye-glass*, but the *object-glass* will, of course, have to be altered at every change of distance of the object.

## THE VERNIER.

The vernier is a contrivance for subdividing, to any extent, the smallest division in a graduated scale, varying according to the scale to be subdivided, and the extent of subdivision, but having the same principle in all cases.

The following explanation of that of the 5-inch theodolite, of Simms, which is given in the plate, and which is a very convenient and accurate little instrument, will serve for all.

In this, the lower circle is divided into half degrees or 30 minutes; and the vernier so arranged as to read off to one minute.

Twenty-nine divisions of the graduated scale, which is twenty-nine half degrees of the lower circle, are taken; and are divided, upon the vernier, into thirty divisions: now, as thirty divisions are compressed into the space of twenty-nine, each of these thirty divisions is one-thirtieth less than those of the twenty-nine: or, as the whole arc, in the graduated scale, is equal to 29 half degrees, or 870 minutes, and these are subdivided into 30 divisions, in the vernier scale, each of these subdivisions is equal to 29 minutes—therefore, if the zero point of the limb correspond to the broad arrow of the vernier, the first division line of the vernier is one minute to the right of the corresponding division on the limb; the second division on the vernier, two minutes to the right; the third, three minutes; and, so on, till the thirty divisions of the vernier, exactly coinciding with the twenty-ninth division, is just 30 minutes to the right of the thirtieth, or corresponding division in the limb, and therefore corresponds to the twenty-ninth division.

If the vernier, therefore, be moved, till its first division corresponds to the first division of the limb, the broad arrow of the vernier will be removed one minute from the zero of the limb. If the second division of the vernier be made to correspond with the second division of the limb, the broad arrow of the vernier will be two minutes removed from the zero of the limb. If the third corresponding divisions, coincide, the zero's are three minutes removed, and so on.

Hence, *to set the instrument* at any angle, of any number of minutes, between 0 and 30, and 30 and 60, move the vernier, until the broad arrow becomes in such a position, *within the required half degree*, as, that the number of the line, on the vernier, coinciding with the same numbered line on the limb, shall correspond to the number of minutes required. And, *to ascertain the number of degrees and minutes*, that there are in a given angle, observe where the broad arrow of the vernier is; if, between a full degree and a half degree, so many degrees and as many minutes, as are denoted by the number of the first division line of the vernier, (reading onwards as the degrees number,) that coincides with the corresponding division in the limb; or, if between a half degree and a whole one, so many degrees and 30 minutes, *plus* the broken number of minutes, as denoted by the coincidence of the corresponding lines of the vernier and limb.

The principle of the above subdivision of the vernier of Simms's theodolite, is very simple, and is universally applicable to any subdivision; viz., divide the value of the division, in the graduated scale, by the number of divisions in the vernier, and the quotient will be the value of each of these subdivisions.

Thus, in the theodolite the graduated division is 30 minutes ; the number of vernier divisions 30 ; the instrument (29 being subdivided into 30) reads to one minute.

In the sextant, the graduated division is 10 minutes, or 600 seconds ; the vernier subdivisions are 60 ; the reading of the instrument is to 10 seconds, (59 of these 10 minutes are subdivided into 60.)

In the circumferentor, in this country, the common graduation of the plate is to one degree, or 60 minutes ; the vernier subdivisions are 20, making the reading of the instrument only 3 seconds, (19 of these divisions being subdivided into 20) ; in this case, nine and a half on each side of the zero being divided into 10.

Hence, the value of the graduated division being given, and the extent of subdivision of reading required, to ascertain the number of requisite subdivisions.

By the rule above, where  $x + 1 =$  the required number of the subdivisions in the vernier, and  $x$  that of the graduated arc ;  $V$ , the value of the graduated division ; and  $v$ , that of the required subdivision, we have

$$\frac{V}{x+1} = v, \text{ or } x = \frac{V-v}{v}$$

thus in the sextant  $V=10$  minutes or 600 seconds ;  $v=10$  seconds

$$x = \frac{600-10}{10} = \frac{590}{10} = 59 \text{ divisions.}$$

Again, if, in the circumferentor,  $V = 1$  degree or 60 minutes ; and  $v = 3$  minutes,

$$x = \frac{60-3}{3} = \frac{57}{3} = 19 \text{ divisions of the plate, to be subdivided into } x+1, \text{ or } 20.$$

## CHAP. II.

## THE THEODOLITE.

*Its Application.*

HAVING given an explanation of the nature, adjustment, and method of using this Instrument, we will now proceed to show its application to the purposes of surveying.

It is an instrument calculated for extreme accuracy. It is an instrument that should always be used, when, as in the Ordinance Survey, quality and not quantity is the desideratum; when the correctness of the result, and not the rapidity of execution, is the object.

I would here beg to caution my young readers against falling in this profession, as is often the case in many others, into a mere system of exclusively advocating this or that particular system of surveying, whether of the Chain, Theodolite, Sextant, or Circumferentor; they are all useful in their way, and each, under certain circumstances, has the advantage; and there is scarcely any survey, of any extent, but what all of the three may advantageously be brought into use.

*In broad and extensive flats*, though the triangulation were better carried on by the chain, as the chain is indispensable for determining the cross hedges, yet the long lines of the triangulation must be run in by the theodolite; the common method of ranging by the poles would not be sufficiently accurate.

In broken and hilly countries, where the chaining could only be obtained by an application of the angles, taken by the theodolite, to the determining of the comparative lengths of the hypothenusal to the horizontal lines, this instrument is indispensable.

The correct length of one side of a triangle, together with the measures to minutes, half-minutes, obtained with the accuracy of which a good theodolite is susceptible, of its two adjacent angles, will always more certainly determine the position of a third point, when hills intervene, than the incorrectly measured distances of the two other lines. And, in both the above cases, the circumferentor,\* or the sextant, under certain limitations, can be advantageously introduced for the filling in.

There are two methods of using this instrument, generally adopted; the first by the *needle*: the second by the *back angle*.

The first, (*by the needle*,) I will briefly describe—The broad arrow of the vernier, and the zero point of the horizontal limb, are, by means of the adjusting-screw, made carefully to coincide; *always with the magnifying glass*; the needle is then released, and allowed freely to play upon its agate; and the whole instrument, with the two circles, firmly clasped together, turned round until the north end of the needle coincides, *as nearly as the eye can tell*, to the north point or zero in the graduated circle in the compass box. The whole is then clamped, and if in clamping, any error has arisen, it is carefully corrected by the large adjusting-screw (*T*). Now, if the two plates be detached, and the vernier plate turned round to the object, the angle read by the vernier, will be the angle made at the station, between the first object and the north end of the needle.

If the vernier plate be again unclamped, and turned round to the second object, the vernier will, in this case, also denote the angle made between this second station and the same north point.

\* The third part will be devoted to the uses and application of this instrument.



The difference between the first and second reading will be the measure of the angle required.

This is the method of taking an angle *by the needle*.

It is, however, a method that I consider *very objectionable*. It is after all only an imperfect circumferentor. The graduation of the inner plate of the compass is too contracted, the diameter too small, and the needle too clumsy, to admit of any accuracy in the setting of it in the first instance.

I should certainly advise its being used but sparingly. The only case, when it may be used with advantages, is, in the surveying of roads, as the angle made at the commencement of a line, between the other end and the north point, can always be checked by the angle at the end of the line, made between the commencement and the south point.

The second method, *by the back angle*.—In this case, the instrument is placed at the second station, the bearing between the first and second being assumed as a base line, determined in position, and the angles are all based upon that line; thus, supposing the starting point to be A, the first station B, the base line AB, and the several stations C, D, E, F, &c., then the angle at B is that between A and C; the angle at C, that between B and D, and so on.

The disadvantage of this method is, that, as in the case of the division of a many-sided field into triangles, the error of each angle is not confined to itself, but is increased by the sum of the errors of all the preceding.

While, in the first method, by the *needle*, each angle is confined to itself, being the angle made between each line severally, and certain meridian lines, that are theoretically considered parallel. These meridian or North and South lines, however, depending upon so small a needle, and so confined a graduation as that of the theodolite, can scarcely be made practically parallel. For myself, I should consider, that there would be a greater risk of error in two consecutive readings of the needle, than would ever occur in the reading of the vernier, by the back angles.

And, in the first method, there are double the errors of plotting, to what there would be in the second. In the first, there are the errors of the reading of the angle, which is common to the second method, and then there is the error of the non-parallelism of the needle, which is peculiar to the first.

## CHAP. III.

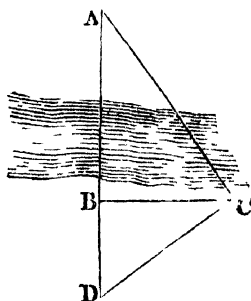
## HEIGHTS AND DISTANCES.

IN measuring a base line across a river, the following problem will be found useful.

## PROBLEM I.

Let DA be the direction of the line which has been measured up to the river. It is required to ascertain the distance BA at once upon the ground; so as to continue the measurement of the line.

On the line DA, take any point, B, whence a perpendicular, BC, can be taken, which will be free from obstruction, so that BC can be accurately measured; carry the range on across the river, and at A set up a flag; on BC, take any point, C, whence A can be seen; and at C erect a perpendicular to AC, intersecting the line AD in D.



Measure BD; then, because ACD is a right angle

$$\frac{BC^2}{BD} = AB, \text{ (Theorem 9, page 13.)}$$

These angles can either be taken by a cross-staff or by the chain, with the distances of 30 links, 40 links, and 50 links; 50 being the hypotenuse of a right angled triangle, when the base and perpendicular are respectively 30 and 40.

**EXAMPLE.**—Was engaged in the measurement of a base line, that unfortunately crossed a river, too wide for the chain—measured up to the river 261 chains 45 links. Sent a man across in a boat, with a flag to carry on the range, and to plant the flag in the line on the other side. At 261 chains, at right angle with the base line on the right, measured 4 chains 50 links to the water's edge, whence only, on account of the trees near the river's side, the flag was visible.

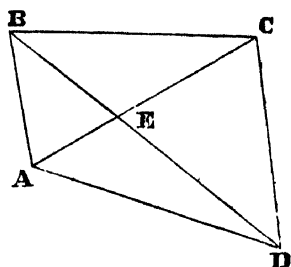
At this point, at right angles with an imaginary line to the flag, measured to a point on the base line, which on trial I found to be 259 chains 25 links: required, the width of the river?

Width of river, 11 chains 57 links.

## PROBLEM II.

*To determine the area of a quadrilateral figure, whose diagonals are known, and the angle between them.*

Let ABCD be the quadrilateral, having the diagonals AC and BD, and the angle AEB given. Let  $\angle AEB = \theta$  and diagonals  $= D$  and  $d$ , viz.,  $AC = D$ , and  $BD = d$ .



Now, the area of  $\triangle AEB = \frac{D}{2}(EB \sin. \theta)$

and of  $\triangle CED = \frac{D}{2}(ED \sin. \theta)$

The area of the two therefore  $= \frac{D \sin. \theta}{2} (EB + ED) = \frac{D \cdot d \sin. \theta}{2}$

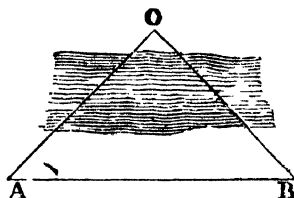
**EXAMPLE.**—Placed, the theodolite in the middle of a field, and taking the angle between its diagonals, 56 deg. 28 min., measured therefrom to the 4 corners, 6.80; 8.27; 5.49; 7.34 chains, what is the area of the field?

	A.	R.	P.
Ans.	7.	3.	39.

## PROBLEM III.

*To measure the width of a river by a base line alongside of it.*

Take at either end of the base line, with a theodolite, the angle made between the base line and a flag placed at the edge on the other side of the river. Compute the length of the sides by the first of the three cases of trigonometry.



Then  $AO, \text{nat. sine } \angle A = x$ , the perpendicular width of the river.

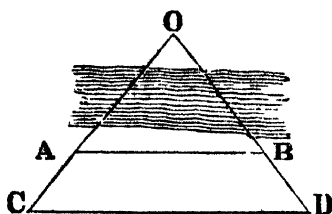
**EXAMPLE.**—Took a base line by the side of a river, 12 chains, and observed the angles at its ends to the flag on the other side, found them  $25^\circ$  and  $55^\circ$ : what is the perpendicular width?

Ans. 4.22. chs.

## PROBLEM IV.

*To find the distance of one object from another, where a river divides them, without using the theodolite.*

Let O and A be the given objects, and AO the distance required. From A draw AB, at any angle to AO, and produce OA to C, measuring AC about  $\frac{1}{2}$  the length of AB; from C measure CD, parallel to AB;\* and such that OBD may be in one straight line.



Then, because AB and CD are parallel, the triangles are similar, and therefore  $CD : AB :: CO : AO$ ; and  $CD - AB : AB :: CO - AO$ ; or  $:: CA : AO$   $\therefore AO = \frac{AB \cdot CA}{CD - AB}$ , the distance required.

**EXAMPLE.**—Took a line, 6 chains, alongside a river, and having had a flag placed on the other side, in the direction I was desirous of going, measured in a range with it from one end of the line, 4.50 chains, then took a second line parallel to the former, 8 chains, to such a point that I covered the flag and the second station of the first line.

What is the width of the river in the required direction?

*Ans.* 13 chains 50 links.

### PROBLEM V.

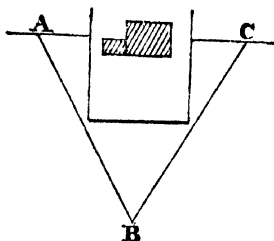
*To find the distance between two places, not visible from each other, but visible and accessible from a third point.*

Chain the length of the sides; take the included angle by the theodolite. The solution comes under the second of the three cases.

### PROBLEM VI.

*To continue the measurement and direction of a given line, when any obstacle stands in the way, which cannot be crossed, but can be avoided by going to the right or the left.*

At any point (A), on the given line AC, take an angle with the given line of  $120^\circ$ , if you would turn to the left, or  $240^\circ$ , if to the right, as in this case, and proceed measuring to B, till an angle of  $60^\circ$  made with this line, towards the first line AC, will carry you clear of the obstacle.



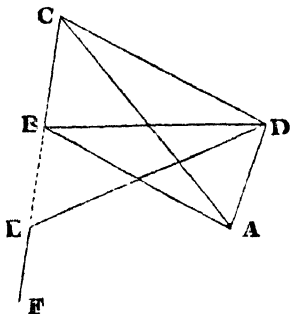
\* To make CD parallel to AB: at A and B erect equal perpendiculars, which can be done by means of the unit proportion of the sides of a right angled triangle, viz., 3, 4, and 5, or any multiples of them whatever.

Take this angle,  $ABC$ , 60 degrees, and measure  $BC$  the same distance as  $AB$ ; the point  $C$  shall be in the given line, and  $AC$  shall be equal to  $AB$  or  $BC$ . By taking an angle of 240 degrees with the line  $BC$ , the range of the line can be continued.

### PROBLEM VII.

*It is often desirable to be able to produce a given line  $BC$ , which is inaccessible and invisible at the required point of production  $B$ .*

Take any stations,  $A$  and  $D$ , whence  $B$  and  $C$  are visible, and at  $A$  and  $D$ , take several angles required (see Chap. VI.), on the determining of the length of a new base line, by angles taken from an old line,) and determine  $BC$ ,  $CD$ , and the angle  $BCD$ : through  $D$ , draw  $DE$ , at any angle with  $DC$ .



Now, because the line  $DC$  and the angle  $ECD$  have been computed, and  $CDE$  has been assumed, the triangle,  $DEC$ , comes under the first case, and the angle  $CED$  becomes determinable, as well as the required length of  $DE$ ; measure this computed length to  $E$ ; and at  $E$  lay off  $EF$ , making the angle  $DEF$  equal to the supplement of the computed angle  $CED$ .

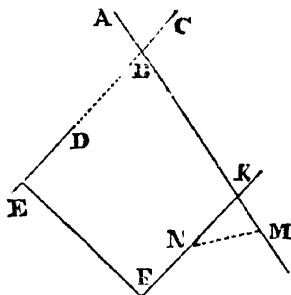
**EXAMPLE.**—Being desirous of continuing an inaccessible line on the opposite side of a river, I measured a base  $AD$ , of 12 chains 50 links, and took the several angles made at each end of it, between this new base and two points that were visible on the given line, viz., at  $A$ ,  $50^\circ$ ,  $37^\circ$ ; and at  $B$ ,  $85^\circ$ , and  $126^\circ$ . I also measured, from the end of the new base line, nearest the river, another line, making an angle of  $52^\circ$ , with the new base. What must be the length of this line, and what the angle between this line and the production of the given line?

### PROBLEM VIII.

*To measure the angle made between two inaccessible lines, as the angle of a fort, or the salient angle of a bastion.*

Let  $B$  be the salient angle of the bastion  $ABC$ , whose faces are  $AB$  and  $BC$ . It is required to measure the angle  $ABC$ .

Take any point D, out of musket shot, and produce DE in a line with the face CB. At E, erect EF perpendicular to EC, about equal to ED, and also the perpendicular FK, such that AB, the other face, and K, shall be in the same straight line; produce the direction BK, to any point M. Now, because DEF and KFE are two right angles, EB and FK are parallel, and, therefore, the outward angle FKM is equal to the inward opposite angle EBM, which is equal to the required angle ABC. Measure, therefore, the angle FKM, by the theodolite or the chain, and you obtain the measurement of the required angle.

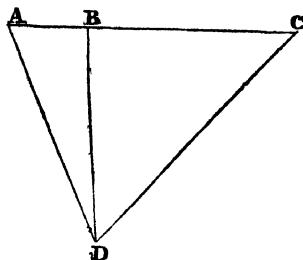


## PROBLEM IX.

*It sometimes happens in determining the position of two objects, (visible, but inaccessible from each other,) that both together are also inaccessible from any new station.*

The only plan that can, in such cases, be adopted, is to select that station, at which the angle between them can be taken by the theodolite, and whence the most favourable lines, to clear the obstructions, can be drawn on either side, as *new bases*, to determine the lengths of the unmeasurable sides, that include the angle taken.

*First.*—Let B and C be the given objects, whose distance BC is required. Let D be the station, whence B and C can be seen, and the angle BDC measured; but BD and DC cannot be measured.



Now, if the stations B and C are of such a kind, and so situated, that, though AB cannot be measured, the line of direction of CB can be produced to any point A, whence D can be seen; by taking the angles at A and D, and measuring AD, you obtain the triangle ABD, and, therefore, BD and the angle ABD, which being the outward angle to the triangle BCD,

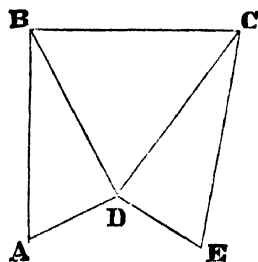
is equal to the angles BDC and BCD. BDC being known, BCD, therefore, becomes known, and BD is known; hence, the distances required become determinable, under the first of the three cases of trigonometry.

**EXAMPLE 1.**—Wanting to know the distance between two forts, on each side of the entrance to a harbour, I measured a base line, 35 chs. 20 links, along the beach, from a point on the beach in the produced range of the flags of the fort, 8 chains 30 links from the nearest flag, to another point, which was nearly opposite to the centre of the entrance, and found, at this second point, the angles made between this line and the two flags of the fort, to be  $8^{\circ} 29' 40''$ , and  $45^{\circ} 11' 20''$ .

*Secondly.*—When CB cannot be produced, it becomes necessary to consider DB and BC, as two new unknown distances required.

Select any points A and E, visible from D, and such, that B is visible from A, and C visible from E.

At D, take the angles ADB, BDC, and CDE; measure DA and DE; and, at A, take the angle BAD, and at E, the angle CED,



The triangles ADB and CDE, have one side and two angles given, and come under the first of the three cases.

Hence, BD and DC are determinable, and the triangle BDC, having now the two sides BD and DC, and the included angle known, comes under the second case.

**EXAMPLE 2.**—On another occasion, wishing to obtain the distance between two forts, similarly situated, at the mouth of a river, I found that, in consequence of the high ground in the rear, the line of direction could not be produced.

I was, therefore, under the necessity of adopting another plan. Placed my theodolite at the head of the harbour, and took the several angles, made between the flags of the fort and two new stations, whence these flags were also visible, viz., the angle ADB,  $58^{\circ}$ ; BDC,  $42^{\circ}$ ; and CDE,  $72^{\circ}$ ; measured to these stations, DA, 32 chains; DE, 35 chains; and at A, and E, found the angles to be  $83^{\circ}$  and  $86^{\circ}$ .

*Ans.* 1433 yds.





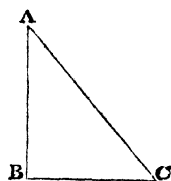
## H E I G H T S.

## PROBLEM I.

*To ascertain the height of an inaccessible object.*

Measure any distance from the base of the object, as nearly equal as possible to the height, and take the angle of elevation by the theodolite.

Let BA be the object; measure BC, and take the angle BCA, then BA is determinable by the case of right-angled triangles.



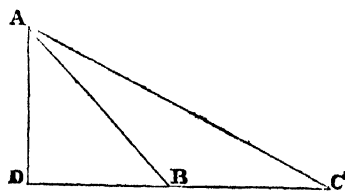
**EXAMPLE 1.**—What is the height of a tower, whose top, at the distance of 5 chains 75 links, subtends an angle of  $33^{\circ} 17'$ ?

If the object be inaccessible at the base, determine the distance of any favourable station from it, by one of the methods, described in the CHAPTER ON DISTANCES, and take the angle of elevation, as in the preceding example.

Thus, measure a base line, at a convenient distance from the inaccessible object; at each end of this line, take the horizontal angle between the other end of it and the given object, and at some point within the given line, measured from either end, take the angle of elevation.

**EXAMPLE 2.**—*Another method may, however, be more conveniently adopted, where practicable, viz :—*

From D, take any point C, in a line with DB; measure BC; and at B and C, take the angles of elevation. The angle DBA being the outward angle, is equal to the angle at C, + the vertical angle BAC, and BC is measured; therefore the triangle ABC, in the vertical plane, comes under the first case; and AB, being thus determined, and the angle ABD, at the base, being known, the triangle ABD is



These are merely the same figures in a vertical plane, instead of a horizontal one.

determinable by the case of right-angled triangles, and AD becomes known.

**EXAMPLE.**—Wanted the height of a tower, and the width of a moat around it, when the angle subtended by the top of it at the edge of the moat was  $64^{\circ} 20'$ ; and, at 4 chains 50 links off, was  $25^{\circ} 54'$ .

## PROBLEM II.

*To ascertain the height of a hill above the level of the Country, wherethe ground is so broken that a horizontal line cannot be measured, and the only angles, that can be taken, are from a point in the rising ground of another hill, which does not admit of measuring the horizontal distance from the object, and from the bottom of the first hill.*

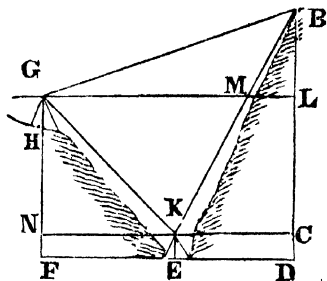
Let EBD be the hill, whose vertical height is DB. Let E and H be the two stations, for taking the vertical angles, in the same vertical plane with the object BD.

At E, take the vertical angle BKC, and from E, measure the distance HE along the slope of the hill: at H, take the angle of elevation BGL, and the angle of depression LGK, setting a flag KE, at the point E, equal to the height GH. The heights required are DB, and FG.

Now, in the triangle BKC, because the angle BKC is known, if KC or KB were known also, the triangle were determinable.

But in the triangle BGK, because  $\angle$  BGK is equal to the sum of the angles of elevation and depression, and because GN is parallel to LC, the angle MKC is equal to the angle GMK, which, being the outward angle, is equal to the angles BGM and GBM; that is, the vertical angle GBM is equal to the angle of elevation at K, minus the angle of elevation at G. The angle GBM thus being known, the triangle GBK has the side GK, and two angles, known, and is therefore determinable, and KB becomes known; and, therefore BC and KC and GN, is determinable, because GK is known, and the angle GKN equals the angle of depression, MGK. Add CD, the height of the theodolite, to obtain the height of BD.

The difference between the heights of the two stations, B and H,



is found by subtracting NH from CB, their respective heights above the common *datum* line NC.

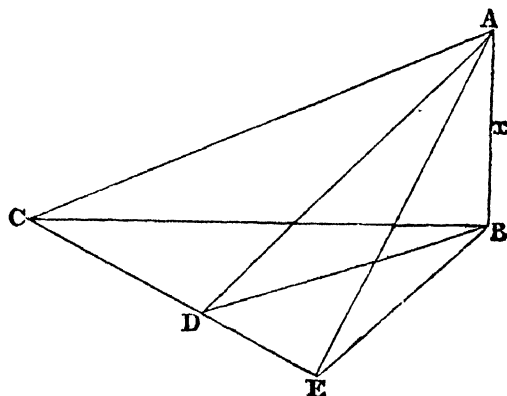
**EXAMPLE.**—Given at G, the angle of elevation BGL,  $12^{\circ} 15'$ , and the angle of depression LGK,  $42^{\circ} 29'$ ; the hypotenusal distance GK, 8.25; and, at K, the angles of elevation NKG,  $38^{\circ} 20'$ ; CKB,  $64^{\circ} 15'$ . Required the relative heights of the stations G and B.

### PROBLEM III.

*To determine the height of an inaccessible object by a sextant.*

Select three points *in a straight line*, whence a distinct view of the object can be obtained, measure their distance from each other, and take the angles of elevation of the object at these several points.

Let AB = the height =  $x$ ; and the angles ACB, ADB, and AEB, or the angles of elevation =  $\beta$ ,  $\gamma$ ,  $\delta$ , respectively, and  $cd = a$ , and  $DE = b$ ; also let the angle CDB =  $\phi$ .



$$\text{Now the cos. or angle CDB.} = \frac{a^2 + DB^2 - CB^2}{2a \cdot DB}$$

$$\left. \begin{array}{l} \text{and—cos. } \phi \\ \text{or the angle BDE} \end{array} \right\} = \frac{b^2 + DB^2 - BE^2}{2b \cdot DB}$$

$$\therefore b(a^2 + DB^2 - CB^2) = -a(b^2 + DB^2 - BE^2)$$

$$\text{but } DB^2 = \cot^2 \gamma \cdot x^2$$

$$CB^2 = \cot^2 \beta \cdot x^2$$

$$BE^2 = \cot^2 \delta \cdot x^2$$

$$\therefore b(a^2 + x^2(\cot^2 \gamma - \cot^2 \beta)) = -a(b^2 + x^2(\cot^2 \delta - \cot^2 \gamma))$$

$$x^2(\cot^2 \gamma \cdot b - \cot^2 \beta \cdot b + \cot^2 \delta \cdot a - \cot^2 \gamma \cdot a) = -(a b^2 + a^2 b)$$

$$x^2(b \cot^2 \beta + a \cot^2 \delta - (a+b) \cot^2 \gamma) = (a+b)ab$$

$$x = \sqrt{\frac{(a+b)ab}{b \cot^2 \beta + a \cot^2 \delta - (a+b) \cot^2 \gamma}}$$

Where the height equals the root of the product into the sum, divided by the sum of each distance into  $\cot.$  of its remote angle *minus* the sum of the distances into the  $\cot.$  of the middle angle.

When  $a = b$

$$x = a \sqrt{\frac{2}{\cot.^{\circ} \beta + \cot.^{\circ} \delta - 2 \cot.^{\circ} \gamma.}}$$

**EXAMPLE.**—Took three stations in the same straight line, at some distance from an object, whose height was required, and, by means of a pocket sextant, took the angles of elevation  $12^{\circ} 45'$ ,  $14^{\circ} 15'$ , and  $18^{\circ} 7'$ . The distances between the stations were 12 chains, and 15 chains. What is the height of the object ?

## CHAP. IV.


### SURVEYING BY THE THEODOLITE.

HAVING given the theories of solution of most of the practical cases, that can occur in the measurement of inaccessible heights and distances, by means of the theodolite, we will now proceed to explain the measurement of a field, by this instrument, and the method of keeping the field notes.

In the survey of a field, by the theodolite, as well as in the survey of roads, as explained in Chap. II., there are likewise two methods adopted: first, by selecting one of the *sides* as the base, and using the back angle; and secondly, by two stations, by making the largest diagonal line the base of a number of triangles, and computing the position of the several corners, considered as vertices of triangles, whose angles at the base are determined by the theodolite, at the two stations.

The annexed field notes refer to both methods.

FIELD NOTES No. 1, (*By the back angle.*)

from $\Delta$ 25·13 on 25·13	60·47	to 0 on base line 2200
from 18·76 on 25·13	9·83	to 487 on 487
from 487	571	to 1848 on 26·13
from 25·13	487	$\Delta$ to fence corner
25+0 $\Delta$	25·13	R 45+12
R 27+9	22·73	R 43+12
	18·48	$\Delta$
	13·76	$\Delta$
R 25+53+16	7 00	3+16
between 765 on 793	1·00	
at 1200	171°49'	and 25·13
17+38+16	1200	$\Delta$
R 18+48+6	11·90	
10+10	10·60	
	260	
between 709 on 790	1·50	20+10
at 705 on 793	199°39'	and 1200
to paling 25+20	793	18+11
	7·65	$\Delta$
	6·90	
	6·50	45 to house
	610	38 to house 
2	300	10+45
20	1·00	33
between 697 on 707		and 765 on 793
at 709 on 790	*	
R 12+32 $\Delta$	7·90	R
2	7·09	6 R
14+20	5·00	10+35
	1·50	
R 40+30	1·00	R 11
between 11·11 on 11·11	161°11'	and 709 on 790
at 697 on 7·07		
hedge—	7·07	—X
$\Delta$	6·97	12 to end of fence
	6·65	6+post+6
12+5	509	30
12+13	1·00	20
between 844 on 8·80	158·28	and 697 on 707
at $\Delta$ 11·11		

12+32	11.11	3
12+8	500	33
<sup>R</sup>		<sup>R</sup>
12+14	1.00	22
between 9.79	157.50.30	and 11.11
at 844 on 880		
<sup>P</sup> <sup>R</sup>		
12+37	8.80	
	8.44	△ 8+D
	5.80	22+7 to G. P.
	4.80	20+7 to G. P.
10+27	2.00	6
<sup>P</sup> <sup>R</sup>		<sup>R</sup>
to fence 10+32	1.00	0
between 447 on 447	104.6	and 844 on 880
at △ 979		
hedge—	9.79	—X
road—	9.30	—X
21	9.10	
	9.05	34 to wall —
11	8.84	
(12 wide) to style 17	6.93	
gate—	6.90	—X 20 to fence
to parish stone 15	6.76	
	6.40	27 to corner —
between 151 on 151	180.22	and 979
at △ 447		
	4.47	
other side—	2.05	—X
fence of enclosure—	1.30	—X
between 2188 on 2200	201.3	and 447 on 447
between 8.72 on 2200	1°58'30"	and 2188
at △ 151 on 151		
	1.51	△
<sup>D</sup> —	1.45	—X
other side—	1.30	—of hedge
road and hedge—	1.05	—X
paling and road—	62	—X
┌		
from 21.88 on 2200		
gate	22.00	post
9	21.88	△
49	18.50	
120	17.00	
23	10.00	
└ to fence 4	8.94	

BASE LINE

No. 2, (*By two Stations.*)

road—	14.42	to $\Delta$ G.		34.00	
H—	14.00	—X	D—	27.80	—X
to tree blazed 1	13.73	—X		27.66	$\Delta$
D—	13.72		path—	17.90	—X
	286	—X	D—	16.63	—X $\Delta$
	264	$\Delta$	D—	10.36	—X
	1.04	to stile	7	8 768	0 to gate post
from 16.63			10	600	+gate
sign post +	23.41	to $\Delta$ G	30	400	
D+16			30	200	
R			to hedge 100	000	
8+30+10	21.63	12 X D	from 473 on		
	21.06	21 to G.P.	473		
	2090	20 to G.P.	producing FL		
R	1900	6	D+5 p + 25	4.73	to a point in
D+28 x 30 + 20			D+10	200	FL.
	1800	0	path—	1.00	
	1700	6 + D	bridge + 1	54	—+
road +	1600		bridge	.53	
road + D—	15.40	—X	11+5+5	.31	
hedge—	15.35	—X	bank of stream	.30	—X
hedge—	1510	—X	from $\Delta$ B		
	94		producing D B		
	38				
	28		between $\Delta$ B	36° 43'	and $\Delta$ L
D—	5.02	—X	at $\Delta$ F		
to G.P. and 25	4.33		between $\Delta$ F	230° 10'	and $\Delta$ L
D+22	1.00		at $\Delta$ G		
D+21 path—	0.83	—X	D+28	1.86	to $\Delta$ G
	324.00	and $\Delta$ G	H—	0.20	—X
between 16.63	304.38'	and top of St. Paul's		270° 45'	and $\Delta$ ranging
at $\Delta$ D			between $\Delta$ C	215° 46'	with adproduce
	308° 23'	and $\Delta$ C	at $\Delta$ F		
	129° 10'	and $\Delta$ F	D+63	2.69	to $\Delta$ F
	127° 55'	and $\Delta$ (G) in Maiden Lane.	D+63	1.00	
	36° 17'	and $\Delta$ E	between $\Delta$ D	275° 14'	
between 2766	21° 10'	and $\Delta$ D	at $\Delta$ C		
at 16.63 on					
3400					
	350° 48'	and $\Delta$ F			
	324° 32'	Ball of St. Paul's			
	284° 51'	and $\Delta$ E			
	242° 22'	and $\Delta$ D			
between 1663	42° 40'	and $\Delta$ C, being			
at 27.66		11.97 or 1197			

N.B.—The plan of the first portion of these notes is purposely omitted.

The notes, in the first examples, are those of a survey of a road *by the back angle*. In the second example, they are those of a field surveyed *by two stations*.

The student will observe carried out, in these notes, the principle before recommended, of taking the angles always from left to right.

For instance, at station C, between station D, and station F, the angle is 275 degrees 14 minutes; that is, at the station C, the direction of F (CF), is, measuring from left to right, not only 180 degrees, but more than 270 ( $275^{\circ}$ , 14'), extending into the fourth quadrant of the circle. The angle, reading the other way, from right to left, might have been taken, or the instrument might have been set to F, making CF the base line. In the former case, the work would only have to be done in the field, which is now left to be done at home: in the latter, every station, making from left to right a greater angle than 180 degrees, must be made a new station; the entering of the field notes would be more complicated, and errors without number would result.

It is oftentimes requisite to take several angles at the same station, by considering the base line as fixed and constant, and determining the relative directions of the other points to this; the arrangement of the note becomes simple and clear. As, for instance, at 16.63 (see page 156), on line 3400, denotes that all the following angles (*reading upwards*) are taken *at that* station, with the theodolite unmoved. And between 27.66, on 3400, shows that the station, whence the angles are taken, is on the line between which they are measured; and the following lines, in the margin to the left, being left vacant, proves



that all the angles following are taken at the same station, 1663, and measured from the same line.

I have preferred the phraseology of ("between—and") to that of ("from — to"): there is less ambiguity about it; an angle is made *between* two lines, *at* a certain point.

The angles taken at this point are all of them, with the exception of the first, in the third or fourth quadrant. There is no possibility of error in this arrangement, and there is certainly nothing unmathematical in admitting an angle greater than 180 degrees.

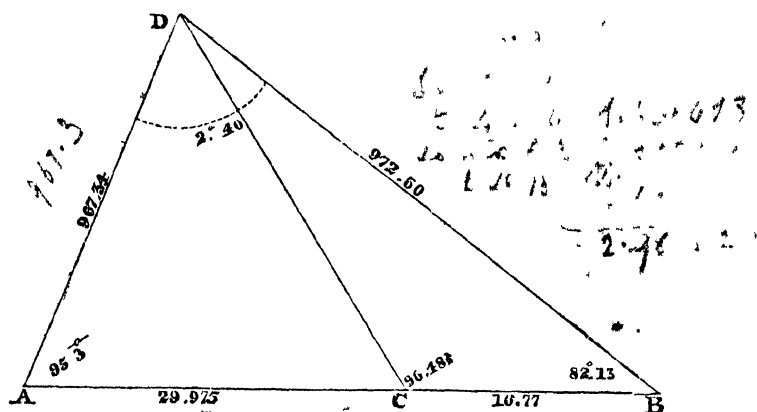
In taking several angles at a station, when it is only desired to measure along the thus ascertained direction of a new line, this angle should be taken last. and placed immediately before the chain distances. Thus, at  $\Delta$  F, the angle between  $\Delta$  C, and  $\Delta$  G, although nearer to the base line FC ( $215^{\circ} 46'$ ), is placed after that of  $\Delta$  F ( $270^{\circ} 45'$ ), because the following distances, 20 links, 1.86 links, are distances on the line FG (to  $\Delta$  G).

The field notes No. 2, (page 156,) are those of the survey by the theodolite of the same field, which was before surveyed by the chain, the plan of which is given at p. 89. The same points are determined in this case as then; but the offsets have been omitted as unnecessary.

I hope the student will understand, that I have not introduced the example of surveying (by the theodolite) the same field which was before surveyed by the chain, as conveying an opinion, that it is indifferent, which method is adopted under similar circumstances: far from it. I have merely given this, as an example of how it is to be done, should it be necessary. A theodolite would be quite out of place, in so small a survey as this.

## FIELD NOTES, No. 3.

Being notes of a survey, actually made at Blackheath, explaining the method of keeping the field book, and the practical arrangement and computation of the angles.



Between A on base line |  $83^{\circ} 10'$  | and Knockholt Beeches.  
at 16.77 on base line

The base here was so disproportionate to the unknown sides, that it became necessary to take a check-angle to prevent inaccuracy.

vertical angle	$0^{\circ} 22' 0''$	to top of Spire
348°30'	348° 30'	and Charlton Church.
337°0'		
vertical angle	$0^{\circ} 33'$	to Summit
162°29'	162° 29'	small bush right of large
324°58'		Tree on Forest Hill.
vertical angle	$0^{\circ} 33'$	to top of Spire
108°20'		
216°38'	$108^{\circ} 19'$	and Lee Church.
324°57'		
vertical angle	$0^{\circ} 30' 0''$	to Summit
82°12'		
164°26'	$82^{\circ} 13'$	and Knockholt Beeches.
246°39'		

9. 445780  
1. 668015

vertical angle	1° 30'	to top of Spire
53·2	53° 2' 20"	and Blackheath New Church.
106·5		
vertical angle	1° 15'	to top of Tower
21·29	21° 29' 20"	and pole at top of Sevendroog Castle.
42·58		
64·28		
between base line at $\Delta$ B		
345·33	345° 33'	and bush on right of large tree on Forest Hill.
123·37		
vertical angle	0° 36'	top of Spire of Church.
325·41	325° 41' 20"	and Lee Church.
83·54		
202·6		
vertical angle	2° 8' 0"	to top of Spire
284·46	284° 45' 40"	and Blackheath New Church.
209·31		
vertical angle	0° 34'	to Summit
264·57	264° 57'	and Knockholt Beeches.
169·54		
vertical angle	1° 42'	to top of Turrets
207·29	207° 29'	and Sevendroog Castle.
54·54		
between base line AB at $\Delta$ A		
	46·56	to $\Delta$ A
	49·71	to 14·43 on 14·43
	10·77	$\Delta$
from $\Delta$ B		
the distances were		

*Correcting for the 2½ inches.*

This chain had been previously carefully measured at the standard chain length at Somerset House, and was 2½ inches too long.

46·70½	to $\Delta$ A
40·83	to 1443 on 1443
16·82	$\Delta$

from  $\Delta$

The chain, which was used the first day, having been mislaid, another chain was used for the second measurement.

*End of first day's work.*

from 1400 on 3952	30·46	to 5·82 on 47·31
from 585	7·40	to 600 on 3952
	47·51	to 585 on 585
	46·51	$\Delta$ B

from $\Delta$ A, by side of Shooter's Hill Road.	38.98	$\Delta$ by side of road
	5.82	to $\Delta$ 14.43 on 1443
from S. E. corner of park, ranging with easterly wall.	14.43	$\Delta$
	1.35	to 39.52
side of —	38.52	$\Delta$
	38.57	—road
to wall 37	30.00	
3	20.00	
	14.00	$\Delta$
to park gate 0	13.98	
	6.00	$\Delta$
0	1.96	
to wall 4	1.27	
from 65 on 585		along wall of park
$\Delta$	5.85	
in a line with	0.65	park wall on left
to wall 0	0.62	

from S. W. corner of Greenwich Park, at Blackheath, near the Princess Augusta's.

*To find the value of this check-angle by computation.*

The base AB being given, 46.56, and the angles at the base  $95^{\circ} 3'$ , and  $82^{\circ} 13'$ , the other two sides were found to be 967 chains 34 links, and 972 chains 60 links.

Now, in the triangle ADC, we have the sides AD, AC, and the included angle at A; hence, by the second case of trigonometry, the other angles were computed, and that, at the base, ACD, found to be  $83^{\circ} 11' 12''$ , the supplement of which is  $96^{\circ} 48' 48''$ ; the measure of the angle DCB being only 12 seconds less, than the observed angle  $96^{\circ} 49'$ .

The length DB, being the distance from the Knockholt Beeches to  $\Delta$  B, was found to be 972 chains 60 links. To this was added the distance of B, from the S W. corner of the park, viz.,  $\sqrt{5.85^2 + 1.00^2}$ , or 5 chains 93 links, and the total distance, obtained thus, was 978 chains 53 links, or 12 miles, 1 furlong, and 8 chains 53 links. The distance on the ordnance map is somewhat less than 12 miles 2 furlongs.

## CHAP. V.

## TRIGONOMETRICAL PROBLEMS.

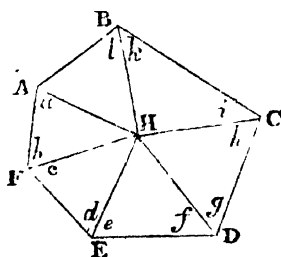
## PROBLEM I.

*Given the position of two known points, to determine the length of a new base line, by means of angles taken from its extremities with the given base.*

(Resolved analytically.)

Before proceeding to the solution of this Problem, it must first be demonstrated, that if any polygon be taken, and lines be drawn from a point, either within or without it, to the several angles of this polygon, the continued produced of the sines of one set of alternate angles, made by these lines, and the sides of the polygon, will be equal to the product of the sines of the other set.

Let H be the point when the lines HA, HB, HC, &c., are drawn to the angles of the Polygon.



Now, in the triangle AHF,

$$\frac{FH}{AH} = \frac{a}{b}; \frac{EH}{FH} = \frac{c}{d}; \frac{DH}{EH} = \frac{e}{f}, \&c.$$

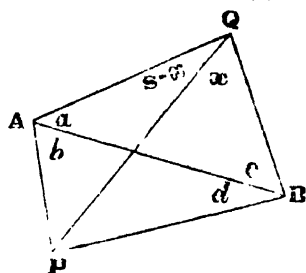
$$\frac{FH}{AH} \times \frac{EH}{FH} \times \frac{DH}{EH} \&c. \times \frac{AH}{BH} = \frac{\sin. a}{\sin. b} \times \frac{\sin. c}{\sin. d} \times \frac{\sin. e}{\sin. f} \times \frac{\sin. g}{\sin. h} \&c.$$

$$\text{and } \therefore \frac{\sin.a. \sin.c. \sin.e. \&c.}{\sin.b. \sin.d. \sin.f.} = 1, \text{ or}$$

$$\sin.a. \sin.c. \sin.e. \sin.g. = \sin.b. \sin.d. \sin.f. \&c.,$$

The same result would obtain, where *H* is *without* the Polygon.

Let *QP* be a line determined in position; having selected two stations *A* and *B*, it is required to find their distance from each other, this distance cannot be measured, but the angles *a*, *b*, *c*, *d*, can be taken by a theodolite.



Let  $s = AQB = (AQP + PQB)$ , or,  $180 - (a + c)$  [the other.  
 $(\sin. -b. \sin. s \overline{x. \sin. (c+d)} = \text{one set}; \sin. (\overline{a+b}) \sin. x. \sin. d =$

$$\sin. b. \sin. c+d. \sin. s-x = \sin. d. \sin. a+b. \sin. x.$$

$$\sin. b. \sin. (c+d) (\sin. s. \cos. x - \sin. x. \cos. s) =$$

$$\sin. d. \sin. \overline{a+b} \sin. x;$$

$$\text{dividing by } \sin. x.] \sin. b \sin. (c+d) (\sin. s \cot. x - \cos. s) =$$

$$\sin. d \sin. (a+b);$$

$$\sin. b \sin. (c+d). \sin. s \cot. x - \sin. b \sin. (c+d). \cos. s =$$

$$\sin. d \sin. \overline{a+b};$$

$$\therefore \sin. b \sin. (c+d) \sin. s \cot. x =$$

$$\sin. b \sin. (c+d) \cos. s + \sin. d. \sin. \overline{a+b}.$$

getting rid of the coefficient of the cot. *x*; we have

$$\cot. x = \cot. s + \sin. d. \sin. \overline{a+b}. \operatorname{cosec}. b. \operatorname{cosec}. \overline{c+d}. \operatorname{cosec}. s.$$

This will give the value of *x*.—

$$\text{Now } \sin. s : AB :: \sin. a : QB;$$

$$\therefore AB = \frac{\sin. s}{\sin. a} QB = \operatorname{cosec}. a. \sin. s. QB;$$

$$\text{and } \sin. (x+c+d) : QB :: \sin. (c+d) : PQ;$$

$$\therefore \sin. \frac{(x+c+d)}{\sin. (c+d)}. PQ = BQ.$$

and, therefore, by substituting this value of *QB*, we have

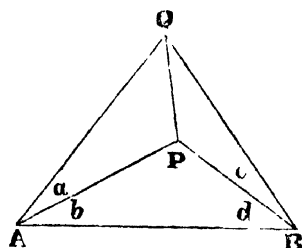
$$AB = \operatorname{cosec}. a. \sin. s. \operatorname{cosec}. (c+d). \sin. (x+c+d). PQ;$$

$$\text{and } \cot. x = \cot. s + \sin. d. \sin. (a+b.) \operatorname{cosec}. b. \operatorname{cosec}. (c+d). \operatorname{cosec}. s.$$

Now  $(a+b)$  and  $(c+d)$  are the angles opposite *PQ*, therefore,

## PROBLEM. II.

If the point, P, should fall within the triangle, and it is required to find AB as before.



We shall have

$$\cot. x = \cot. s + \sin. d. \sin. a. \operatorname{cosec}. b. \operatorname{cosec}. c. \operatorname{cosec}. s.$$

$$\text{when } s = 180 - (\text{the angles } QAB + QBA)$$

$$\text{otherwise} \quad = \quad 180 - (a + b + c + d)$$

$$\sin. b. \sin. s - x \sin. c = \sin. d. \sin. x. \sin. a.$$

$$\sin. b. \sin. c. (\sin. s. \cos. x - \sin. x. \cos. s) = \sin. d. \sin. x. \sin. a.$$

$$\sin. b. \sin. c. (\sin. s. \cot. x - \cos. s) = \sin. d. \sin. a.$$

$$\sin. b. \sin. c. \sin. s. \cot. x = \sin. d. \sin. a + \sin. b. \sin. c. \cos. s.$$

$$\therefore \cot. x = \frac{\sin. b. \sin. a.}{\sin. b. \sin. c. \sin. s.} + \frac{\cos. s.}{\sin. s.} (\cot. s.)$$

$$\text{and } \therefore \cot. x = \cot. s + \sin. d. \sin. a. \operatorname{cosec}. b. \operatorname{cosec}. c. \operatorname{cosec}. s.$$

$$\text{And the required } AB = \operatorname{cosec}. (a + b) \sin. s. \operatorname{cosec}. c. \sin. (x + c) = \operatorname{cosec}. (\text{opp. } \angle \text{ to } BQ), \sin. s. \operatorname{cosec}. (\angle \text{ opp. to } PQ) \sin. (x + QBP) PQ$$

$$\text{For } \sin. s : AB :: \sin. (a + b) : BQ$$

$$\therefore AB = \sin. s. \operatorname{cosec}. (a + b). BQ$$

$$\text{and } \sin. c : PQ :: \sin. (x + c) : BQ$$

$$\therefore BQ = \sin. (x + c) \operatorname{cosec}. c. PQ$$

and therefore, by substituting the value of BQ, we have

$$AB = \operatorname{cosec}. (a + b) \sin. s. \operatorname{cosec}. c. \sin. (x + c) PQ$$

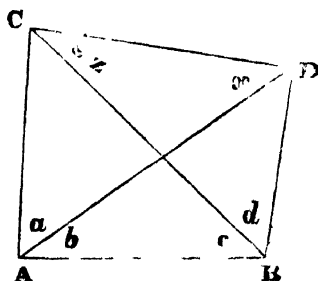
And, whether P fall within or without the triangle ABQ—

the required distance AB will always be

$$AB = PQ (\operatorname{cosec}. BAQ. \sin. AQB. \operatorname{cosec}. QBP. \sin. QPB.)$$

## PROBLEM III.

Let CD be given in position, it is required to obtain a *correct base* AB, the angles  $a, b, c, d$ , being taken by the theodolite.



Now let  $b+c=s=CEA=EDC+ECD$  and let  $EDC=x$ ,  
therefore  $DCB=s-x$

Now

$$\sin. x. \sin. c. \sin. ADB. \sin. (s-x)=$$

$$\sin. b. \sin. d. \sin. x. \sin. ACB.$$

but  $\sin. ADB=\sin.$  of supplemental  $\angle (b+c+d).$

and  $\sin. ACB=\sin.$  supplemental  $\angle \sin. (a+b+c).$  (see p. 162)

$$\therefore \sin. a. \sin. c. \sin. (b+c+d). \sin. (s-x)=$$

$$\sin. b. \sin. d. \sin. x. \sin. (a+b+d).$$

$$\text{or, } \sin. b. \sin. c. \sin. (b+c+d). (\sin. s. \cos. x - \sin. x. \cos. s)=$$

$$\sin. b. \sin. d. \sin. x. \sin. (a+b+c).$$

dividing by  $\sin. x) \sin. a. \sin. c. \sin. (b+c+d). (\sin. s. \cot. x - \cos. s)$

$$= \sin. b. \sin. d. \sin. (a+b+c).$$

$$\text{or } \sin. a. \sin. c. \sin. (b+c+d). \sin. s. \cot. x =$$

$$\sin. b. \sin. d. \sin. (a+b+d) + \sin. a. \sin. c. \sin. (b+c+d). \cos. s$$

hence, by transposition and getting rid of, as before, the coefficient of  $\cot. x$ , we have,

$$\cot. x = \cot. s + \sin. b. \sin. d. \sin. (a+b+c). \operatorname{cosec}. a.$$

$$\times \operatorname{cosec}. c. \operatorname{cosec}. (b+c+d). \operatorname{cosec}. s.$$

whence  $x$  and  $s-x$  are known.

Then as  $\sin. (a+b+c) : AB : \sin. (a+b) : BC$

$$\therefore AB = \sin. (a+b+c). \operatorname{cosec}. (a+b). BC.$$

again as  $\sin. (d+s-x) : BC :: \sin. d : DC$

$$\therefore BC = \sin. (d+s-x). \operatorname{cosec}. DC.$$

$$AB = \sin. (a+b+c). \operatorname{cosec}. (a+b). \sin. (d+s-x). \operatorname{cosec}. (d). DC.$$

$$= \operatorname{cosec}. (a+b). \sin. (d+s-x). \operatorname{cosec}. d. \sin. (a+b+c). DC.$$

$$\therefore AB = \operatorname{cosec}. CAB. \sin. CDB. \operatorname{cosec}. CBD. \sin. ACB. DC.$$

or, by using the former letters of P for D, and A for C,

$$AB = \operatorname{cosec}. QAB. \sin. QPB. \operatorname{cosec}. QBP. \sin. AQB. QP$$

Which is the same result as the two preceding cases, where the relative positions of the given and required bases materially differ from the present.

Hence, if the distance between two objects be correctly known, by previous surveys, and a **BASE LINE** be required, the actual measurement of which cannot be accurately depended upon, at the same time that perfect accuracy is indispensable in determining it, as it is to become the base of other triangulations, by selecting two stations, from each of which the two given objects and the other station can be seen, whatever may be the relative position of the given and required bases, the length of the *new base* can be always determined.



## CHAP. VI.

## PROBLEM I.

*Given the position of three known points, to ascertain the position of a new station, whence those points can be seen.*

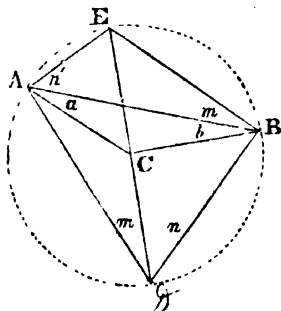
(Resolved geometrically.)

Having previously determined the position of three different places, A, B, and C, not in the same straight line, and being desirous of connecting with them a new survey, which is at some distance, but from which the three objects can be seen, it is required to determine the position of any point D, within the new survey, by means of angles taken at that point between the three given points.

Let A and B be the extreme stations; join AB, AD, and BD. Now the other given point, C, can either fall on the line AB, or within or without the triangle ABD.

1st. *Let it fall within*; about the triangle ABC; describe a circle, join DC, and produce to E; join AE and EB.

Let the  $\angle ADE = m$ , and the angle  $BDE = n$ , (in all the three cases,) then the angles ABE, BAE, will be also respectively equal to  $m$  and  $n$ , being upon the same segments of a circle, AE and BE.



Now, therefore, there are in the triangle AEB, the base AB, and the angles BAB and ABE, known.

AE and EB are thus determined.

Again, the sides AC, and AE, being known, and the included angle, the angle AEC can be obtained  $= d$ , which is also an angle of the triangle DAE, whose other angle ( $m$ ) at D has been taken, and therefore the angle DAE; or, because the angle CAE,  $(a+n)$  is

known, their supplement becomes known, and there are, in the triangle DAC, the angles ADC, and DAC, and the side AC, given to determine the sides AD and DC; and in the triangle DCB, you have now DC and CB, and the angle CBD, ( $n$ ), known.

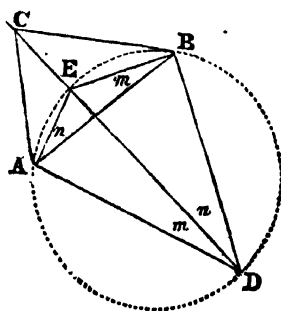
## PROBLEM II.

Now let  $C$  fall without the triangle  $ABD$ .

Describe a circle as before; join  $AE$ , and  $BE$ ; then the angle  $BAE = n$ , and  $ABE = m$ ; and the angle  $CAE = a - n$ ; and the angle  $EBC = b - m$ .

Determine as before the sides  $AE$ , and  $EB$ , and, having the sides  $AB$  and  $BC$  known, you have two sides  $AC$  and  $AE$ , and their included angle, whence to obtain the angle  $ACD$ .

The after process is the same.

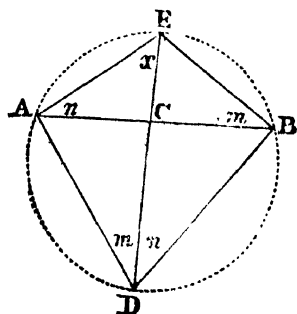


## PROBLEM III.

Lastly, let  $A$ ,  $B$ , and  $C$ , be in the same straight line.

Let  $\angle ADC = m$ , and  $CDB = n$ , as before, and describe round  $ABD$  a circle; produce  $DC$  to  $E$ , and join  $AE$  and  $EB$ .

$AE$  and  $EB$  are determined, and the angle  $E$ ;  $AC$  also is given; therefore the two sides  $CA$ ,  $AE$ , and the included angle,  $DAE$ , are known, and, therefore, the angle  $AEC$ ; and in the triangle  $AED$ , the angle  $DAE$ , the supplement of the other known angles is known also; whence, in the triangle  $ADC$ , there are two angles and a side known, and also in  $DBC$ . The required distances  $AD$ ,  $CD$ ,  $BD$ , are thence found.



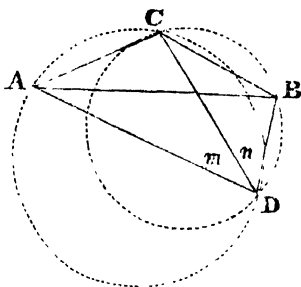
The above are examples of solution, when the student is not acquainted with analytical trigonometry.

*There is an easy method of constructing the figure, and determining upon paper the position of the required station.*

Let ABC be the given triangle, and D the given station, whence the angles ADC, and CDB are taken.

*To find the point D.*

Upon AC describe a segment of a circle, having an angle ( $m$ ) equal to ADC, and upon CB the segment of a circle, having an angle ( $n$ ) equal to CDB, the point of intersection of the circles shall be the station D required.



These cases may be proved analytically, if the student should prefer it, in the same way as the Problems in Chapter V.

*In the first case, where AB is given, and two angles  $a$  and  $b$ .*

Make  $s$  = the difference between the known angles and 180, and  $s$ ; and  $s-x$ , are the unknown angles required.

Equate the product of the sines of the alternate angles, as in Problem—, resolve the  $\sin. (s-x)$  into its equivalent values of the simple sines.

Arrange the known and unknown values on their proper sides of the equation.

Divide by the coefficient of  $\cot. x$ , and you obtain the value of one of the unknown angles.  $S-x$  = the value of the other. The sides are obtained by the formulæ, for the relative values of the sides, and the sines of their opposite angles.

*In the second case, where AC and CB are given, and the included angle, ACB.*

Let  $s = 360 \text{ degrees} - (\angle ABC + m + n)$ ; let the angle  $CBD = x$ , then the angle  $CAD$  will be equal to  $S-x$ ; ED is the common base of the two triangles.

Equate the value of ED in the two triangles, thus—

$$\begin{aligned} \text{As sin. } m &: AC : \text{sin. } s-x : DC \\ \text{as sin. } n &: BC : \text{sin. } x : DC \\ \therefore \frac{AC, \text{sin. } s-x}{\text{sin. } m} &= \frac{BC, \text{sin. } x}{\text{sin. } n} \end{aligned}$$

Or, AC.  $\text{sin. } n. \text{sin. } (s-x) = BC. \text{sin. } x. \text{sin. } m.$

AC.  $\text{sin. } n. (\text{sin. } s. \cos. x - \cos. s. \text{sin. } x) = BC. \text{sin. } x. \text{sin. } m.$

AC.  $\text{sin. } n. (\text{sin. } s. \cot. x - \cos. s) = BC. \text{sin. } m.$

AC.  $\text{sin. } n. \text{sin. } s. \cot. x = BC. \text{sin. } m. + AC. \text{sin. } n. \cos. s.$

and  $\therefore \cot. x = \frac{BC}{AC} \text{sin. } m. \text{cosec. } n. \text{cosec. } s + \cot. s.$

## CHAP. VII.

### TRIGONOMETRICAL SURVEY.

THE following field notes are given for examples to the Student. A base line was taken on Hampstead Heath, and angles were taken between it and the Churches of Highgate, Hampstead, and St. Paul's. There was also an angle taken to a Stone Monument, at some distance on the Heath, on the ascent of a hill, whence a second angle could be taken to St. Paul's, which was not visible from both stations on the *base line*.

The distance between St. Paul's and Highgate was subsequently calculated, and this distance being assumed as an *old line*, a new line was taken at Streatham, whence angles, at each end, were taken to this old line, for the purpose of ascertaining the relative position of the new line, preparatory to determining the relative position of the two surveys at Streatham and Hampstead.

### FIELD NOTES.

Between $\Delta P$ $\Delta Q$	$86^{\circ} 50' 0''$ $69^{\circ} 8' 27''$	and St. Paul's and Highgate Church
betwn. Highgate Church $\Delta P$	$108^{\circ} 27' 0''$ $19^{\circ} 48' 40''$	and $\Delta Q$ and St. Paul's

	<b>32.10</b>	$\Delta$ a
fence—	4.81	—x
side of Reservoir 5	4.42	
side of Reservoir 5	0.61	$\Delta$ p near new Church, opposite Crown and Sceptre

## AT STREATHAM.

$\angle$ elevation	0° 19'	ball of St. Paul's
	96° 2'	and St. Paul's
between $\Delta$ A	11° 36' 35"	and Highgate Church
at $\Delta$ c		
angles of { elevation	0° 18' 0"	of St. Paul's
{ elevation	3° 15' 5"	of Highgate Church
{ depression	3° 15' 0"	of bottom of hill on B. L.
	306° 3' 40"	and Stone
	293° 49'	and Hampstead Church
	181° 51' 40"	and St. Paul's
between $\Delta$ A	74° 47'	and Highgate
at $\Delta$ B		
	278° 11'	and Highgate Church
	94° 29'	and Stone
between $\Delta$ B	81° 0' 8"	and Hampstead Church
at $\Delta$ A		
from $\Delta$ A	24.74 chs.	to $\Delta$ B twice measured

## BASE LINE

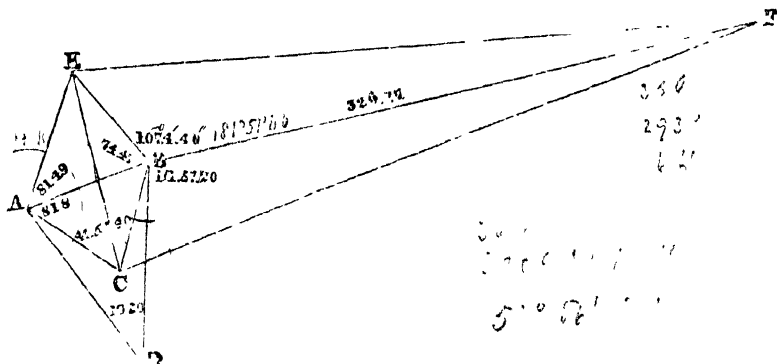
## CALCULATIONS OF ANGLES TAKEN AT HAMPSTEAD.

Let AB be the base line, 24 chains 24 links long; E, Highgate Church; D, Hampstead; F, St. Paul's; and C, the Stone Monument by the road side.

At station A the following angles were taken—to the Stone, 81° 8'; to Hampstead Church, 94° 29'; computing, always, from the base line towards the right. The first angle is the angle BAC; the second is the angle BAD; the angle CAD is equal to the difference of these two, or 13° 21'; the other angle is 278° 11', or an angle in the fourth quadrant; the angle BAE is the difference between this and 360° or 81° 49'.

At station B, the first angle taken, is to Highgate Church, ABE, 74° 47'; the second angle, to St. Paul's, 181° 51' 40", being the angle ABE, 74° 47' + the angle EBF, which is therefore 107° 4' 40". The next, to Hampstead Church, 293° 49', making the angle ABD

the difference between this and  $360^\circ$ , or  $66^\circ 11'$ . The next angle is that to the Stone, which is  $306^\circ 3' 40''$ , making the angle ABC,  $53^\circ 56' 20''$ .



The angles of elevation were taken, in case of their being wanted. And, lastly, at the Stone C, the angles were taken to Highgate Church and St. Paul's,  $11^\circ 36' 35''$ , and  $96^\circ 2'$ .

The distances required, were EF, BF, and CF.

Calculations are annexed, for the sake of example, of a more accurate method of calculation, than is required in this case.

#### CALCULATIONS OF ANGLES TAKEN AT HAMPSTEAD.

Arrange, systematically, the work to be done, by dividing it into triangles, and apportioning the proper lines to be determined in each.

In the triangle AEB, find AE and BE.

In the triangle ACB, find AC and BC.

In the triangle CAE, find CE.

In the triangle CBE, find CE.

Their equality proves the work.

In the triangle CBF, find CF and BF.

In the triangle EBF, find EF<sup>1</sup>.

In the triangle ECF, find EF<sup>2</sup>.

EF<sup>1</sup> and EF<sup>2</sup> must agree.

To find BE in the triangle AEB.

As sine $\angle E = 23^\circ 24'$	= 9.5989523
is to AB = 24.74 chs.	= 1.3933997
so is sine $\angle A = 81^\circ 49'$	= 9.9955552
	11.3889549
	9.9589523
to BE = 61.66 chs.	= 1.7900026

*To find AC, in the triangle ACB.*

As sine $23^{\circ} 24'$	=	<u>9.5989523</u>
is to AB = 24.74 chs.	=	<u>1.3933997</u>
so is sine $\angle A = 74^{\circ} 47'$	=	<u>9.9845004</u>
		11.3779001
		<u>9.5989523</u>
to AC = 60.1113 chs.	=	<u>1.7789478</u>

*To find BC, in the triangle ACB.*

As $\angle C = 44^{\circ} 55' 40''$	=	<u>9.8489368</u>
is to AB = 24.74 chs.	=	<u>1.3933997</u>
so is sine $\angle A = 81^{\circ} 8'$	=	<u>9.9947788</u>
		11.3881785
		<u>9.8489368</u>
BC = 34.61 chs.	=	<u>1.5392417</u>

*Again, to find AC in the triangle ACB.*

As sine $\angle C = 44^{\circ} 55' 40''$	=	<u>9.8489368</u>
is to AB = 24.74	=	<u>1.3933997</u>
so is $\angle B = 53^{\circ} 56' 20''$	=	<u>9.9076207</u>
		11.3010204
		<u>9.8489358</u>
to AC = 28.3195 chs.	=	<u>1.4520836</u>

*In triangle CAE, to find the unknown angles.*

As sum of sides (AC + AE), 88.43	=	<u>1.9465996</u>
is to their diff. = AC - AE, 31.79	=	<u>1.5202905</u>
so is tan. of $\frac{1}{2}$ sum unknown $\angle$ s E and C		
or $8^{\circ} 31' 30'$	=	<u>9.1757930</u>
		10.6780135
		<u>1.9465996</u>
to tan. of $\frac{1}{2}$ difference, $3^{\circ} 5' 4''$	=	<u>8.7314839</u>

*The angles being known, to find EC.*

As cos. of $\frac{1}{2}$ diff. of unknown $\angle$ s E and C or, $3^{\circ} 5' 4''$	=	<u>9.9993704</u>
is to cos. of $\frac{1}{2}$ sum, $8^{\circ} 31' 30''$	=	<u>9.9951749</u>
so is sum of sides (AC + AB), 88.4 chs.	=	<u>1.9465996</u>
		11.9417745
		<u>9.9993704</u>
to EC = 87.58 chs.	=	<u>1.9424041</u>

*In the triangle ECB, to find the unknown angles, and thence the side EC.*

As sum of sides (CB+BE) 96·27 chs. =  $\frac{1\cdot9834910}{1}$

is to difference (CB-BE)=27·05 chs. =  $\frac{1\cdot4321673}{1}$

so is tan. of  $\frac{1}{2}$  sum of unknown

$\angle s E+C = 25^{\circ} 38' 20'' = \frac{9\cdot6812000}{11\cdot1133673}$

$\frac{1\cdot9834910}{1}$

to tan.  $\frac{1}{2}$  diff. =  $7^{\circ} 40' 49'' = \frac{9\cdot1298763}{9\cdot9960862}$

And as cos.  $\frac{1}{2}$  diff. of angles =  $7^{\circ} 40' 49'' = \frac{9\cdot9960862}{9\cdot9549845}$

is to cos.  $\frac{1}{2}$  sum,  $25^{\circ} 38' 20'' = \frac{9\cdot9549845}{1\cdot9834910}$

so is sum of sides 96·27 chs. =  $\frac{1\cdot9834910}{11\cdot9384755}$

$\frac{9\cdot9960862}{1}$

to EC =  $87^{\circ} 58'$  as before =  $\frac{1\cdot9423893}{1}$

*In triangle CBF, to find CF and BF.*

As sine  $4^{\circ} 41' 40'' = \frac{8\cdot9129747}{1\cdot5392417}$

is to 34·61 chs. =  $\frac{1\cdot5392417}{9\cdot8911489}$

so is  $51^{\circ} 6' 20'' = \frac{9\cdot8911489}{11\cdot4303906}$

$\frac{8\cdot9129747}{1}$

to BF, 329·17 chs. =  $\frac{2\cdot5174157}{8\cdot9129747}$

And, as sine  $4^{\circ} 41' 40'' = \frac{8\cdot9129747}{1\cdot5392417}$

is to 34·61 chs. =  $\frac{1\cdot5392417}{9\cdot9175478}$

so is sine  $124^{\circ} 12' = \frac{9\cdot9175478}{11\cdot4567895}$

$\frac{8\cdot9129747}{1}$

to 342·80 chs. CF =  $\frac{2\cdot5438146}{1}$

*In triangle EBF, to find angle s.*

As sum or sides = 390·83 =  $\frac{2\cdot5919879}{2\cdot4273400}$

is to diff. = 267·51 =  $\frac{2\cdot4273400}{9\cdot8685922}$

so is tan.  $\frac{1}{2}$  sum  $\angle s = 36^{\circ} 27' 40'' = \frac{9\cdot8685922}{12\cdot2959322}$

$\frac{2\cdot5919879}{1}$

to  $\frac{1}{2}$  diff. =  $26^{\circ} 49' 42'' = \frac{9\cdot7039443}{1}$

$\angle BEF = 63^{\circ} 17' 22''$

$\angle BFG = 9^{\circ} 37' 58''$



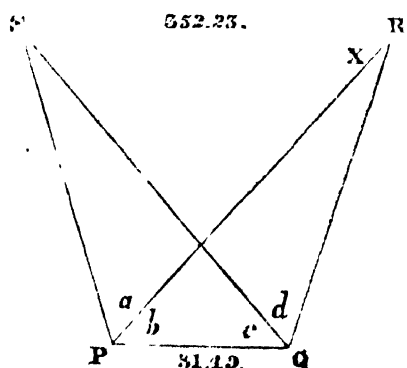
As cos. of diff.	$26^{\circ}49'42''$	$=$	$9.9505413$
is to cos. $\frac{1}{2}$ sum,	$36^{\circ}27'40$	$=$	$9.9053967$
so is sum of sides,	$390.83$ chs.	$=$	$2.5919879$
			$12.4973846$
			$9.9505413$
to $352.24$ chs. $=$ (EF)		$=$	$2.5468433$

*In triangle ECF, to find EF.*

As sine $14^{\circ} 19' 38''$	$=$	$9.3935204$
is to $87.58$ chs.	$=$	$1.9424041$
so is sine $84^{\circ} 25' 29''$	$=$	$9.9979409$
		$11.9403450$
		$9.3935204$
to $352.23$ chs. $=$ EF	$=$	$2.5468236$

*Calculations to determine the length and the relative position with the Hampstead Survey of the base line at Streatham.*

As it was doubtful, whether, in the observation taken at station Q, the angle PQS was taken to the proper object, from the extreme fogginess of the morning, which prevented the spire of the Church (Hampstead) from being distinguished from many others in the neighbourhood, though that observation, if to the right object, was, in itself, correctly and carefully taken, it was checked by assuming the distance PQ (taking its measured distance), as known, and *calculating* what the angle should be, between the proper object and the given base line.



To find the angle PQS, being the angle at the point Q, made between the base line and Highgate Church.

First find PR, in the triangle PQR.

$$\begin{array}{rcl}
 \text{As sin. } 4^{\circ} 31' 40'' & = & 8.8973097 \\
 \text{is to } 31.49 \text{ chs.} & = & 1.4981727 \\
 \text{so is sin. } 86^{\circ} 50' & = & 9.9993364 \\
 & & 11.4975091 \\
 & & 8.8973097 \\
 \text{to PR. } 398.29 \text{ chs.} & = & 2.6001994
 \end{array}$$

To find the angle PSR, in the triangle PRS.

$$\begin{array}{rcl}
 \text{As } 352.23 \text{ chs.} & = & 2.5468246 \\
 \text{is to sin. } 17^{\circ} 48' 40'' & = & 9.5300976 \\
 \text{so is PR } 398.29 \text{ chs.} & = & 2.6001994 \\
 & & 12.1302970 \\
 & & 2.5468246 \\
 \text{to sin. PSR, } 22^{\circ} 32' 4'' & = & 9.5834724
 \end{array}$$

To find PS in the same triangle.

$$\begin{array}{rcl}
 \text{As sin. } 19^{\circ} 48' 40'' & = & 9.5300976 \\
 \text{is to } 352.23 \text{ chs.} & = & 2.8283272 \\
 \text{so is sin. SRP, } 137^{\circ} 32' 16'' (42^{\circ}.20'.44'') & = & 9.8283972 \\
 & & 12.3752218 \\
 & & 9.5300976 \\
 \text{to PS, } 700.045 \text{ chs.} & = & 2.8451242 \\
 \text{As PS+PQ, } 731.53 \text{ chs.} & = & 2.8642311 \\
 \text{is to PS-PQ, } 668.55 \text{ chs.} & = & 2.8251339 \\
 \text{so is tan. } \frac{1}{2} s = 35^{\circ} 46' 30'' & = & 9.8576700 \\
 & & 12.6828039 \\
 & & 2.8642311 \\
 \text{to tan. } \frac{1}{2} d = 33^{\circ} 21' 27'' & = & 9.8185728
 \end{array}$$

$$\begin{array}{r}
 \text{the required angle } 69^{\circ} 8' 27'' \\
 \quad \quad \quad 2^{\circ} 34' 33'' \\
 \quad \quad \quad 108^{\circ} 27' 00'' \\
 \hline
 \quad \quad \quad 180^{\circ} 0' 0''
 \end{array}$$

which was the same as that observed by the theodolite.

The following calculations were made for the purpose of verifying the correctness of the several angles taken at the two stations P and Q, by comparing the computed with the measured length of PQ.

Let  $s=b+c$ , where  $b$  is the angle  $RPQ$ ,  $88^\circ 38' 20''$ , and  $c$  is the angle  $PQS$ ,  $69^\circ 8' 27''$ ;  $b+c$  being, together, equal to the unknown angles  $QSR$  and  $SRP$ .

Let  $SRP=x$ , then  $QSR$  will be equal to  $S-x$ .

Now, by Problem III., Chap. V., let  $s=b+c$

$\cot. x = \cot. s + \sin. b. \sin. d. \sin. (a+b+c). \operatorname{cosec}. a \operatorname{cosec}. c. \operatorname{cosec}. (b+c+d). \operatorname{cosec}. s$

$$\begin{aligned}\sin. b &= 88^\circ 38' 20'' = 9.9998774 \\ \sin. d &= 17^\circ 41' 33'' = 9.4827426 \\ \sin (a+b+c) &= 177^\circ 35' 27'' = 8.6236120 \\ \operatorname{cosec}. a &= 19^\circ 48' 40'' = 10.4699023 \\ \operatorname{cosec}. c &= 69^\circ 8' 27'' = 10.0294406 \\ \operatorname{cosec}. (b+c+d) &= 175^\circ 28' 20'' = 11.1026903 \\ \operatorname{cosec}. &= 157^\circ 46' 47'' = 10.4223198 \\ &\text{to rad. (10 Index)} = \overline{70.1305849} \\ &\quad -70\end{aligned}$$

$$(\text{to rad. 1.}) 1.3507831 = \overline{0.1305849}$$

again  $\cot. S = \cot. 157^\circ 46' 47''$  and being in the second quadrant  $= -\cot. 22^\circ 13' 13''$ .

$$-(\log. \cot. 22^\circ 13' 13'') = -(10.388892) \text{ to rad. 10.}$$

subtract 10

$$\begin{aligned}(\text{to rad. 1.}) -(2.4483819) &= -(0.388892) \\ &1.3507838 \text{ (see above)} \\ &\underline{-2.4483819}\end{aligned}$$

$$-(42^\circ 30') = \text{natural cot. } x = -1.0975971$$

$$\begin{array}{rcl} \text{rad. cot. } 42^\circ 31' &= 1.0977020 & 5971 = \text{given angle} \\ \text{rad. cot. } 42^\circ 30' &= 1.0970609 & 0609 = 42^\circ 30' \\ &6411 & \underline{5362} \\ & & 60 \\ & & \hline & 6411 \mid 311720 \mid 48 \end{array}$$

$$= \cot. -(42^\circ 30' 48'') = \text{its supplement } 137^\circ 39' 12'' = x$$

but  $PQ = \operatorname{cosec}. SPR. \sin. PSQ. \operatorname{cosec}. PQS. \sin. SRP (x)$

$$\text{viz. log. SR. } 352.23 \text{ chs.} = 2.5468246$$

$$\text{log. cosec. } 19^\circ 48' 40'' = 10.4699023$$

$$\text{log. sin. } 2^\circ 21' 33'' = 8.6336120$$

$$\text{log. cosec. } 69^\circ 8' 27'' = 10.0294405$$

$$\text{log. sin. } 137^\circ 39' 12'' = \overline{9.8284115}$$

$$\text{the length of PQ} = \log. 31.4912 \text{ chs.} = 1.4981909$$

## CHAP. VIII.

## TO REDUCE ANGLES TO THE CENTRES OF STATIONS.

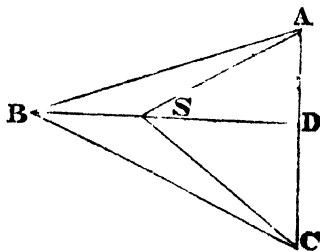
IN large trigonometrical surveys, as those objects, which, from their position, are more commanding, and are furthest visible, are generally so situated, as to prevent the theodolite being placed immediately in the centre, it has been found necessary to calculate expeditiously the angle of reduction to the centre, or the difference between the angle, as taken from a station (S) near the centre of this object, and the angle from the centre.

You have, for instance, determined the position of a church and tower, and taken angles to the vane of the church, or the pole of the tower, but on wishing to base upon the line connecting them a further triangulation, you find you cannot place your instrument at the centre of either, you are therefore compelled to take your angles as near to it as possible. This angle, thus taken, will fall either within this new triangle, or without, or upon one of its sides.

If it falls within, it will be greater than the true angle; if without the triangle, but within the circumscribing circle, it is greater; if upon the circumference, equal; if without it, less.

The amount of this excess or deficiency we are now to ascertain.

1st. Let it (S) fall within the triangle ABC; the angle taken will be  $\angle ASC$ .



To find its excess, or, to reduce it to ABC, produce BS to D.

$\angle ASD = \angle ABS + \angle SAB$ ,  $\angle CSD = \angle CBS + \angle SCB$ , and  $\angle ASC = \angle ABC + \angle BAS + \angle BCS$ ; but BC, and AB, and BS are known distances, and the angle BSC is known also.

Now, in the triangle BCS,

as  $BC : \sin. BSC :: BS : \sin. \text{angle } BCS$ ,

therefore  $\angle BCS = \frac{BS}{BC} \sin. BSC$ , and for the same reason,

$\angle SAB = \frac{BS}{BA} \sin. BSA$ , and, therefore,

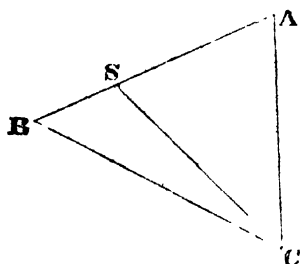
$$\angle ABC = \angle ASC - BS \left( \frac{\sin. BSC}{BC} + \frac{\sin. BSA}{BA} \right)$$

2nd. Let  $s$  fall upon the side AB,

then  $\angle ABC = \angle ASC - BS \left( \frac{\sin. BSC}{BA} \right)$

$$+ \left( \frac{\sin. 180^\circ}{BA} = 0 \right)$$

$$\therefore \angle ABC = \angle ASC - BS \left( \frac{\sin. BSC}{BC} \right)$$



3rd. Let  $s$  fall *without* the triangle ABC, and let E be the point of intersection of AS and BC.

Now the  $\angle AEC = \angle ABC + \angle BAS = \angle ASC + \angle BCS$ ,

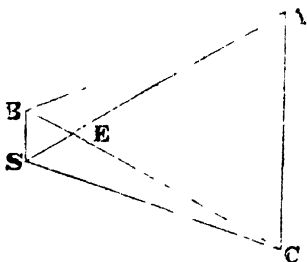
therefore  $\angle ABC = \angle ASC + \angle BCS$

$- \angle BAS$ ; but

$$\angle BCS = \frac{BS}{BC} \sin. (BSA + ASC)$$

$$\text{and } \angle BAS = \frac{BS}{AB} \sin. (BSA)$$

$$\therefore \angle ABC = \angle ASC + BS \left( \frac{\sin. (BSA + ASC)}{BC} - \frac{\sin. BSA}{AB} \right)$$



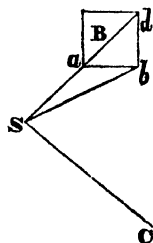
In the case where BC and AB are infinite, each of the expressions, where they occur, will vanish; and  $\angle ABC$  becomes equal to  $\angle ASC$ , which is the case when the objects are heavenly bodies.

This obtains also, when the station falls upon the circumference of the circumscribing circle.

It is not always possible, however, to measure BC, or take the angle CSB.

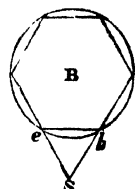
As it is desirable that the instrument should be placed close to the centre of the station, the vane of the steeple, or the flag pole of the tower, may be invisible from it.

When this is the case, if the centre be the vane of a church, as in the diagram, when the tower is squared, select  $S$  in a line with the diagonal. Measure  $Sa + \frac{a^2}{2} = SB$ ; and take the angle  $CSa = CSB$ .

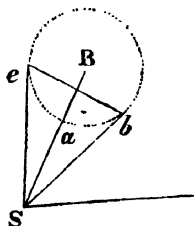


Again, let the tower be a hexagon.

Produce outward two of the sides to  $S$ , where place the instruments; ( $ebS$ ) is an equilateral triangle, and  $SB = eb\sqrt{3}$  \* and  $\angle CSB = \angle CSb + \frac{bSe}{2}$



If the tower be circular, the  $\triangle CSB = \angle CS + \frac{bSe}{2}$ , but  $\angle SB = bS \sec. \sqrt{\frac{bSe}{2}}$ . or if the circumference of the tower can be taken  $= Sa + \text{radius}$ .



#### EXAMPLES OF CALCULATIONS OF THE REDUCTION OF ANGLES TO THE CENTRE.

EXAMPLE 1.—Determining the angle made at the centre of the spire of Highgate Church, between the spire of Hampstead Church and St. Paul's, from angles taken in the Church-yard, between St. Paul's and Hampstead, and St. Paul's and Highgate.

\* The interior angle of a hexagon is  $120^\circ$ , the angle at  $e$   $120^\circ$ , and  $Be$  bisects it, and the  $\angle B eb$ , is  $60^\circ$ , but the angle  $S eb$  is also  $60^\circ$ , and therefore  $BS$  is bisected by  $eb$ , but each of its bisections  $= \tan. 60^\circ \times \frac{eb}{2}$ .

$\therefore$  The whole  $BS = \tan. 60^\circ \times eb$ , and, as  $\tan. 60^\circ = \sqrt{3} = eb \cdot \sqrt{3}$ !

from $\Delta$ s	2.56	to Highgate Church Spire
	<u>156.16'</u>	and Highgate Church
between St. Paul's	85° 25'	and Hampstead Church
at $\Delta$ s		

Let  $x$  = the unknown angle DEF.

Now, by the preceding formula,

$$\angle DEF + \angle EDS = \angle EFS + \angle DSF.$$

$$\therefore \angle DEF \text{ or } x = \angle DSF + \angle EFS - \angle EDS$$

but the sine of the angle EFS =  $ES \left( \frac{\sin. \angle ESF}{EF} \right)$

where EF is the distance from St. Paul's to Highgate Church ;

and the  $\sin. \angle EDS = ES \left( \frac{\sin. \angle ESD}{DE} \right)$ , where DE

is the distance from Hampstead to Highgate Church.

Hence the values of the angles EFS, and EDS can be obtained ; and, by adding their difference to the observed angle DSF, you obtain the value of the corrected angle DEF, from the centre of the previous point of observation.

To find the distances EF and DE, which are at present unknown, having the data given in the preceding chapter.

In the triangle ADB (see Diagram p. 171), to find the distance DB,

$$\text{As } \sin \angle ADB * (19^\circ 20') \quad = 9.5199112$$

$$\text{is to AB, } 24.74 \text{ chains} \quad = 1.3933997$$

$$\text{so is } \sin \angle DAB, (94^\circ 29') \quad = 9.9986691$$

$$11.3920688$$

$$9.5199112$$

$$\text{to DB} = 74.50 \text{ chains,} \quad = 1.8721576$$

In the triangle EDB, to find ED.

Having calculated the length of DB, we have two sides and the included angle, therefore—

$$\text{As sum of sides (DB+BE) } 74.50 + 61.66 \quad = 2.1340495$$

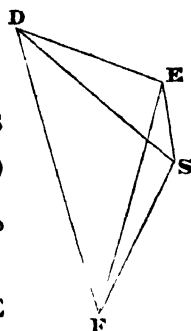
$$\text{is to their difference, } 74.50 - 61.66 \text{ chs.} \quad = 1.1085650$$

$$\text{so is tan. } \frac{(x+y)}{2} \quad 19^\circ 41' \quad = 9.5535477$$

$$10.6621127$$

$$2.1340495$$

$$\text{tan. } \frac{x-y}{2}, 1^\circ 30' 18'' \quad = 8.4194982$$



\* The angle ADB is the supplement of the observed angles DAB, and DBA.

$$\begin{array}{r}
 19^{\circ} 41' \\
 1^{\circ} 30' 18'' \\
 \hline
 21^{\circ} 11' 18'' = \text{greater angle DEB} \\
 18^{\circ} 10' 42'' = \text{smaller angle EDB}
 \end{array}$$

Again, in the same triangle,

$$\begin{array}{rcl}
 \text{As sin. } \angle \text{ EDB, } 18^{\circ} 10' 42'' & = & 9.4941207 \\
 \text{is to EB, 61.66 chains,} & = & 1.7900026 \\
 \text{so is sin. } \angle \text{ EBD, } 39^{\circ} 22' & = & 9.8022816 \\
 & & \hline
 & & 11.5922842 \\
 & & 9.4941207 \\
 \hline
 \text{to ED, 125.36 chains,} & = & 2.0981635
 \end{array}$$

Now substitute their proper values in the two equations, viz.,

$$\sin. \angle \text{ EFS} = \text{ES} \frac{(\sin. \angle \text{ ESF})}{\text{EF}}$$

$$\text{and } \sin. \angle \text{ EDS} = \text{ES} \frac{(\sin. \angle \text{ ESD})}{\text{DE}}, \text{ thus:—}$$

$$\begin{array}{rcl}
 \log. \sin. \angle \text{ EFS} & = & \log. \text{ES} + \log. \sin. \angle \text{ ESF} - \log. \text{EF}, \\
 \text{and } \log. \sin. \angle \text{ EDS} & = & \log. \text{ES} + \log. \sin. \angle \text{ ESD} - \log. \text{DE}, \\
 \log. \text{ES, 2.56 chs.} & = & 0.4082400 \\
 + \log. \sin. \angle \text{ ESF, } (156^{\circ} 16') & = & 9.6030166 \\
 & & \hline
 & & 10.0112566 \\
 - \log \text{EF,} & = & 2.5468246 \\
 \log. \sin \text{EFS } (0^{\circ} 10' 1'') & = & 7.4644320 \\
 \text{and } \log. \text{ES 2.56} & = & 0.4082400 \\
 + \log. \sin. \angle \text{ ESD } (70^{\circ} 51') & = & 9.9755394 \\
 & & \hline
 & & 10.3837794 \\
 - \log. \text{DE, 125.36 chains} & = & 2.0981630 \\
 \log. \sin. \text{EDS } (1^{\circ} 6' 22'') & = & 8.2856159 \\
 \text{but } \angle \text{ DEF or } x = \angle \text{ DSF} + (\angle \text{ EFS} - \angle \text{ EDS}) \\
 \angle \text{ EFS} & = & 0^{\circ} 10' 1'' \\
 - \angle \text{ EDS} & = & 1^{\circ} 6' 22'' \\
 \hline
 \text{their difference} & = & 56' 21''
 \end{array}$$

therefore  $85^{\circ} 25' (\angle \text{ DEF}) + (-56' 21'') = (84^{\circ} 28' 39') = \angle \text{ DEF} = x$   
 which is the angle at the spire of Highgate Church, between Hampstead Church and St. Paul's.

**EXAMPLE 2.**—Having measured the distances between two objects AB 2 miles, 6 furlongs, and taken the angles at the base line, to a third object, viz.,  $59^{\circ} 25' 40''$  and  $78^{\circ} 24' 20''$ . It was required to observe the remaining angle at the third station, as a check upon the other two angles; planted the theodolite at a distance of 3 chains from it, and found one of the angles  $80^{\circ}$ . What must the other be, so that the previous observations should be correct?



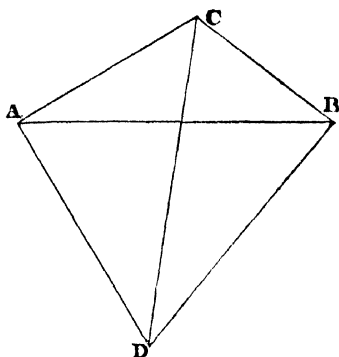
## CHAP. IX.

## ON TRIANGULATION.

IN extensive surveys, carried on by a continued system of triangulation, the most important part is the proper selection of a base line, proportionate to the intended extent of the survey. This base line should be measured on a nearly level surface of country. In hilly countries the plane of a valley is generally selected for that purpose.

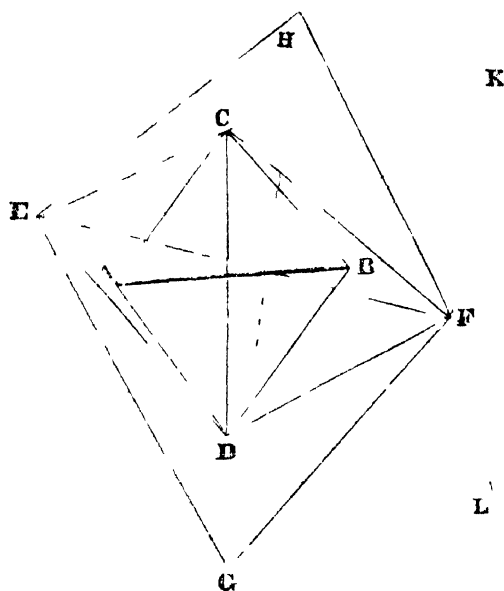
Let AB be the base, measured in a valley, and CD, two prominent objects, on neighbouring hills, which are visible from A and B.

Take at C, the angles ACD, BCD; and at D, the angles ADC, BDC; and the stations, C and D, are determined, by the formulæ given in Chap. V., Prob. I. DC thus becomes a new base line, of longer extent and equal accuracy with AB; or, by taking at A and B, when B is visible from A, the angles between the base, and C and D respectively; C and D can be more directly and more accurately determined, as in either triangle its length will be the same. The three angles should, in all cases, be taken, if practicable, as from their sum, which should be equal to 180 degrees, the accuracy of the work can be tested.



The sides of the triangle should be, as nearly equilateral as possible, or the angle at the new station should not differ, materially, from 90 degrees. All the sides should be calculated and plotted from their determined lengths, and not protracted from their angles, as the smallest error of an angle would be of injurious effect, in determining the position of the new station.

The accompanying diagram will exhibit the method of extending a system of triangulation, and of obtaining, between inaccessible stations, a base line, commensurate with the extent of the survey.



Let GH be the base line required, and inaccessible ; AB, the only favourable base line that can be measured.

By determining from AB the position of C and D, CD becomes a new station ; from CD determine E and F, EF again becomes a new base for the stations G and H, which, by observing the angles EHF and EGF, and comparing their calculated with their observed values, becomes a base, as much to be depended upon as the first AB.

It is not, of course, necessary that the triangulation should be carried on in the regular manner, exhibited in the figure, as it might be branched off in any direction that might be required ; for after having, from AB, determined C and D, CB might be taken as the new base, as correctly as DC, and the triangulation extended towards K ; or, after having determined E and F, it might be extended towards L ; and by a similar process, in any other direction whatever.

Having carried the triangulation in the direction, and to the extent required, it becomes desirable, for the sake of testing the accuracy of the work, to make one of the lines, of the last triangles, a *base of verification*, by selecting for it a level position, along the slope of a

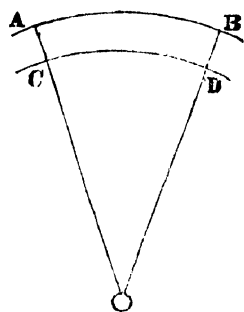
hill, or in the bottom of a valley. This base, as it can be computed trigonometrically, being compared with its actual length by measurement, is a test, not only of its own accuracy, but of all the various triangles that subserve to its determination.

As the triangulation goes on, the sides increase in length, and the angles taken are between objects of some miles distant. It becomes then imperative to call in the aid of science to make the objects distinct. For this purpose, various contrivances have been, at different times, adopted, such as plane mirrors, disks of tin, plane convex lens, parabolic reflectors, to receive the rays of artificial light, that were thrown upon them, by means of Argand lights, balls of burning lime, &c. These latter were introduced by Captain Drummond, who managed to send through a flame of alcohol, a powerful stream of oxygen gas, upon the lime, which was placed in the focus of a parabolic reflector.

By these means, angles have been taken to objects from 40 to 60 miles off.

#### REDUCTION OF A BASE LINE, TO THE LEVEL OF THE SEA.

Let  $CD$  be the length of the line at the level of the sea  $= x$ ,  $AB$ , the measured line above it. Let  $AC$  be the height of the measured line upon the level of the sea  $= h$ , and  $CO$  be the radius of the earth  $= R$ . It is required to find the length of  $CD$ .



As the arcs of different circles, subtending the same angle, are proportional to their radii, we have,

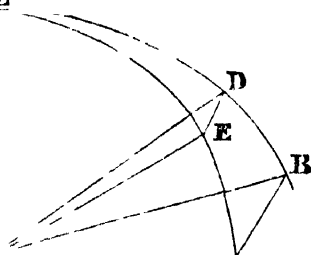
$$AO : CO :: AB : CD, \text{ or} \\ R+h : R :: L : x$$

$$\therefore x = \frac{R}{R+h} \cdot L; \text{ and } \therefore L-x = L - \frac{R}{R+h} \cdot L = \frac{h}{R+h} \cdot L$$

Or the excess of any line  $AB$ , measured above the level of the sea, is equal to the length of the line, into its height above that level, divided by its distance from the centre of the earth. Now the radius of the earth is 21,008,000 feet, therefore  $\log. L$  in feet  $+$   $\log. h$  in feet  $- \log. (21,008,000 + h)$  in feet  $= \log. \text{ excess of measured base above the true base at the level of the sea.}$  This base is the arc  $CD$ .

*To reduce angles, taken in a plane inclined to the horizon, to angles in the horizontal plane.*

Let DAE, the angle taken at the point D, between the objects D and E, be in a plane, inclined to the horizon. Let BAC be the horizontal plane, and Z the zenith of the observer at A, then AZEC, AZDB will be portions of planes, of large circles, passing through the radius AZ, and the horizontal angle will be BAC.



Let CAE be the angle of elevation of the station E, and DAB that of D, of which EC and EB are the measures; ZE and ZD are the zenith distances, or measures of the complements of these angles of elevation; and DE is the measure of the angle DAE.

In the triangle ZDE, there are ZE, ZD, and DE given, hence the sine  $\frac{1}{2}$  BAC or  $\frac{1}{2}$  DZE =  $\sqrt{\frac{\sin. \frac{1}{2} (S-ZE) \sin. \frac{1}{2} (S-ZD)}{\sin. ZE. \sin. ZD}}$   
 S, being the sum of the sides of the triangle ZED; or  $\sin. \frac{1}{2} Z$   

$$= \sqrt{\frac{\sin. \frac{1}{2} (\angle DAE + \angle BAD + \angle CAE) \cdot \sin. \frac{1}{2} (DAE + \angle CAE - \angle BAD)}{\cosine \angle BAD. cosine \angle CAE}}$$

If the angle BAD = angle CAE, or the objects be of the same altitude, then,

$$\sin. \frac{1}{2} Z = \sqrt{\frac{\sin^2 \frac{1}{2} DAE}{\cos.^2 CAE}} = \frac{\sin. \frac{1}{2} DAE}{\cos. CAE}$$

In the first case, when the angles of elevation differ slightly, and when each of them is but small, not exceeding  $2^\circ$  or  $3^\circ$ , as the cosines of angles vary slowly, the following formulæ may be safely adopted, viz.,  $\sin. \frac{1}{2} Z = \frac{\sin. \frac{1}{2} DAE}{\cos. (H+h)}$  when H and h are the respective angles of elevation, which becomes a convenient logarithmic formulæ, viz.,  $\log. \sin. \frac{1}{2} Z = 10 + \log. \sin. \frac{1}{2} DAE - \log. \cos. \frac{1}{2} (H+h)$ ; but the angle BAC is the measure of the angle Z, or the plane angle made between the two planes.

## SPHERICAL EXCESS.

The angles taken between any three points on the surface of the earth, by a theodolite, are, strictly speaking, spherical angles, and their sum must exceed  $180^\circ$ ; and the lines bounding them, are not the chords, as they should be, but the tangents to the earth.

This excess is inappreciable in common cases, but in the larger triangles it becomes necessary to allow for it, and to *diminish* each of the angles of the observed triangle, by one third of the spherical excess.

*To calculate this excess.*

Divide the area of the triangle in feet, by the radius of the earth in seconds, and the quotient is the excess, viz.,

$$\text{excess} = \frac{\text{area}''}{R^2} \text{ or,}$$

because the radius = as above, 21,008,000 feet, and one second of space = 101.43 feet, then  $101.43^2$  feet = the area in a square second, and radius = 206.264 seconds of space  $\therefore$  excess in seconds =  $\frac{\text{area in square feet}}{(101.43)^2 \times 206264}$  or,  $\log. \text{excess} = \log. \text{area} - 9.3267737$ .

*When two sides and the included angle are given.*

$$\cot. \frac{1}{4} \text{ excess} = \frac{\cot. \frac{1}{4} a - \cot. \frac{1}{4} b + \cos. C}{\cos. C}.$$

*When the three sides are given.*

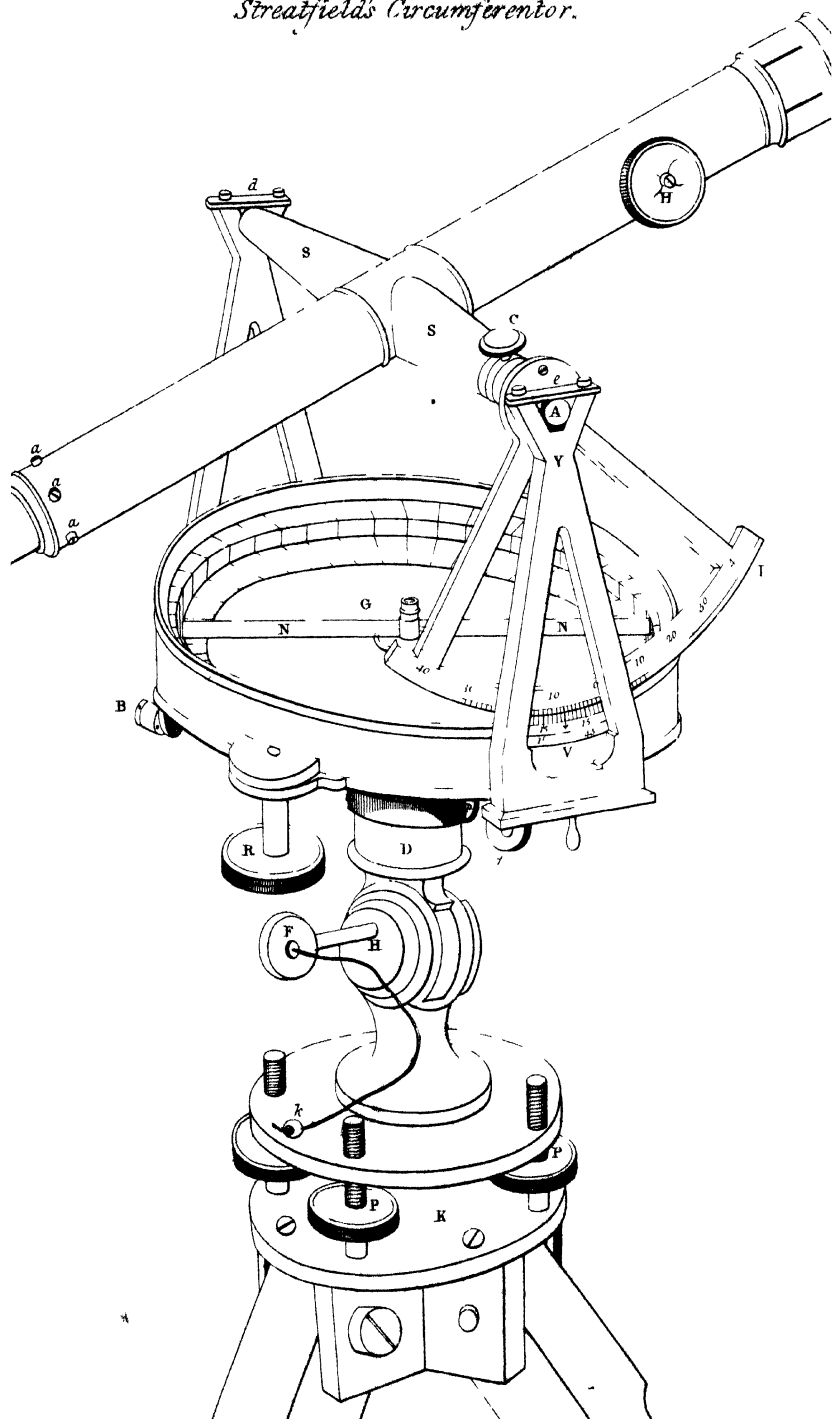
$$\tan. \frac{1}{4} \text{ excess} = \left( \frac{\tan. a+b+c}{4} \right) \cdot \left( \frac{\tan. (a+b-c)}{4} \right) \\ \left( \frac{\tan. a-b+c}{4} \right) \left( \frac{\tan. b+c-a}{4} \right)$$

*To allow for this excess.*

In any triangle take the three angles, find their sum. Calculate from these angles the spherical excess, by the above rule; the sum of the angles taken should amount to  $180^\circ$  + this spherical excess; if not, the difference must be divided among the three observed angles, so as to make them, when thus corrected, equal to  $180^\circ$  + the excess; then subtract one third this excess from each of the angles, and their sum will be reduced to  $180^\circ$ , the correct measure of a plane triangle.



*Streatfield's Circumferentor.*



# LAND SURVEYING.

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## Part the Third.

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### THE CIRCUMFERENTOR.

THE Circumferentor is the mariner's compass differently divided, and furnished with sights, standing upon one or three legs, and capable of a horizontal motion, by means of the usual parallel plates, or a ball and socket. In new countries, where expedition is required, the circumferentor is generally used with a ball and socket, and with one leg or staff, strongly shod with iron. After taking the angle at any station, the head part is taken off, and is carried by the Surveyor under his arm, the staff is seized in his hand, and the next station is proceeded to. The mould of the woods is soft, and easily penetrated; and as for the purpose of blazing\* the trees the general direction alone is wanted, which may be blazed within 10 yards on either side of the line, there is no delay arising from the placing of flags and driving of stakes, the axemen, being

\* Blazing, in American phraseology, is taking a slice off the bark of a tree into the wood.



generally in advance or close upon the heels of the Surveyor, and the chain men overtaking him, before he has taken his next sight.

All the *LINE* trees, as trees are termed, which stand directly in the line, in addition to being *blazed*, are *witnessed* with three notches, and being marked where the line strikes them, become permanent boundary marks—of so permanent and distinctive a character, that the very year almost of their being blazed is distinguishable by an experienced hand. The scar seldom grows over entirely: never without a seam, which, if carefully cut, will, by the number of rings, that every year has added to it, show the exact year of the incision.

The three legged staff, however, is very useful; especially in old countries, or in the surveying of roads in frosty weather, and under various other circumstances, when the single staff cannot penetrate, or when walls intervene, upon which the short legs of the staff can be advantageously placed.

#### STREATFIELD'S CIRCUMFERENTOR.

This instrument consists of a compass box (G); divided into degrees, and, by means of a vernier, subdivided into three minutes.

It stands upon three legs, and, by means of a pair of parallel plates (Kk,) is capable of a truly horizontal position, which is determined by a level, placed, so as not to interfere with the reading, under the compass box (the end of it alone can be seen in the accompanying plate at B).

This compass box, (G,) has an absolute horizontal

motion round its centering at D, and is fastened by the clamp screw (a side of which is visible) at *p*.

When this is clamped, by detaching the pin *f*, which passes through the two plates of the compass box, the brass one, which, with the vernier attached, works round the inner one, on which are divided the degrees, is capable of a relative motion, and thus partakes of the character of the theodolite; this motion is communicated to it by a rack and pinion at R.

There is, however, one thing wanting in this instrument, which is indispensable in a new country. There is no means of clamping the vernier, except when it is at zero of the dividing-plate. In running lines, of a given direction, through the woods of a new country, where the bearings are often odd minutes, it would be impossible, at every fresh station, to be altering the vernier.

In the circumferentors that are used in America, they have placed the vernier outside, and by that means have contrived to clamp in to any position; there is also a tangent-screw for fine adjustment, so that being properly adjusted for the odd minutes, or the variation of any kind, and clamped, the bearing of the full number of degrees need alone be referred to throughout the whole line. None but a practical man can be aware of the immense advantages resulting from this arrangement.

YY are two frames, or supports, capable of being taken on and off, on the Y's of which rests the arm (Ss) of a telescope, so contrived, as to move in a truly vertical position, when the instrument is horizontal. The telescope has its usual adjustments for the object and eye-glasses, and for the line of collimation.

To one end of the cross piece of the telescope is attached a graduated arc of a circle, with a vernier fastened to the supports of the Y, for taking vertical angles. This instrument has also, in addition to the telescope (not shown in the plate), the usual pair of sights, whose position would be, in the meridian line of the instrument, in the same plane as the telescope moves in.

The introduction of the telescope was forced upon me, from the constant inconvenience I experienced in the back woods of America, (where I was engaged in government surveys,) from the almost uselessness of the common sights, in surveying up and down hills, across steep but narrow vallies; all these difficulties were remedied by this contrivance, the advantages of which I have tested practically in the woods. The telescope is capable of a vertical motion either way of 45 degrees, and moving, truly vertically, enables you to carry a line of a given direction either down or up a hill, and from its magnifying power to secure, at the same time, a check position at the top of the ascent beyond; so that, after having descended and ascended, you may prove your correctness.

The instrument, also, has other contrivances; the pin F, when taken out, allows the whole instrument to be turned upon its side, and the spirit level at B, being then in a horizontal position, the instrument is made capable of a vertical motion, reading off to three minutes, by means of the vernier.

This may sometimes become a fair substitute for a sextant on a clear night; though I should myself, in all cases, prefer each instrument being kept to its

especial purposes, as, the more simple an instrument is, the more accurate.

#### DIVISION OF THE CIRCUMFERENTOR.

The line of sights is made the north and south end of the *instrument*, and from each of these the circular rim is graduated toward the east and west points, from  $0^{\circ}$  to  $90^{\circ}$ .

On the right of the north of the instrument, looking to the north point of it, should be lettered west, and on the left should be lettered east; and any point between the north and west point of the instrument, read by the north end of the needle, is read north, so many degrees west.

When the needle is released, and is allowed to play freely, it points toward the magnetic north. The north of the instrument points to the object whose bearing is required, the angle made between these two must necessarily give the relative position of the line of the object, and the magnetic north and south line; and the bearing of the object, by reading off the number of degrees to which the needle points on the graduated circular rim, is thus obtained.

As the needle points to the north, should the object bear to the west, the line of sights would be on the left of the needle, and the north end of the needle would be on the right of the line of sights; if, therefore, on the right of the instrumental north, looking northwards, were marked east, the needle, which should at once, with the number of degrees, read off the bearing, would give east, but the bearing is west. Having been, as was shown above, marked

west, the needle reads west, that is, north so many degrees west. By this arrangement the needle gives at once the degrees and bearing.

As the accuracy of the bearing depends, in a considerable degree, upon the goodness of the needle, great care should be observed in using it, and in marking whether it continues vibrating, or soon settles.

In the latter case there is some radical defect, arising either from the diminution of magnetic power in the needle, or from the wearing away of the centre on which it plays; this must be corrected immediately.

A really good needle is actually wearisome in its tardiness to settle.

### DEFINITIONS.

Meridian lines are due north or south lines, and strictly considered, are arcs of large circles of the earth meeting at the poles; these arcs, however, are subtended by so small an angle, and are so infinitely small in comparison of the whole circle, as to admit of their being assumed as parallel.

*The distance of a line* is its horizontal measurement, as a tangent to the earth, not following the surface of the ground, and is usually calculated in chains and links.

*The angle of bearing of any line*, is the angle of bearing made between that line and a meridian line running through the point, where the instrument is placed; and is measured always from north or south, eastward or westward.

Thus a line is said to bear *magnetically* north 16 degrees west, when the needle points to 16 degrees on the graduated circle, and when the direction of the line is to the west or north.

*The reverse bearing* of a line, is merely the bearing taken in a contrary direction.

The reverse bearing of a line, bearing north 38 degrees east, is south 38 degrees west; that of south 75 degrees west, is north 75 degrees

east. The reverse bearing, therefore, is measured by the same angle,\* as the direct bearing, only taken in opposite directions: from south bearing northward, from east westward.

*Difference of latitude*, or *northing* and *southing*, is the distance that the end of the line is further north or south than the beginning.

The *difference of longitude*, or *departure*, is the distance that the end of the line is east or west from the beginning.

In changing our position from one point on the earth's surface to another, in a direction making any angle with the meridian, we at once change our latitude and longitude—the one is the northing and southing, the other the easting and westing.

*To obtain the latitude and departure geometrically, when the length and bearing of a line are given.*

Draw a meridian line through either end, and let fall a perpendicular from the other. This perpendicular is the departure; and that portion of the meridian line, intercepted by it, is the difference of latitude.

The *latitude*, † *departure*, and *distance*, form the three sides of a right-angled triangle, whose angle at the vertex is the angle of bearing; whose hypotenuse, or radius, is the distance; the latitude, the cosine; and the departure, the sine of the angle of bearing.

The meridian distance of any station, is the distance of that station from the meridian line passing through the first or any other assumed point, and is equal to the difference between the sums of the eastings and westings from that point; and is east or west, as the eastings or westings predominate.

## THE TRAVERSE TABLE

Is a table giving the latitudes and departures to any distance and bearing, to the extent of the number of minutes to which it may be calculated.

The tables, which are appended to the present work, are calculated to every three minutes of the quadrant, and to every number from 0

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\* This is not the case where the lines are long, say two or three miles, and where the observations are taken in very high latitudes; for most practical purposes, it is sufficiently correct to assume the direct and reverse bearings the same.

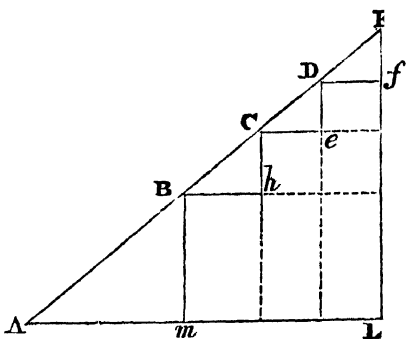
By the term *latitude*, here and elsewhere, is meant *difference of latitude*.

to 10, from 10 to 100, &c. They have been prepared upon the following principle :—

The whole northing or southing of a line, of any bearing, is equal to the sum of the northings and southings of any number of lines of the same bearing, when the sum of the several distances is equal to the one distance of the whole line.

Let  $AE$  be a line, having any bearing whatever ; through  $E$  draw the meridian line  $EL$  ; and, from  $A$ , draw  $AL$  perpendicular to  $EL$ .  $AL$  is the departure, and  $EL$  is the latitude of the line  $AE$ .

Now, divide  $AE$  into  $AB$ ,  $BC$ ,  $CD$ ,  $DE$ , and through  $B$ ,  $C$ ,  $D$ , draw the several meridian lines,  $Bm$ ,  $Ch$ ,  $De$ , and  $Bh$ ,  $Ce$ ,  $Df$ , at right angles to them.



$Bm$ ,  $Ch$ ,  $De$ ,  $Ef$ , are the latitudes of the several parts of the line  $AE$ , and are together equal to  $EL$ , that is, the sum of the northings of the parts is equal to the one northing of the whole.

And, in the same way,  $Am$ ,  $Bh$ ,  $Ce$ ,  $Df$ , the sum of the eastings are together equal to  $AL$ , the one easting of the whole line  $AE$ .

Again, if  $AE$  be a multiple of  $AB$ , (by similar triangles,)  $AL$  is the same multiple of  $Am$ , and  $EL$  is the same multiple of  $Bm$ , being but sines and cosines of similar arcs to different radii.

If  $Bm$ , therefore, be the northing of a line, whose distance is  $AB$ , or one chain, then the northing of a line  $AE$ , which is ten times  $AB$ , or 10 chains, is  $LE$ , or ten times  $Bm$ .

This being premised, the understanding of the tables of latitudes and departures, or traverse table, becomes very simple.

**EXAMPLE.**—Let it be required to find the latitude and departure of a line bearing north  $4^{\circ} 3'$  east, and 15 chains 25 links long.

Look in the margin of the pages for the degrees, and down the column on the left hand for the minutes, then on the same line, collaterally with the three minutes, will be found the latitude and departures for any number of links or chains.

*Take the latitude first.*—For the 15 chains, look  
out in the tables for the latitude of 1·00 chains = ·998  
                                multiply by                 10  
                                and you have                 9·98  
the latitude of 5·00 chains         =                 4·99

For the 25 links, look out in the latter for the latitude of 2·00 chains = 1·99  

divide by 10

(because each 10 links are one tenth of a chain) 0·19  

and you have

the latitude of 5·00 chains	=	4·99
divide by		100
(each link being one hundredth of the chain)	=	0·05

Add them together, and you obtain, as the one  
 nothing of the whole line,

—————  
 15·21

Or, by supposing the number at the top of the page not to be confined to 1 chain, 2 chains, &c., for it is not necessary that they should be so limited, but to be 1 chain or 10 chains, 2 or 20, 3 or 30 chains, &c.; or 10 links or 1 link, 20 or 2 links, 30 or 3 links, &c., a more expeditious method is used, in obtaining the same result.

To find the latitude of 15.25 chains to the angle of bearing of 4 degrees 3 minutes.

Look in the tables as before, and for one chain you find 0.993, that is some decimal less than 1; instead of 1 chain, let this be 10 chains, then the northing becomes 9.98, or some decimal less than 10 chains; therefore the latitude

of 10 chains	= 9.98
of 5 chains	= 4.98
	<u>14.96</u>

Of 20 links, by looking in the tables under the head of 2, you find 1.99, that is, as before, some decimal less than 2. This may be 20 as well as 2, and 20 chains or 20 links indifferently. The latitude, therefore, of 14.960

20 links	= 0.199
5 links	= .050
the total of which is as before	15.21

which is the latitude for 15 chains 25 links, at a bearing of 4 degrees and 3 minutes.

**This is north latitude, because the given bearing is north.**



*To find the Eastings or Westings.*

Look in the same place as before, under the head of degrees, and you will find that, at a bearing of 4 degrees 3 minutes, the departure of 1 chain =

		·071
and (by the first method) by multiplying by	10	
the departure of 10 chains	=	0·710
of 5 chains	=	0·353
of 2 chains = 0·141 dividing by 10	=	0·014
of 5 chains = 0·353 dividing by 100	=	0·003
total, east departure,		1·080

which is the required departure of the whole line.

## EXAMPLES.

Required the difference of latitude and departure of a line, which bears south 16 degrees 30 minutes east, 3 chains 47 links.

*Ans.* 3·33 south lat. ; 0·98 east dep.

Given a line, bearing north 13 degrees 30 minutes west, and 6 chains 10 links long, to find the latitude and departure.

*Ans.* 5·93 north lat. ; 1·42 west dep.

What are the latitude and the departure of a line bearing north 41 degrees 9 minutes east, 4 chains 47 links ?

*Ans.* 3·36 north lat. ; 2·95 east dep.

A line bears north 22 degrees 45 minutes west, 27 chains 62 links, required its latitude and departure.

*Ans.* 25·47 north lat. ; 10·68 west dep.

These tables are found to be extremely useful ; they enable you quickly to test the accuracy of the survey of a large extent of country ; for, as the given distances and bearings are all capable of being resolved into their respective latitudes and departures, and as you cannot, from any point, go northwards without (to return to that point) coming back the same distance southward, nor any distance eastward, without remeasuring westward the same distance back, it follows, that, in going completely round a tract of country, the sum of the northings must be equal to the sum of the southings, and the sum of the eastings to that of the westings.

In adding up, however, the northings and southings, and eastings and westings, of bearings and distances, actually taken by the circumferentor, there will be generally found some small discrepancy, which the inaccuracy of the instrument may render inevitable. Within a certain limit this error is allowable. This error should not, however,

exceed one link to every five chains of the sum total of distances. Beyond this, a re-survey becomes necessary. This error must be apportioned among the whole, proportionally to the several distances of each bearing. The method will be explained hereafter.

*The method of finding the bearing of a line by the Circumferentor.*

Having placed the circumferentor over the point of the station, release the needle, and then, having unclamped the body of the instrument, by means of the parallel plates, as in the theodolite, make the whole instrument level. Now, turn the whole round until the *north* end of it lies towards the object, and looking through the sight, at the *south* end, fix the instrument, so that the fine web-line, in the north sight, exactly covers the object; then, when the needle has perfectly settled, (which should be immediately released on setting the instrument,) read off, by the north end of the needle, the number of degrees that it points to, from the north or south division line of the compass-box, according as the north end of the needle is in the north or south semicircle of the instrument; the angle measured by these degrees is called the angle of bearing.

Should the needle not point exactly to any degree, but lie between two of them, turn the instrument carefully, so as to make the needle point exactly to the next lower, and clamping the whole head of the instrument, detach, by withdrawing the connecting pin, the two horizontal plates. The instrument having been altered to suit the needle, the flag is no longer covered by the web. By using the rack and pinion, and carefully bringing the sights back, which are connected with the same plate as the vernier is, to cover the flag, the difference of the angle, in minutes, is denoted by the distance of the broad arrow in the vernier from the  $360^\circ$ , or the zero point of the other plate, which distance, as in the case of the vernier of the theodolite, is read, by observing which line of division in the vernier, reading to the left or the right (as the broad arrow is to the left or the right of the  $360^\circ$ ), first coincides with some line in the graduated circle. In the circumferentor, the broad arrow of the vernier is placed in the centre, and if the distance from the  $360^\circ$  exceed half a degree, it is requisite to carry on the observation, as to which line first coincides, all round the plate, so as to end at the  $360^\circ$ . In some circumferentors the vernier is erroneously marked, as, in taking observations, the  $15'$  may become  $45'$ , and the  $45' 15'$ , they should have been marked  $15'$  and  $45'$ .

Having taken the angle, the two plates must again be brought to their proper position, by making the broad arrow of the vernier coincide with the  $360^{\circ}$ , where the connecting pin keeps them, till they are again required to be separated. The needle must be fixed, and the whole instrument clamped. It might not, perhaps, be out of place to observe here, that the kind of surveying required in a new country, as Australia or New Zealand, is directly the reverse of that which must be in use here. Here, a representation is, in most cases, wanted on paper, of actual boundaries of properties in the field: there, various lines, or certain positions, are to be marked down upon the ground. Here, the course of the hedge of a field is required: there, lines have to be run of given courses. The circumferentor, as constructed in this country, answers exceedingly well for the nature of the country it is generally used for here; but, when wanted to run a particular course, the objection, I have before mentioned, holds specially against it.

Having measured the distances to the next station, before taking the forward angle of the second line, take carefully the reverse bearing of the first, this will verify the last forward bearing, as the two should be the same; it also prevents, in a great measure, the probability of the needle being acted upon, without detection, by any magnetic substance in the neighbourhood; and I would especially recommend the young beginner invariably to adopt it. It is anything but a loss of time, and no survey can be depended upon without it.

#### VARIATION.

Before proceeding, it were better here to observe, that the magnetic meridian may not be the true meridian. The pole-star is not exactly at the pole; the needle seldom points either to the one or the other; two needles seldom point exactly the same way; and the same needle seldom points exactly the same way two years together. There are, therefore, two or three kinds of variation. The variation of the needle, strictly so termed, is the angle made between the magnetic north and the true north, and the variation is called east or west, according as the magnetic north is on the east or west side of the true north.

The measure of this angle, or the variation, is different in different countries, at the same time; and in the same country at different times. The various methods for determining this variation will be hereafter explained.

This general variation is not, however, that variation which principally affects the accuracy of a Surveyor's work; it is not the *general* angle of variation; but it is the *actual* angle made by his own needle and the true meridian line—that is the variation he has more particularly to guard against. This also shall be explained hereafter—observing only, now, that it is but necessary to determine once a true meridian by the method that will be then explained. By determining the true bearing of any fixed line—such as a long wall or fence—once, it becomes a standard of reference afterwards, as, at any time, the difference between its magnetic bearing, taken by an instrument and its true bearing, will determine the positive variation of the instrument required.

No Surveyor should assume that the variation of his instrument agrees exactly with the *general variation of the needle* given. Nor should he attempt to connect a new survey with an old one, after a year's interval, without determining, by the check or standard line, whether any, and what variation had taken place in his needle, and duly allowing for the difference; without this, no new line can be depended upon. In running divisions, or side lines, between lots in a new country, every instrument must be tested by the boundary line between the townships, as a standard line, and the division lines run parallel to it, whatever be the magnetic bearing, or the bearing of the individual instrument. This ever secures the parallelism of the lines.

## CHAP. II.

*To find the true bearing of a line, the magnetic bearing and the variation being given.*

By the variation of the compass, is always meant the variation of the needle, which is the only variable east or west, from the true north, which is ever constant

**RULE.**—*Mark upon paper*, the relative positions of the given line and the *magnetic* north, which represents the north end of the needle, then observe, whether the variation be easterly or westerly; if easterly, the *true* north will be to the left of the magnetic; and

if westerly, to the right; place this also on paper, in its proper relative position.

The angle made between this last line (of the true north) and the given line, is the angle of bearing; the expression of this angle depending, of course, whether, by the variation, it may or may not have been moved into a different quadrant, being often changed from the magnetic N. W. to the true S. W., or N. E.

#### EXAMPLES.

1. Let the line AB bear N.  $24^{\circ}$  E., and the variation be  $23\frac{1}{2}^{\circ}$  W.; required the true bearing of the line.

AB, bearing N.  $24^{\circ}$  E., the needle is on the left of the line, but the needle bears west, the true north is on the right of the needle, therefore the line and the true north, being both on the right of the needle, the line being at the greater angle, the difference of the D angles is the bearing of the line eastwards, N.  $0^{\circ} 30' \text{ E.}$

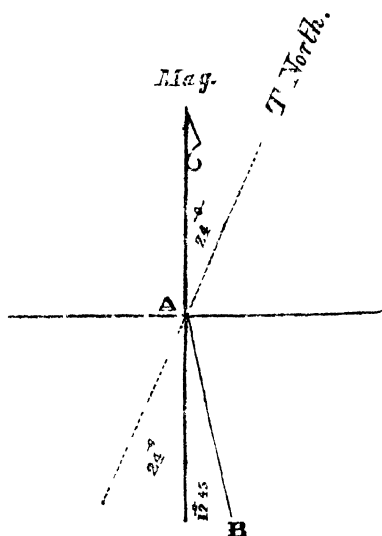
2. Let the reverse bearing of AB be S.  $24^{\circ}$  W., with the same variation, what is the true bearing?

*Ans.* S.  $0^{\circ} 30' \text{ W.}$ , which is the result of the former.

3. Let AB bear S.  $12^{\circ} 45' \text{ E.}$ , and the variation be  $24^{\circ} \text{ W.}$ ; required the true bearing.

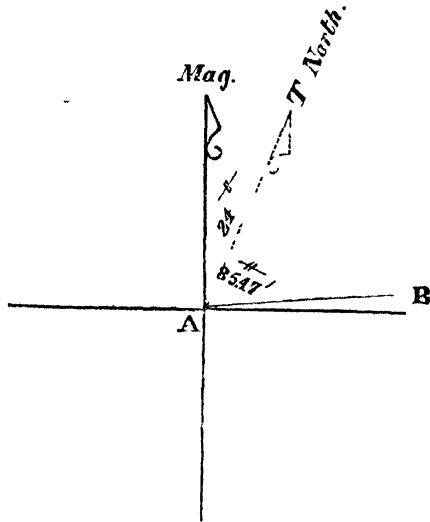
Now the north end of the needle is to the left of the true north, therefore,

$$\begin{array}{r} 24^{\circ} \\ + 12^{\circ} 45' \\ \hline = \text{S. } 36^{\circ} 45' \text{ E.} \end{array}$$



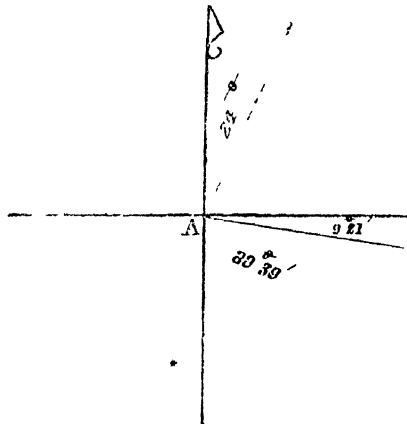
4. Given AB, N.  $85^{\circ} 47'$  E., with the same variation of  $24^{\circ}$  W.; required the true bearing.

$$\begin{array}{r} \text{N. } 85^{\circ} 47' \\ \quad 24^{\circ} 00' \\ \hline \text{N. } 61^{\circ} 47' \text{ E.} \end{array}$$



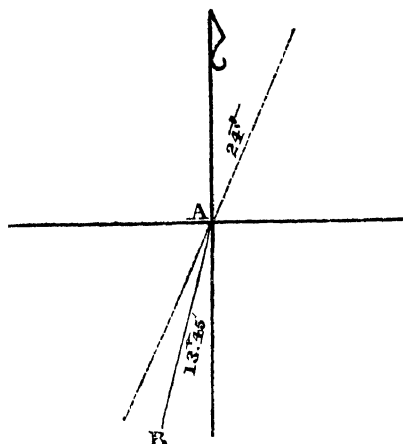
5. Given AB, S.  $80^{\circ} 39'$  E., with same variation; required the true bearing.

$$\begin{array}{r} 90^{\circ} \\ -24 \\ \hline 66^{\circ} \\ \\ 90^{\circ} \\ -80^{\circ} 39' \\ \hline 9^{\circ} 21' \\ 66^{\circ} \\ \hline \text{N. } 75^{\circ} 21' \text{ E.} \end{array}$$



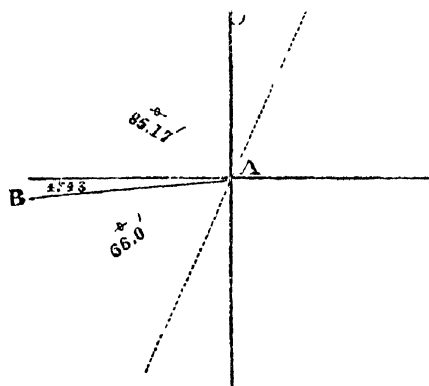
6. Given AB, S.  $13^{\circ} 45'$  W., with the same variation; required the true bearing.

$$\begin{array}{r} 24^{\circ} 00' \\ -13^{\circ} 45' \\ \hline \text{S. } 10^{\circ} 15' \text{ E.} \end{array}$$



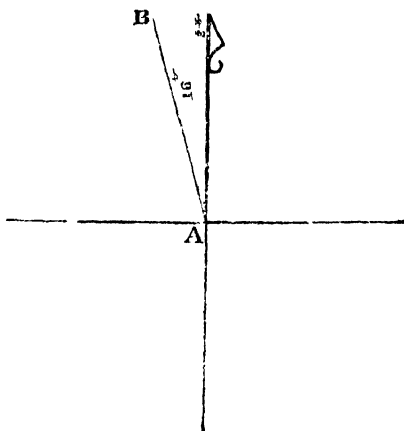
7. Given AB, N  $85^{\circ} 17'$  W., variation as before; required the true bearing.

$$\begin{array}{r} 90^{\circ} \quad 99^{\circ} \\ 24^{\circ} \quad -85^{\circ} 17' \\ \hline 66^{\circ} \quad 4^{\circ} 43' \\ + 66^{\circ} \\ \hline \text{S. } 70^{\circ} 43' \text{ W.} \end{array}$$



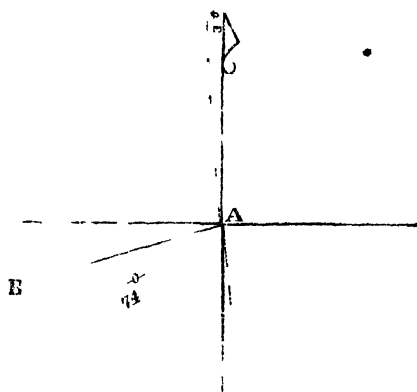
8. Given AB, N.  $16^{\circ}$  W., variation  $3^{\circ}$  east; required the true bearing.

$$\begin{array}{r} 16^{\circ} \\ - 3^{\circ} \\ \hline \text{N. } 13^{\circ} \text{ W.} \end{array}$$



9. Given AB, S.  $74^{\circ}$  W., variation  $3^{\circ}$  east; required the true bearing.

$$\begin{array}{r} 74^{\circ} \\ + 3 \\ \hline \text{S. } 77^{\circ} \text{ W.} \end{array}$$

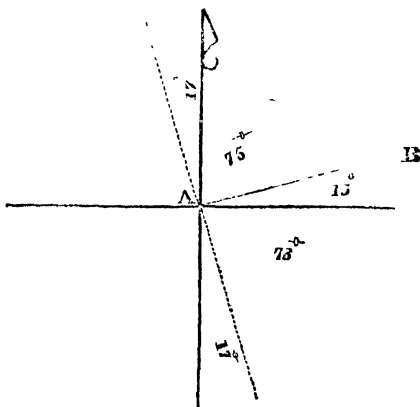




10. Given AB, N.  $75^{\circ}$  E., and the variation of needle  $17^{\circ}$  east; required the true bearing.

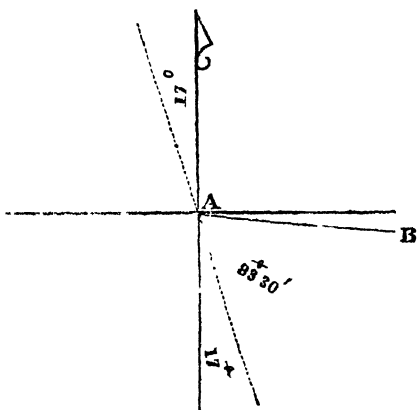
$$\begin{array}{r} 90^{\circ} \\ -75^{\circ} \\ \hline 15^{\circ} \end{array}$$

$$\begin{array}{r} 90^{\circ} \\ -17^{\circ} \\ \hline 73^{\circ} \\ 15^{\circ} \\ \hline S.88^{\circ}E. \end{array}$$



11. Given AB, S.  $83^{\circ} 30'$  E., variation  $17^{\circ}$  east; required the true bearing.

$$\begin{array}{r} 83^{\circ} 30' \\ 17^{\circ} \\ \hline S.66^{\circ} 30'E. \end{array}$$



## CHAP. III.

THE BEARINGS OF TWO LINES BEING GIVEN, TO  
DETERMINE THE ANGLE BETWEEN THEM.

RULE.—First let both these lines run northwards or southwards. If they run on the same side of north or south, whether eastward or southward, this angle will be the *difference* of their angles of bearing ; if on different sides, it will be their sum.

Next, let one run north and the other south.

If they both run on the same side of the meridian, this angle will be the supplement angle of the *sum* of the angles of bearing.

If one be on the east, and the other on the west of the meridian, this angle will be the supplemental angle of the *difference* of their angles of bearing.

In the interior angles of a polygon, as an angle may exceed  $180^\circ$ , the required angle might be the difference between the angle calculated as above, and  $360^\circ$ .

EXAMPLE 1.—Given AB, N.  $16^\circ$  W., and AC, N.  $12^\circ$  E, to find the angle between them.

$$\begin{array}{r} 12^\circ \\ 16^\circ \\ \hline 28^\circ = \text{angle between them.} \end{array}$$

**EXAMPLE 2.**—Given AB, N.  $16^{\circ}$  W., and AC, S.  $12^{\circ}$  W., to find the angle between them.

angle = supplement of sum.

$$\begin{array}{r} 12^{\circ} \\ 16^{\circ} \\ \hline 28^{\circ} \end{array} \qquad \begin{array}{r} 180^{\circ} \\ 21^{\circ} \\ \hline 152^{\circ} = \text{angle between them.} \end{array}$$

**EXAMPLE 3.**—Given AB, N.  $84^{\circ} 20'$  W., and AC, S.  $49^{\circ} 51'$  E., to find the angle between them.

here the angle = the supplement of their difference.

$$\begin{array}{r} 84^{\circ} 20' \\ 49^{\circ} 51' \\ \hline 34^{\circ} 29' = \text{the difference.} \end{array} \qquad \begin{array}{r} 180^{\circ} \\ 34^{\circ} 29' \\ \hline 145^{\circ} 31' = \text{angle required.} \end{array}$$

**EXAMPLE 4.**—Given AB, S.  $25^{\circ}$  E., and AC, S.  $16^{\circ}$  E.; required the angle between them.

angle = their difference.

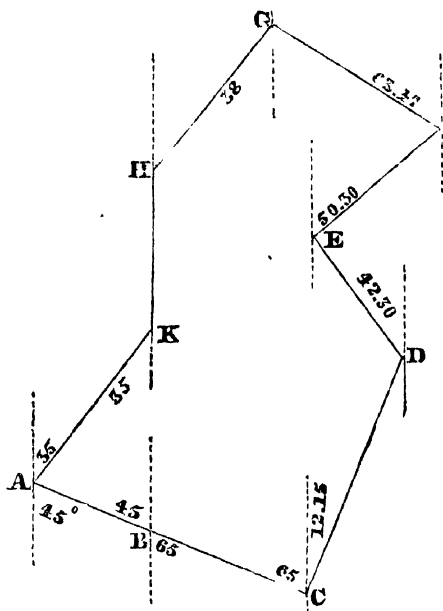
$$\begin{array}{r} 25^{\circ} \\ 16^{\circ} \\ \hline 9^{\circ} = \text{angle required.} \end{array}$$

For further practice, I adjoin the following Example, where the bearings only are given, and the interior angles (being severally the angles between every two adjacent lines of the whole) are required. This Example will establish an important truth, that the sum of the interior angles of any figure, obtained from the *bearings* of its sides, must, *right* or *wrong*, always amount to four less than twice as many right-angles as the figure has sides, and that, therefore, the sum of these angles, can, in no case, be used as a check upon the correctness of the work.

**EXAMPLE 1.**—Given a tract of country, with the bearings of its several boundaries, to find the interior angles, and determine their correctness, viz., AB bears S.  $45^{\circ}$  E.—BC, S.  $65^{\circ}$  E.—CD, N.  $12^{\circ} 15'$  E.—DE, N.  $42^{\circ} 30'$  W.—EF, N.  $45^{\circ} 30'$  E.—FG, N.  $68^{\circ} 47'$  W.—GH, S.  $38^{\circ}$  W.—HK, due S., and KA (S.  $35^{\circ}$  W.) Required each of the angles.

Take first the angle ABC; this is equal to the first angle of bearing,  $45^\circ$  + the supplement of the second angle,  $65^\circ$ , that is, equal to  $45^\circ + 115^\circ = 160^\circ$ .

BCD, the second interior angle, is equal to the second angle of bearing,  $65^\circ$  + the third angle  $12^\circ 15'$ , or  $77^\circ 15'$ .



Thus, ABC being found to be  $= 160^\circ$   
and BCD  $= 77^\circ 15'$

$$\text{CDE} = 125^\circ 15'$$

$$\text{DEF} = 273^\circ 00'$$

$$\text{FGH} = 65^\circ 43'$$

$$\text{FGH} = 101^\circ 47'$$

$$\text{GHK} = 142^\circ 00'$$

$$\text{HKA} = 215^\circ 00'$$

$$\text{KAB} = 100^\circ 00'$$

$$90 | 1260^\circ 00'$$

14 right angles

$$+ \frac{4}{18}$$

18 equals twice the number of

sides of the polygon.

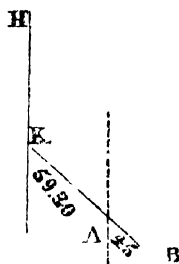
Now let KA, instead of S.  $35^\circ$  W., bear S.  $59^\circ 30'$  E., and find the amount of the present interior angles.

All the former angles, but the last two, remain the same; the sum of the constant angles equals  $945^\circ 00'$

The angle HKA now equals  $120^\circ 30'$

And the angle KAB  $194^\circ 30'$

resulting the same total as before, of  $1260^\circ 00'$



Hence the sum of the interior angles of a polygon, when these angles are calculated from the bearings of the several lines, is no check upon the correctness of the work.

But the equality, or discrepancy of the northings and southings, of the eastings and westings, depending upon the lengths, as well as the bearings of the lines, is a certain test of its accuracy.

The method of applying this test, we will, therefore, now proceed to consider.

**EXAMPLE.**—The bearings and distances of the several sides of a polygon being given, it is required to determine their correctness, and to divide the allowable error, proportionally, among the whole.

Rule a schedule, as in the annexed example, dividing it into columns, headed stations, bearings, distances, northings and southings, eastings and westings.

Find, as above shown, the difference of latitude and departure, in the tables for the several bearings and distances, and place the numbers, thus found in the proper column of northings, southings, eastings, or westings. Add them together, separately, find the difference of the northings and southings, and of the eastings and westings; the former is called the error of latitude, and the latter the error of departure. This error, if it does not exceed one link in five chains in the sum total of the distances, must be apportioned among each of the distances by the following proportions, viz.—*As the sum of all the distances is to the whole error, so is each distance to its correction.*

This must be done for the latitudes and departures, and must be placed in a column appropriated to each, called the north and south correction, and the east and west correction; the correction, thus determined, must be placed, collaterally, with the distance to which it refers, without distinguishing as to north, or south, east, or west.

In making this proportion, the links of the distances need not be taken into account, and frequently, as an approximation is sufficient, the apportionment may be made without reference to calculation.

Having found the several corrections for each of the latitudes and departures, add them together, severally, and see whether their total agrees with the whole error; then draw four other columns, heading them, corrected northings, southings, eastings, westings, and proceed to allot the corrections. If the error be an excess of northings, subtract each correction from its collateral northing, or add it to the collateral southing; if an excess of easting, add to the westing; and

subtract from the easting; the respective sums of their corrected latitudes and departures will now be found exactly to agree.

**EXAMPLE 1.**—Given the bearings and distances as follows, viz.,—  
 S.  $16^{\circ} 30'$  E., 3 chains 47 links; and S.,  $17^{\circ}$  E., 3 chains 2 links; S.  $26^{\circ}$  E., 5 chains 51 links; S.  $31^{\circ} 30'$  E., 7 chains 34 links; S.  $5^{\circ} 20'$  E., 10 chains 55 links; S.  $15^{\circ}$  E., 5 chains 9 links; S.  $8^{\circ}$  W., 4 chains 3 links; S.,  $3^{\circ} 30'$  E., 4 chains 70 links; S.,  $45^{\circ} 30'$  W., 6 chains 50 links; S.  $64^{\circ} 45'$  W., 7 chains 34 links; and N.,  $1^{\circ} 21'$  E., 49 chains 18 links, to the place of beginning, to find their northings and southings, eastings and westings, and hence determine their correctness.

Stations	Bearings	Distances	North Lat.	South Lat.	East Long.	West Long.
		ch. lks.				
1	S. $16^{\circ} 30'$ E.	3.47		3.33	0.98	
2	S. $17^{\circ}$ 0' E.	3.02		2.89	0.89	
3	S. $26^{\circ}$ 0' E.	5.51		4.94	2.41	
4	S. $31^{\circ} 30'$ E.	7.34		6.25	3.84	
5	S. $5^{\circ} 20'$ E.	10.55		10.49	0.97	
6	S. $15^{\circ}$ 0' E.	5.09		4.91	1.31	
7	S. $8^{\circ}$ 0' W.	4.03		3.99		0.56
8	S. $3^{\circ} 30'$ E.	4.70		4.60	0.28	
9	S. $45^{\circ} 30'$ W.	6.50		4.56		4.64
10	S. $64^{\circ} 45'$ W.	7.34		3.13		6.64
11	N. $1^{\circ} 21'$ E.	49.18	49.17		1.16	
		106.73	49.17	49.17	11.84	11.84

The northings being the same as the southings; the eastings the same as the westings.

It is seldom, however, that they agree so exactly, as has been given in this example. As the sum total of the distances amount to 106 chains 73 links, there would have been an allowable difference of 21 links (*i.e.*, of one link in every five chains) between the northings and southings, and the eastings and westings.

As errors will sometimes, notwithstanding the greatest care, occur in a survey of extent, we subjoin an example of two blocks of land, adjoining each other, having a common line, in which a considerable error was suspected, with the method of determining the locality of the error, and the means of correcting it.

**EXAMPLE.**—Given the bearings of the one block, S.  $49^{\circ} 30'$  E., 7 chains 25 links; S.  $73^{\circ}$  E., 8 chains 82 links; N.  $6\frac{1}{4}^{\circ}$  E., 15 chains 41 links; S.  $68^{\circ}$  W., 17 chains 84 links, to the place of beginning;

and the bearings of the second block, reversing the bearing of the last line, which is common to the two, N.  $68^{\circ}$  E., 17 chains 84 links; N.  $67\frac{1}{4}^{\circ}$  E., 6 chains 37 links; N.  $47\frac{1}{4}^{\circ}$  E., 3 chains 86 links; N.  $76^{\circ}$  W., 14 chains 63 links; S.  $32^{\circ}$  W., 19 chains 73 links, also returning to the place of beginning. It is required to calculate the northings and southings, eastings and westings, of each of them separately, as it is imagined an error has been somewhere committed, which it is required to discover.

## 1st. SURVEY.

(Having a common line with the second.)

Stations	Courses	Distance in chains	North Lat.	South Lat.	East Dep	West Dep
1	S. $49^{\circ} 30'$ E.	7.55		4.91	5.74	
2	S. $73^{\circ}$ E.	8.82		2.58	8.43	
3	N. $6\frac{1}{4}^{\circ}$ E.	15.41	15.32		1.68	
4	N. $68^{\circ}$ W.	17.84		6.69		16.54
			15.32	14.18	15.85	16.54
			14.18			15.85
		Excess of N.	2.14	Excess of W.		.69

There is some considerable error here.

## 2nd. SURVEY.

(Having a common line with the first.)

Stations	Courses	Distances in chains	North Lat.	South Lat.	East Dep.	West Dep.
1	N. $68^{\circ}$ E.	17.84	6.58		6.54	
2	N. $67\frac{1}{4}^{\circ}$ E.	6.37	2.61		5.68	
3	N. $47\frac{1}{4}^{\circ}$ E.	3.86	2.61		2.84	
4	N. $76^{\circ}$ W.	14.63	3.54			14.19
5	S. $32^{\circ}$ W.	19.73		16.73		10.46
			15.44	16.73	25.06	24.65
				15.44	24.65	
		Excess	of S.	1.29	.41	Excess of eastings.

There is a considerable error here also.

This error is southerly, the former was northerly; but the bearing to this line, in the present example, is the reverse of the former; as this line forms, in itself, in the first survey, the whole of the northings, and, in the second survey, the whole of the southings, and as it has, in each case, an excess of more than a chain, the

error will most probably be found in this line. To ascertain whether this is the case, make one survey of the two, by throwing out of it this common doubtful line, and proceed to test the correctness of the several other lines and bearings by the usual way.

## 3rd. SURVEY,

(Being the union of the two, omitting the common line.)

Stations	Courses	Distances	North Lat	South Lat.	East Dep.	West Dep.
1	S. $49\frac{1}{2}^{\circ}$ E.	7.55		4.91	5.74	
2	S. $73^{\circ}$ E.	8.82		2.58	8.43	
3	N. $6\frac{1}{4}^{\circ}$ E.	15.41	15.32		1.68	
4	N. $67\frac{3}{4}^{\circ}$ E.	6.37	2.61		5.68	
5	N. $47\frac{1}{2}^{\circ}$ E.	3.86	2.61		2.84	
6	N. $76^{\circ}$ W.	14.63	3.54			14.19
7	S. $32^{\circ}$ W.	19.73		16.73		10.46
		76.37	24.08	24.22	24.37	24.65
	5.76			24.08		24.37
	15		Excess of S.	.14	Excess of W.	.28

As 76.37 is the sum of the distances, the allowable error in this case is 15 links, that being one link in every chain of the sum of the distances.

The excess of southing is less than this, being .14; that of the westings is more (.28), being, however, sufficiently near for our object in proving that there is no material error in any one of the lines that form the present block; and *it may, therefore, be presumed that the bearing and distance, common to the two polygons, must be incorrect.*

Having found the place of error, we will now consider the best method of correcting it.



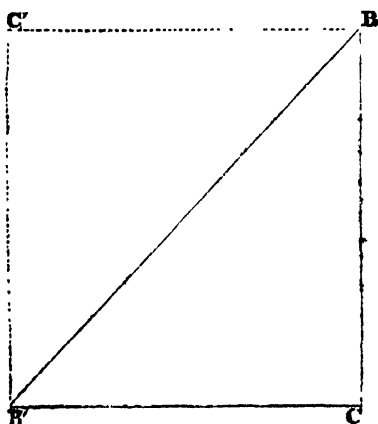
## CHAP. IV.

## ERRORS OR OMISSIONS OF SURVEY.

As it is necessary, for the correction of this common line, when any two of the following data, viz.,—bearing, distance, difference of latitude, and of departure are given, to understand the method of determining the other two; we will first briefly say a few words on the subject.

*Any two of the above data being given, to determine the other two.*

Let  $B'B$ , or  $BB'$ , be the distance of the given line;  $BB'C'$ , or  $B'BC$ , is the angle of bearing;  $BC$ , or  $B'C'$ , is the difference of latitude, and  $B'C$ , or  $BC'$ , is the departure; but  $BC$ , or  $B'C'$ , is the cosine of the angle  $B'BC$ , or  $BB'C'$ , which is the angle of bearing; and  $B'C$ , or  $BC'$ , is the sine, also, of the same angle of bearing; therefore, the latitude, departure, and distance, are respectively the cosine, sine, and radius of a circle,



whose arc is measured by the angle of bearing. By the rules for the cases of right-angled triangles, given in the Chapter on Trigonometre, in the Introduction, having two terms given, the rest can be found.

**EXAMPLE 1.**—A line bears N.  $45^{\circ}$  E., 31 chains, 40 links; required its difference of latitude and departure.

As $\sin 90^{\circ}$	=	10.00000
to 31.41 chs.	=	1.49693
so is $\sin. 45$	=	9.84949
		<u>11.34642</u>
		10.00000
22 chs. 20 lks. N. lat.	=	<u>1.34642</u>
$\sin 45^{\circ} = \cos. 45^{\circ} \therefore \text{dep.} = \text{lat.}$		

**EXAMPLE 2.**—The difference of latitude of a line was 4 chs. 40 lks. N. ; the departure was 25' W. ; required the distance of the line.

*Ans.* 25·57 chs.

**EXAMPLE 3.**—A line bears N. 72° W. ; its departure is 24 chs. 58 lks. : what is its length ?

*Ans.* 25·84 chs.

**EXAMPLE 4.**—A line is 38 chs. 45 lks. in length, and its departure is 14 chs. W. ; what is its angle of bearing, the same being between the north and west ?

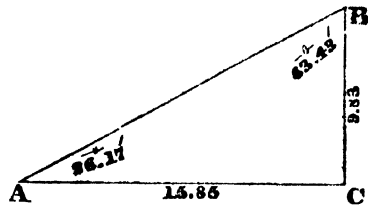
$$\begin{array}{rcl}
 \text{As } 38\cdot45 \text{ chs.} & = & 1\cdot58490 \\
 \text{is to rad.} & = & 10\cdot00000 \\
 \text{so is 14 chs.} & = & 1\cdot14613 \\
 & & \hline
 & & 11\cdot14613 \\
 \text{sine angle of bearing} & = & 1\cdot58490 \\
 21^\circ 21' & = & 9\cdot56123
 \end{array}$$

*To correct the fourth station, which is common to the two polygons.*

**RULE.**—Add up the northings and southings, and subtract them for the northing or southing of the unknown line ; do the same with the eastings or westings to obtain its departure ; then, by the preceding note having the departure and latitude of a line given, the distance and bearing can be determined.

Stations	Courses	Distances	North Lat.	South Lat.	East Dep.	West Dep.
		<i>c. l.</i>				
1	S. 49°30' E.	7 55		4·91	5·74	
2	S. 73° E.	8 82		2·58	8·43	
3	N. 6½° E.	15 41	15·32		1·68	
4	S. W.					
			15·32	7·49	15·85	
			7·49			
	Deficiency of S.		7·83		15·85	Deficiency of westings.

Now AC, the westing or base of the right-angled triangle, being given 15 chs. 85 lks.; and BC, the southing or perpendicular, being 7 chs. 83 lks.; the angle BAC, or the bearing of the line AB, and its distance, which are required, can be determined by the rules of the bases of right-angled triangles.



Thus—take AC as radius, and describe the circle CD; then CB becomes the tangent to angle BAC, and AB is the secant of the same angle; then say—

$$\begin{array}{rcl}
 \text{As } 15.85 & & = 1.20003 \\
 \text{is to radius} & & = 10.00000 \\
 \text{so is } 7.83 & & = 0.89376 \\
 & & \underline{10.89376} \\
 & & 1.20003 \\
 \text{to tan. angle BAC, } 26^{\circ} 17' & = & 9.69373
 \end{array}$$

$$\begin{array}{rcl}
 \text{Also, as radius} & & = 10.00000 \\
 \text{is to } 15.85 & & = 1.20003 \\
 \text{so is secant } \angle A, 26^{\circ} 17' & = & 10.04739 \\
 \text{to hypotenuse, } 17.68 & = & 1.24742 \\
 \text{and the bearing in S. } 63^{\circ} 45' \text{ W.}
 \end{array}$$

## CORRECTED SURVEY.

Stations	Courses	Distances	N.	S.	E.	W.	
1	N. $63^{\circ} \frac{3}{4}$ E.	17.68	7.83		15.85		
2	N. $67^{\circ} \frac{3}{4}$ E.	6.37	2.61		5.68		
3	N. $47^{\circ} \frac{1}{2}$ E.	3.86	2.61		2.84		
4	N. $76^{\circ}$ W.	14.63	3.54			14.19	
5	S. $32^{\circ}$ W.	19.73		16.73		10.46	
		62.27	16.59	16.73	24.37	24.65	
				14.59		24.37	
		Excess	of S.	.14		.28	Excess of westings,

which are the same errors as before.

Let us now divide these errors proportionately among the several distances, by the method previously given, viz.,—by saying, as the whole distance is to the whole error, so is each distance to its particular correction.



## EXAMPLES.

1.—Given the following bearings and distances of an old waggon road, through a new settlement, running from one concession road to another, viz.,—1st. N.  $78^{\circ}$  E., 2·20 chains; 2nd. N.  $45^{\circ} 30'$  E., 14·80 chains; 3rd. N.  $16^{\circ}$  W., 21·75 chains; 4th. N.  $68^{\circ} 20'$  E., 13·90 chains; 5th. N.  $10^{\circ}$  W., 15·60 chains; 6th. N.  $70^{\circ}$  E., 8·96 chains. It is required to lay out a new straight road, connecting the two ends of the present road. What will be its bearing and distance?

2.—Given the boundaries of a tract of land, as follows, viz.:—1st. S.  $16^{\circ} 30'$  E., 3·47 chains; 2d. S.  $17^{\circ}$  E., 3·02 chains; 3d. S.  $26^{\circ}$  E., 5·51 chains; 4th. S.  $31^{\circ} 30'$  E., 7·34 chains; 5th. S.  $5^{\circ} 20'$  E., 10·55 chains; 6th. S.  $15^{\circ}$  E., 5·09 chains; 7th. S.  $8^{\circ}$  W., 4·03 chains; 8th. S.  $3^{\circ} 30'$  E., 4·70 chains; 9th. S.  $45^{\circ} 30'$  W., 6·50 chains; 10th. S.  $64^{\circ} 45'$  W., 7·34 chains; 11th. to the place of beginning. Required the bearing and distance of this eleventh line.

*Ans.* N.  $1^{\circ} 21'$  E., 49 chs. 18 lks.

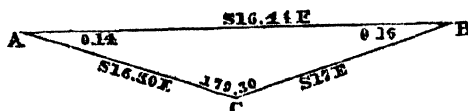
*When the distance of two sides cannot be obtained.*

RULE 1.—Find, by the preceding chapter, the length and bearing of the closing line, connecting the known sides of the survey. This line and the two unknown sides will form a triangle, having a known base, and the bearings known also of all its sides; with these data compute the several angles of the triangle, by the previous rule (page 205), and you have the three angles and one side given; whence the two required sides are determinable by the first case.

EXAMPLE 1.—The boundaries of a tract of land were taken as follows:—1st. S.  $16^{\circ} 30'$  E.; 2nd. S.  $17^{\circ}$  E.; 3d. S.  $26^{\circ}$  E., 5·51 chains; 4th. S.  $31^{\circ} 30'$  E., 7·34 chains; 5th. S.  $5^{\circ} 20'$  E., 10·55 chains; 6th. S.  $15^{\circ}$  E., 5·09 chains; 7th. S.  $8^{\circ}$  W., 4·03 chains; 8th. S.  $3^{\circ} 03'$  E., 4·70 chains; 9th. S.  $45^{\circ} 30'$  W., 6·50 chains; 10th. S.  $64^{\circ} 45'$  W., 7·34 chains; 11th. N.  $1^{\circ} 21'$  E., 49·18 chains, to the place of beginning. Required the distances of the two first stations, which, from local obstructions, could not be measured.



Because AB and AC bear both southwards and eastwards, the angle CAB, is their difference; for the same reason, the angle ABC



is the difference of the bearings of AB and CB. The sides AC and CB can now be calculated by the first case of trigonometry.

$$\begin{array}{rcl}
 \text{As sine } 179^{\circ} 30' & = & 7.94084 \\
 \text{is to } 6.49 \text{ chs.} & = & 2.81224 \\
 \text{so is sine } 0^{\circ} 16' & = & 7.66784 \\
 & & \hline
 & & 10.48008 \\
 \text{to } x \text{ } 3.46 \text{ chs.} & = & 2.53924 \\
 & & \hline
 \text{and so is sine } 0^{\circ} 14' & = & 7.60985 \\
 & & \hline
 & & 10.42209 \\
 \text{to } y \text{ } 3^{\circ} 02' & = & 2.48125
 \end{array}$$

being 3.46, instead of 3.47 (see page 221, example), a difference of one link only in the first distance; and being 3.02, the same as the second distance.

**EXAMPLE 2.**—The following bearings and distances were taken, viz.:—at station 1, the bearing of 2nd station was, S.  $71^{\circ} 24'$  W.; at station 3, the same point bore S.  $46^{\circ} 18'$  E., the distances could not be obtained; at station 3, N.  $15^{\circ} 42'$  E., 6.20 chains; at station 4, N.  $52^{\circ} 18'$  E., 6.75 chains; at station 5, S.  $78^{\circ} 48'$  E., 5.96 chains; at station 6,  $5^{\circ} 51'$  E., 4.84 chains; at station 7, S.  $49^{\circ} 15'$  W., 4.75 chains; at station 8, S.  $4^{\circ} 57'$  E., 3.98 chains, to the place of beginning.

What are the distances of the first and second lines?

**RULE 2.**—*By means of a changed bearing.*—Suppose the whole tract of land to revolve, until it becomes in such a position, as to have one of the unknown sides in a direct meridian line. This will not change the relative position of the sides, as, whatever angle of variation be taken for this line, the same will be taken for all; but, as in the first instance, there

were but two lines, whose departures were unknown, so now, in consequence of one, from having been made due north and south, having no departure, that, of the only one that now remains unknown, is the measure of the difference of the sums of the eastings or westings.

The changing of the bearing is performed in the same way as correcting for the variation, viz.,—by bringing all the given angles to a *new* meridian, which makes so many degrees east or west, with the first meridian.

Arrange, therefore, the bearings and distances, as in the following columns; change the bearings of each, so as to make the bearing of one of the unknown sides a meridian line. Place these changed bearings in their proper column; find the latitude and departure of these bearings, to their several distances. Find the sum of the eastings and the sum of the westings; their difference will be the departure of the second unknown side, in terms of the deficiency.

Calculate the latitude and distance of this second side, from its bearing and departure, which are known, and place them also in their proper column.

Then add the northings together, and the southings together, and their difference will be the distance of the side, which was made a meridian.





When the difference above was 11, which is the sine of the angle of bearing, 30 minutes, the distance, or the radius to that angle, was only 12·6, it cannot now, therefore, be more.

Hence, when the two unknown bearings differ so slightly, as to make the changed bearing of one but a few minutes, this method cannot be adopted; the first method, by the closing line, must, therefore, be used.

I subjoin a correct result by the changed bearings, obtained from the same example, in consequence of the angles of the two bearings, whose distances are unknown, differing considerably.

EXAMPLE.—Make the *seventh* bearing south, and change the other bearings to correspond. The changed bearings have been omitted for practice to the student, but their latitudes and departures are inserted.

Stations	Bearings	Changed Bearing.	Distances	N.	S.	E.	W.
1	S. 16° 30' E.		3·47		3·15	1·43	
2	S. 17° E.		3·03		2·73	1·27	
3	S 26° E.		5·51		4·51	3·08	
4	S. 31° 30' E.		7·34		5·66	4·66	
5	S. 5° 20' E.		10·55		10·26	2·41	
6	S. 15° E.		{ 5·08 }		( 4·67 )	( 1·98 )	
7	S. 8° W.		{ 4·03 }		( 4·04 )		
8	S. 3° 30' E.		4·70		4·60	0·96	
9	S. 45° 30' W.		6·50		5·15		3·95
10	S. 64° 45' W.		7·34		4·02		6·13
11	N. 1° 21' E.		49·18	48·83			5·71
				48·83	44·81	13·81	15·70
				44·11			13·81
Deficiency of southings				4·02,	deficiency of E. 1·98		

As sine 23°	=	9·59188
is to 1·98 chs.	=	2·29667
so is radius	=	10·30000
		12·29667
		9 59188
to 5·08 chs.	=	2·70479
As radius	=	10·00000
is to 5·06 chs.	=	2·70479
so is sin 670	=	9·97403
		12·66882
		10·00000
to 4·67 chs.	=	2·66882

*When the bearings of any two sides of a tract of Survey have been incorrectly taken.*

Arrange the given bearings and distances in their proper columns, and find, as before, the difference of the northings and southings, and of the eastings and westings, for the latitude and departure of a closing line. The line, and the distances of the two unknown sides, form a triangle, with sufficient data to determine its inward angles; and thence, as the bearing of the closing line is known, the required bearings of the two other sides are obtained.

## CHAP. V.

*Given the bearings and distances of a tract of land, to determine its area.*

Arrange the several bearings and distances as before, and find the respective latitudes and departures corresponding to each.

Determine, as in the preceding Problems, the several corrections to each, for latitude and departure, and allow for these corrections, placing the amended latitudes and departures in their proper columns.

Now make five more columns, and head them—East Double Departure; West Double Departure; Multipliers; North Areas; and South Areas.

Take the sum or difference of every two consecutive departures; adding them, if of the same kind, and subtracting them, if of different; and place this sum, or excess, in the column to which it belongs; of west, in the column of W.D.D.; of east, in that of E.D.D.

If the sum of the first and last departures, east or west, be found equal to the first corresponding double departure, east or west, the working has been correctly performed.

Commence from any angle whatever of the survey, and assume any line, either close to, or at any distance from it, as a multiplier, and place it in the column of multipliers collateral with that angle, terming it either east or west, according as the principal double departures are east or west. The sum of this assumed multiplier, and that of the double departure, corresponding to the next side, if they are of the same kind, or their difference, if of opposite kinds, in terms of the greater, will be the next multiplier; proceed with the second multiplier, and the following double departure, until multipliers have been found for all the sides.

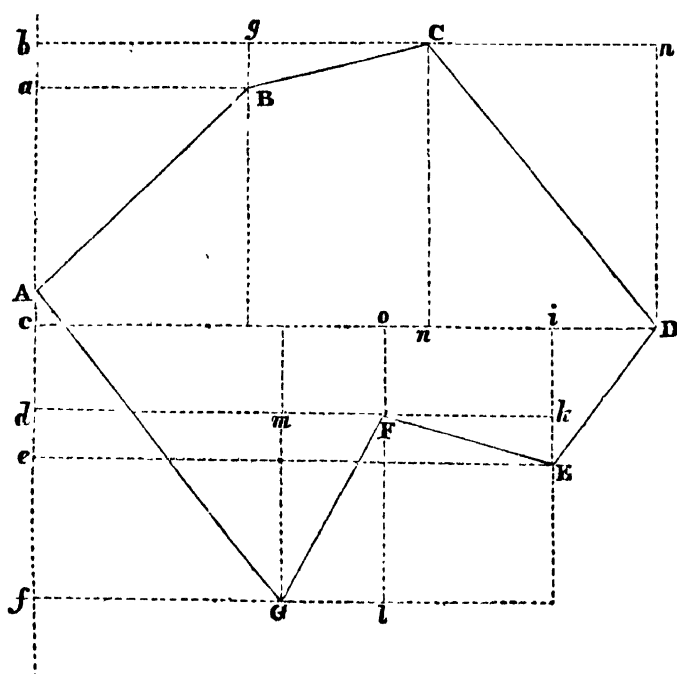
The difference, if of opposite kinds, or the sum, if of the same, of the last multiplier and the first double departure, will be equal to the assumed multiplier, if the work is right.

Multiply each of the corrected differences of latitude by its collateral multiplier, and place the product in the north or south area columns, observing, that whenever the multipliers are of the same name as the assumed, those products are to be placed in the column of areas, which is of the same name as the latitude; if of different names, in the opposite column.

Half the difference, between the sum of the north and south areas will be the area of the survey.

## DEMONSTRATION.

Let  $ABCDEF$  be a tract of country whose area is required ;



through  $A$  draw a meridian line  $fb$ , and through the several points,  $B$ ,  $C$ ,  $D$ ,  $E$ ,  $F$ , and  $G$ , draw  $Ba$ ,  $Cb$ ,  $Dc$ ,  $Ee$ ,  $Fd$ , and  $Gf$ , perpendicular to this meridian, and also  $gB$ ,  $Cn$ ,  $Dh$ , and  $C$ , parallel to it.

Let the assumed multiplier be  $AB$ .

Now the area of the figure,  $bCDEFGf$ , is equal to the figure  $bCBAGf$  + the required area of the tract. And the difference of latitude and departure of the several sides will be found as in the following table. Thus  $Aa$  is the northing of  $AB$ , and  $aB$  is its easting;  $ab$  is the northing of  $BC$ , and  $gC$  its easting, and so on.

Also, according to the rule,  $aB + gC$  or  $bC$ , is the east north departure of  $BC$ , as they are of the same kind, and  $fG - aB$ , is the W.D.D. of  $aB$ .

Having obtained the double departures for the several sides, assume the multiplier  $aB$ , and find, by the rule, the several other multipliers  $ab + bc$ ,  $bl$ , observing carefully, whether these multipliers

are east or west. And place the areas of the product of the multipliers into their collateral northings or southings, in the columns of north or south areas, according to the proper relations between them; multipliers and northings is given in the rule. These areas are double the required areas—being each of them double the areas of the several trapezoids into which the figure has been divided.

Half the difference, therefore, of the north and south areas, will be the required area.

	N.	S.	E.	W.	E. D. D.	W. D. D.	Multiplier	N. Areas	S. Areas
AB	Aa		aB			$fg - aB$	$aB$ E		
BC	ab		gC		bC		$Bbc$ E	$2abCB$	
CD		bc	Ch		gh		$bC + cD$ E		$2bCDc$
DE		ce		Di	hi		$cD + ci (cE)$ E		$2cDEe$
EF	ed			kF		Do	$eE + dF$ E	$dFEe$	
FG		df		Fm		km	$dF + dm (fg)$ E		$2dFGf$
GA	Af		Gf			co	$dm$ E	$AGf$	

Now " $dm$ " the multiplier last found—the first W. D. D., i. e.  $dm$  or  $fg - (fg - aB) = aB$ , the assumed multiplier.

**EXAMPLE I.**—Given the bearings and distances of a tract of country, to find the area, viz.:—1st, S.  $16^{\circ} 30'$  E., 3.47 chains; 2nd, S.  $17^{\circ}$  E., 3.02 chains; 3rd, S.  $26^{\circ}$  E., 5.51 chains; 4th, S.  $31^{\circ} 30'$  E., 7.34 chains; 5th, S.  $5^{\circ} 20'$  E., 10.55 chains; 6th, S.  $15^{\circ}$  E., 5.09 chains; 7th, S.  $8^{\circ}$  W., 4.03 chains; 8th, S.  $3^{\circ} 30'$  E., 4.70 chains; 9th, S.  $45^{\circ} 30'$  W., 6.50 chains; 10th, S.  $64^{\circ} 45'$  W., 7.34 chains; 11th, S.  $60^{\circ} 15'$  W., 4.98 chains; 12th, N.  $22^{\circ} 45'$  W., 27.62 chains; 13th, N.  $67^{\circ}$  E., 2.40 chains; 14th, N.  $13^{\circ} 30'$  W., 6.10 chains; 15th, S.  $62^{\circ} 30'$  W., 4.16 chains; 16th, N.  $27^{\circ} 30'$  W., 8.24 chains; 17th, N.  $41^{\circ} 10'$  E., 4.47 chains; 18th, N.  $4^{\circ}$  W., 16.40 chains; 19th, N.  $84^{\circ}$  E., 4.03 chains; 20th, S.  $69^{\circ} 30'$  E., 18.21 chains—to the place of beginning.

Station.	Bearings	Dist.	N. Lat.	S. Lat.	E. Long.	W. Long.	Corrections of Latitude	Cor. N.	Cor. S.	Cor. E.	Cor. W.	E. D. D.	W. D. D.	Multiplicars	N. Areas	S. Areas
1	S. 16°30' F.	3.47		3.33	0.98		—		3.33	0.98		18.04		00.00	E.	000
2	S. 17° E.	3.02		2.89	0.89		—		2.89	0.89		1.87		E.	E.	5.4043
3	S. 26° E.	5.51		4.95	2.41		1		4.94	2.41		3.30		E.	E.	25.5398
4	S. 31°30' E.	7.34		6.26	3.84		1		6.25	3.84		6.25		E.	E.	71.3750
5	S. 5°20' E.	10.55		10.51	0.97		2		10.49	0.97		4.87		E.	E.	170.2527
6	S. 15° E.	5.09		4.92	1.31		1		4.91	1.31		2.28		E.	E.	90.8841
7	S. 8° W.	4.03		3.99		0.56	—		3.99		0.56	0.75		E.	E.	70.3474
8	S. 3°30' E.	4.70		4.67	0.28		—		4.69	0.28			0.28	E.	E.	89.0162
9	S. 45°30' W.	6.50		4.56		4.64	1		4.55		4.64		4.36	E.	E.	66.5210
10	S. 64°45' W.	7.34		3.14		6.64	1		3.13		6.64		11.28	E.	E.	10.4542
11	S. 60°15' W.	4.98		2.47		4.33	1		2.46		4.33		10.97	W.	18.7098	
12	N. 22°45' W.	27.62	25.47			10.68	5	25.52			10.68		15.01	W.	W.	577.7728
13	N. 67° E.	2.40	0.94					0.94		2.21			8.47	W.	W.	29.2434
14	N. 13°30' W.	6.10	5.93			1.42	1	5.94			1.42	0.79		W.	W.	180.1008
15	S. 62°30' W.	8.16		1.92		3.69	—		1.92		3.69		5.11	W.	6.80256	
16	N. 27°30' W.	8.24	7.31			3.80	1	7.32			3.80		7.4	W.	W.	314.1744
17	N. 41°10' E.	4.47	3.36					3.56		2.95			0.85	W.	W.	147.0672
18	N. 4° W.	16.40	16.36			1.15	3	16.39			1.15	1.80		W.	W.	687.8883
19	N. 84° E.	4.03	0.41					0.41		4.01		2.86		W.	W.	16.0351
20	S. 69°30' E.	18.21		6.37	17.06		4		6.33	17.06		21.07		W.	W.	114.1932
		154.16	59.78	60.00	36.91	36.91	22	59.88	59.88	36.91	36.91				200.9880	2558.5701
			59.78	59.78												200.9880

22 deficiency of northings.

5154.16

31 links of allowable error.

A. R. P.

Ans. = 117 3 20

22357.5881

101178.7944

117.8794

Required the area of a tract of land, whose bearings and distances are as follows, viz.:—1st, N.  $15^{\circ} 42'$  E., 6·20 chains; 2d, N.  $52^{\circ} 18'$  E., 6·75 chains; 3d, S.  $78^{\circ} 48'$  E., 5·96 chains; 4th, S.  $5^{\circ} 51'$  E., 4·84 chains; 5th, S.  $49^{\circ} 15'$  W., 4·75 chains; 6th, S.  $4^{\circ} 57'$  E., 3·98 chains; 7th, S.  $71^{\circ} 24'$  W.,      chains; and 8th, N.  $46^{\circ} 18'$  W.,      to the place of beginning.

Given the following bearings and distances to find the area, viz.:—1st, N.  $10^{\circ} 21'$  W., 4·50 chains; 2d, N.  $9^{\circ} 48'$  E., 5·20 chains; 3d, N.  $75^{\circ}$  W., 3·00 chains; 4th, N.  $20^{\circ} 3'$  E., 4·86 chains; 5th, S.  $45^{\circ}$  E., 5·20 chains; 6th, N.  $3^{\circ} 18'$  W., 2·50 chains; 7th, E. 700 chains; 8th, S.  $12^{\circ}$  W., 3·94 chains; 9th, S.  $43^{\circ}$  E., 4·15 chs.; 10th, S.  $46^{\circ} 57'$  W., 8·20 chains; 11th, S.  $29^{\circ}$  E., 9·15 chains; 12th, S.  $48^{\circ}$  W., 4·56 chains; 13th, N.  $19^{\circ}$  W., 3·42 chains; 14th, S.  $28^{\circ}$  W., 8·54 chains; 15th, N.  $53^{\circ}$  W., 4·60 chains; and 16th, —,—, to the place of beginning.

## CHAP. VI.

### DIVISION OF LAND.

#### PROBLEM I.

*To lay out a given quantity of land in a square form.*

Bring the given quantity into square chains, and take the root which will be in chains.

**EXAMPLE 1.**—It is required to lay out 400 acres of land, in a square form.

$$400 \text{ acres} = 4000 \text{ square chains.}$$

$$\sqrt{4000} = 63\cdot24 \text{ chs. the length of the side required.}$$

**EXAMPLE 2.**—What will be the side of a square, which contains 496 acres, 2 roods, and 32 perches?



## PROBLEM II.

*To lay out a given area in a rectangular form, having the length to the breadth in a given ratio.*

Let  $A$  = the area in square chains, and  $x$  = the common unit measure of the sides, and  $m$  and  $n$  the ratio ; then,  $mx$  = one side, and  $nx$  = the other, and  $mnx^2 = A$ , and  $x = \sqrt{\frac{A}{mn}}$  or the unit of measure of the sides = the root of the given area divided by the product of the ratios.

EXAMPLE 1.—There are 2000 acres of land to be laid out in a rectangular form, whose sides are to each other, as 4 is to 5 ; what will their lengths be ?

$$m = 4, n = 5, \text{ and } mn = 20. \quad A = 20,000 \text{ sq. chains}$$

$$\therefore x = \sqrt{\frac{20000}{20}} = \sqrt{1000} = 31.62 \text{ chs.}$$

$$4x = 31.62 \times 4 = 126.48 \text{ chs.} = \text{one side}$$

$$5x = 31.62 \times 5 = 158.10 \text{ chs.} = \text{other side}$$

EXAMPLE 2.—One side of a rectangular field is double the other, what are the sides, when the area is 20.018 perches.

*Ans.* 10.03 chs. and 20.06 chs.

EXAMPLE 3.—A man has a farm of 150 acres, of a rectangular form, the depth of the farm is 50 chains. He is desirous of adding 50 acres to it, having the same depth, to make up 200. What will be the length of frontage of his farm, after the addition ?

*Ans.* Half a mile.

## PROBLEM III.

*When one of the sides is a certain length longer than the other,*

Let  $x$  be the unit of measure as before, and  $m$  the excess of the one side, then  $x$  and  $x+m$  will be the two sides, and their product, or  $x^2+mx$ , will be the given arc ( $A$ ) in square chains : completing the equation we have  $x = \sqrt{A + \frac{m^2}{2}} - \frac{m}{2}$  ; or the following rule,

*To obtain the smaller side ( $x$ ), square half the difference of the two sides, add it to the given area, and taking the root of their sum, from this root subtract half the difference.*

**EXAMPLE.**—Given 464 acres, it is required to lay it out in a rectangular form, the one side being 6 chains longer than the other.

Let  $x$  = the one side,  $x+6$  = the other

$$x^2+6x = 4640 \text{ square chains}$$

$$x^2+6x+9 = 4649$$

$$x+3 = 68.18$$

$$x = 65.18$$

$$x+6 = 71.18$$

*Ans.* One side = 65.18 chs.; the other = 71.18 chs.

#### PROBLEM IV.

*From a given block of land, with parallel sides, the angle of inclination of the front and sides being given, to cut off a given area, by a line parallel to the sides.*

**RULE.**—Find first the perpendicular depth of the given block, (which is the nat. sin. of the angle of inclination  $\times$  the length of the side), then, as all parallelograms upon equal bases, and between the same parallels, are equal, divide the given area by this depth for the frontage required.

**EXAMPLE 1.**—The concession road, of a certain township, bears N.  $74^\circ$  E., while the side lines bear N.  $9^\circ$  W. The length of the side lines is 66.66 chains, and the fronts of the lots are 30 chains. Required the frontage that must be taken, to cut off 100 acres, by a line parallel to the side lines.

The side lines, bearing N.  $9^\circ$  W., and the fronts, N.  $74^\circ$  E., the included angle, or angle of inclination, of the front to the sides, is  $9^\circ+74^\circ$  or  $83^\circ$ .

$\therefore$  the depth of the lots equals nat. sin.  $83^\circ$  into 66.66 chs.

$$\log. \sin. 83^\circ = 9.99675$$

$$+\log. 66.66 = 1.82387$$

$$11.82062$$

$$-\log. \text{radius} = -10.$$

$$\log. \text{perpend. depth } 66.66 = 1.82062$$

$$100 \text{ acres} = 1000 \text{ chains}$$

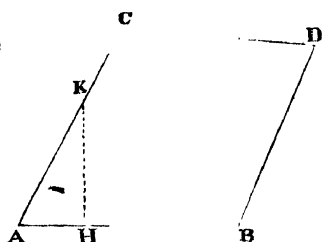
$$\frac{1000}{66.16} = 15.12 = \text{the frontage required.}$$

**EXAMPLE 2.**—When the fronts of the lots bear S. 80° W., and the side lines N. 15° E., the side lines being 101·50 chains long, and the concession road frontage 20·00 chains, what will be the frontage required, to divide the whole lot between A and B, giving A 50 acres more than B, and what quantity will each have ?

### PROBLEM V.

*With the same data as the preceding, except that the dividing line is not to be parallel to the sides, but at right angles to the concession roads.*

Let ACDB be the given lot ; from C draw CE, at right angles to its front AB ; CE is the perpendicular depth of the lot ; find this, as in the preceding Problem, and determine AE, which is the cosine of the given angle of inclination, to which CE is the sine ; find the area of the triangle AEC, which is equal to  $\frac{AC}{2} \sin. \theta. \cos. \theta.$



If this be greater than the required area, then the dividing line will fall within the triangle AEC as Hk, and the triangle AHK will be similar to AEC ; but similar triangles are as the squares of their similar sides, (see Theor. 19, p. 15,) and, therefore, making  $Ak = x$ , we have the following proportion to determine it, viz.:—

As the whole area AEC : the given area ::  $AE^2 : x^2$

**EXAMPLE 1.**—The data of the preceding example being assumed, it is required to cut off six acres, by a line at right angles to the front of the lot.

We found in the last example  $CE = 66\cdot16$

$$\log. \cos. CAE = 9\cdot08589$$

$$+ \log. AC = 1\cdot82387$$

$$\log. 8\cdot12 = 0\cdot90976$$

$$\text{area of AEC} = \frac{66\cdot16 \times 8\cdot12}{2} = 268\cdot61 \text{ sq. chs.}$$

$$\text{By proportion, — As log. } 268\cdot61 \text{ chs.} = 2\cdot42911$$

$$\text{is to log. } 60 \text{ chs.} = 1\cdot77815$$

$$\text{so is } 2 (\log. 8\cdot12) = 1\cdot81912$$

$$3\cdot59727$$

$$2\cdot42911$$

$$2 \mid 1\cdot16816$$

$$AK = \log. 3\cdot63 \text{ chs. frontage} = 0\cdot58408$$

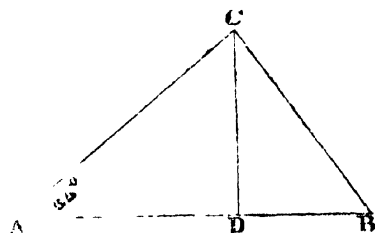
**EXAMPLE 2.**—It is required, with the same data, to find the length of the side HK, and determine the actual area of AKH.

*Ans.* HK = 31·28 chs. ; and the area of AEC = 5 3 39 A. R. P.

### PROBLEM VI.

*To lay out a given area in the form of a triangle, when the base, and the angle at the base, are given.*

Let ABC = the given triangle, having the base AB, and the angle BAC, given ; and let the area of the triangle = A ; it is required to find, first, AC, and thence AD, and DC.



By the nature of triangles,

$$A = \frac{AB}{2} \cdot DC = \frac{AB}{2R} \cdot \sin. \angle A \cdot AC$$

$$\therefore \frac{AC}{2} = \frac{\text{Area} \times R}{AB \sin. \angle A}$$

$$\text{then } DC = AC \sin. \angle A$$

$$\text{and } AD = AC \cos. \angle A$$

**EXAMPLE 1.**—There are 46 acres of land to be laid out in the form of a triangle, whose base is 15 chains, and the angle 35 degrees ; required the lengths of the other two sides.

Here AB = 15, and the angle CAD = 35°,

$$\text{now } \frac{AC}{2} = \frac{\text{Area}}{AB \sin. \angle A}$$

$$\log. \text{area (460 square chains)} = 2.66276$$

$$+\log. \text{rad.} = 10. \quad \underline{\hspace{1cm}} \\ 12.66276$$

$$\log. AB (15 \text{ chs.}) = 1.17609$$

$$+\log. \sin. \angle A (35^\circ) = 9.75859$$

$$10.93468 = \underline{\hspace{1cm}} 10.93468$$

$$\frac{AC}{2} = \log. 53.47 = 1.72808$$

$$\underline{\hspace{1cm}} 2$$

$$\text{chs. } 106.94 = AC$$

**EXAMPLE 2.**—Having the same data, to find the length of BC, in the previous example, and also DC ; and thence to determine, whether, with these lengths, the triangle will contain the given area.

## PROBLEM VII.

*From a given triangle, to cut off any given area, by a line drawn from the vertex to the base.*

Triangles, of equal altitudes, are proportional to their bases (see Theorem 17, page 15); therefore, making  $A$ , the given area of the triangle;  $a$ , the part to be cut off; and  $x$ , the required portion of the base, we have

$$x = \frac{a \cdot \text{base}}{A}$$

**EXAMPLE 1.**—There is a Gore of land between two townships, whose area is 425 acres, and base is 85 chains. It is required to cut off 400 acres by a line drawn from the vertex.

As 435 : 400 : 85 chains.

$$\begin{array}{r} 400 \\ 425 \overline{) 34000} \end{array} \begin{array}{l} 80 \text{ chains} \\ 34000 \end{array}$$

length of new base = 80 chains

**EXAMPLE 2.**—From a Gore of land, having a base of 40 chs., containing 125 acres, to cut off 50, by a line from the vertex, required the base.

**EXAMPLE 3.**—From a triangle, with a base of 74.54 chs., containing 35 acres, to cut off 860 square yards, required the base.

I have annexed a few questions of a more practical nature, and worked them out fully, so that the reader may make himself acquainted with the means of ensuring that practical accuracy, which is indispensable for success, in a profession to which so much responsibility is attached, and where the consequences of inaccuracy are so serious and lasting as in the survey of a new country. In Canada, where I was engaged for some time, one third (I believe I am speaking within compass) of the common law cases were either disputed surveys, or originated in the bad feeling engendered from encroachments, or supposed encroachments upon each other's property.

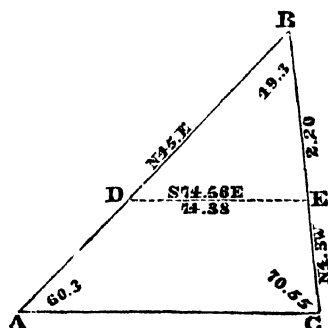
## PRACTICAL EXAMPLES.

1st. There is a Gore of land between two townships, the boundary line of one being N. 45° E., and that of the other N. 4° 3' W.; the length of the first line, 240.00 chs., that of the second 220 chs.

It is required to cut off, at the vertex end, 300 acres, by a line parallel to the base of the Gore.

Because the one line bears N. 45° E., and the other N. 4° 3' W., the angle between them is 49° 3'.

$$\frac{240 + 220 + \sin 49^\circ 3'}{2 R} = \text{area.}$$



log. $\frac{240}{2}$	= 120 chains	= 2.07918125
log. 220 chains		= 2.34242268
log. sin. 49° 3'		= 9.87810900
		14.29971293
		— 10
		4.29971293

1994 acres = 1994000 sq. chains = 4.29971293

Now, as similar figures are proportional to the squares of their homologous sides in the whole, the whole area is to the given area as the square of one of the given sides is to the square of that portion of the side, that is to be cut off: that is,

As 1994 sq. ch. : 3000 sq. ch. ::  $240^2$  :  $x^2$   
 when  $x^2$  = length to be cut off.

As log. 1994	=	4.29971293
is to log. 3000	=	3.47712130
so is log. $240^2$	=	4.76042240
	=	8.23754370
	=	4.29971293
to log. 93 chs. 09 lks.	=	2.393783077
		196891538

The other side BE will by the same process be found to be 85.33 chs.

Having obtained the lengths of BD and BE, the two sides of the piece to be cut off, that will contain 300 acres, together with the included angle 49° 3', we can calculate the angles at the base by the second case of trigonometry.

These angles will be found to be as follows, viz.: — 70° 55' = greater angle, and 60° 3' = the less.

To prove the calculation.

Because the area =  $\frac{DB \cdot BE \cdot \sin 49^\circ 3'}{2 R}$

300 area =  $\frac{85.33 \times 93.09 \times \sin 49^\circ 3'}{2 R}$

log. 42.66 chs.	= 1.630029
+ log. 93.09 chs.	= 1.968915
+ log. sin. 49° 3'	= 9.878109
	<u>13.477053</u>
	10
3000 square chains	= 3.477053

Having verified the correctness of the calculated sides, we proceed to obtain the length and bearing of the base.

As the sin. 60° 03'	= 9.9377492
is to 85.83 chs.	= 1.9311260
so is sin. 49° 03'	= 9.8781090
	<u>11.8092350</u>
	9.9377492
DE = 74.38 ch.	= 1.8714858

*To obtain the bearing.*

AB bears N. 45° 3'
add 60° 3'

105° 6' is the angle made with the north, because it is greater than a right angle; subtract it from 180, and its supplement, 74° 54', is the angle made with the south point of the needle, being, therefore, S. 74° 54' E.

In order, therefore, to cut off 300 acres from the Gore ABC, measure from B to A, 93.08 chs., and at that point run a line, bearing S. 74° 54' E, which will be parallel to the base of the Gore, and should be 74.38 chs. in length.

*The same data being given*, it is required to cut off 300 acres towards the township, whose boundary line bears N. 45° E, by a line drawn from the vertex to the base.

Using whatever calculations may have been made above, that may be useful to our purpose, first ascertain the base of the Gore.

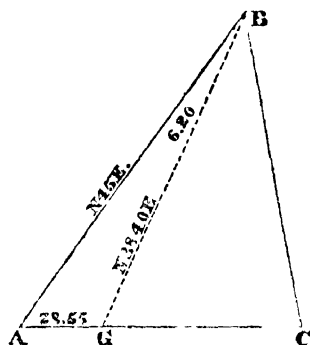
As sin. 60° 3'	= 9.9377492
is to 220 chs.	= 2.3424227
so is sin. 49° 3'	= 9.8781090
	<u>12.2205317</u>
	9.9377492

to 191.17 chs. = AC length of base of Gore = 2.2827825

Now triangles of equal altitudes are to each other as their bases; therefore, the whole Gore is to the piece to be cut off, as the whole base is to the base of the piece cut off.

As 19940 sq. chs.	= 4·27991293
is to 3000 sq. chs.	= 3·47712130
so is 191·77 chs.	= 2·28272250
	<hr/>
	5 75990380
	4·29971297
	<hr/>
to AG = 28·85 ch.	= 1·46019087

Next find the angles of the triangle ABG, by the rule for the second case of trigonometry, so as to determine the length and bearing of BG.



These angles will be found as follows, viz.:— $113^{\circ} 36'$  = larger angle AGB, and  $6^{\circ} 20'$  = the smaller angle ABG.

BA is S. $45^{\circ}$ W.	$45^{\circ} 0'$
subtract	$6^{\circ} 20'$
	<hr/>
	$38^{\circ} 40'$

the bearing, therefore, is S.  $38^{\circ} 40'$  W.

To find the length of the line BG.

As sine $6^{\circ} 20'$	= 7·04262490
is to 2885 chs.	= 1·46019087
so is sin. $66^{\circ} 3'$	= 9·93774920
	<hr/>
	11·39794007
	9·04262490
	<hr/>
to 226·65 chs. = BG	= 2·35531517

To lay off the line BG, cutting off 300 acres on the side AB, proceed to the vertex B, and run BG south  $38^{\circ} 40'$  west, till you come to the base, and measure the distance GA, which should be 28·85 chs.; if the actual distance be found to be the same within a few links, the work is finished, but if not (*let it be 29·85 chs. instead—*



that is, one chain too much to the east), it becomes necessary to correct the bearing of the line by the compass, and run it over again.

*To find the angle of correction.*

As 226·65 chs.  
is to sin.  $90^\circ$  or radius  
so is 1·00 chs.  
to the sin. opposite  $\angle$

As 226·65 :  $57^\circ 3'$  : : 1·00 :  $x$  when  $x$  = length of the arc in degrees, and  $57^\circ 3'$  is the angle, whose length of arc equals radius.\*

57·3		
<u>1·00</u>	present bearing	S. $38^\circ 40'$ W.
57·3	add	<u>15'</u>
<u>60</u>	corrected bearing	S. $38^\circ 55'$ W.
226)343·80(15'		

### PROBLEM VIII.

*The bearings of three sides of a triangle being given, to cut off a given area by a line parallel to the base.*

**RULE 1st.**—These three lines form a triangle, whose angles are all known, make one side = unity, and calculate the proportionate lengths of the other sides, making this side =  $x$ , and multiplying by the other two sides; all the sides are known in terms of  $x$ ; find the area, which will be equal to the given area; the solution of the equation gives the answer: or,

**RULE 2nd.**—Say, As the rectangle of the sines of the angles at the base : the rectangle of radius into the sine of the opposite angle : : twice the area : square of the side required. The other sides may be found in the same way.

**EXAMPLE 1.**—Let one of the boundary lines of a Gore, between two townships, bear N.  $38^\circ$  W., and the other N.  $16^\circ 30'$  E.; it is required to cut off 325 acres from the Gore end, by a line, bearing N.  $88^\circ 15'$  E.

$$* 2 \pi \text{ rad.} = \text{circumference} = 360 \text{ degrees.}$$

$$\pi \text{ rad.} = 180 \text{ deg.} : \text{rad.} = \frac{180^\circ}{-} = 57^\circ 3'.$$

**RULE 1st.**—First, find the interior angles, and let  $AB = 1$ , or unity.

*To find BC or AC*

$$\begin{array}{rcl}
 \text{As sine } 53^{\circ} 45' & & = 9.90657 \\
 \text{is to } 1^{\circ} & & = 0.00000 \\
 \text{so is sine } 71^{\circ} 45' & & = 9.97759 \\
 \text{to } 1.178 \text{ chs. BC} & & = 0.07102
 \end{array}$$

$$\begin{array}{rcl}
 \text{As sine } 53^{\circ} 45' & & = 9.90657 \\
 \text{is to } 1^{\circ} & & = 0.00000 \\
 \text{so is sine } 54^{\circ} 30' & & = 9.91069 \\
 \text{to AC} = 1.01 & & = 0.00412
 \end{array}$$

$$AB = 1.000$$

$$AC = 1.010$$

$$BC = 1.178$$

Let fall a perpendicular BD.

$$\text{Area} = \frac{AC \cdot AB \sin. \angle CAB}{2}.$$

$$\begin{array}{rcl}
 \log. AB, \text{ or } 1.000 & & = 0.00000 \\
 \log. AC, \text{ or } 1.010 & & = 0.00412 \\
 \sin. \angle CAB & & = 9.97757 \\
 -\log. \text{rad.} & & = 10.00000 \\
 2 \mid .9588 & & = 1.98171
 \end{array}$$

.4794 square chains.

Because similar figures are as the squares of their homologous sides.

$$\begin{array}{rcl}
 \text{As } .4794 \text{ sq. chs.} & & = 1.68070 \\
 \text{is to } 3250 \text{ sq. chs.} & & = 3.51188 \\
 \text{so is } 1^2 & & = 0.00000 \\
 \text{to } x^2 & & = 2 \mid 3.83118 \\
 82.34 = AB & & 1.91559
 \end{array}$$

which is the measure of the side taken as the unit side of the three.

## RULE 2nd.—

$$\begin{array}{rcl}
 \text{As sine A. sine B} & \left\{ \begin{array}{l} 9.97759 \\ 9.91069 \end{array} \right\} & = 19.88828 \\
 \text{is to rad. sine C.} & \left\{ \begin{array}{l} 9.90657 \\ 10.00000 \end{array} \right\} & = 19.90657 \\
 \text{so is twice the area (6500 sq. chs.)} & & = 3.81291 \\
 & & \underline{23.71948} \\
 & & 19.88828 \\
 & & \underline{\hspace{1.5cm}} \\
 & \text{to } x^2 = 2 & | \ 3.83120 \\
 82.34 = \text{AB.} & = x = & \underline{1.91560}
 \end{array}$$

the same as before.

To find AC and BC, the other two sides (by the first rule); multiply each of the sides of the unit triangle by this common unit of measurement, or marking A = the actual area, and  $a$ , the unit area, and  $x$ , as before, the length of any other side, in the unit triangle  $a$ , which was taken; then we have, by similar triangles,

$$a : A :: x^2 : y^2.$$

$$i. e. \frac{Ax^2}{a} = y^2 \text{ where } y \text{ is the actual length of the same side.}$$

$$x \sqrt{\frac{A}{a}} = y \text{ or } \log. x + (\log. \sqrt{\frac{A}{a}} = 1.91560) = \log. y.$$

where  $\sqrt{\frac{A}{a}}$  or 1.91560 is the actual length of the unit side (AE).

$$\text{Let } x (= 1.010) \text{ then } \log. x = 0.00412$$

$$\log. \sqrt{\frac{A}{a}} = 1.91559$$

$$\text{AC} = 83.12 \text{ chs.} = 1.91971$$

Again,

$$\text{Let } x = 1.178 = 0.07102$$

$$\log. \text{unit of measurement} = 1.91559$$

$$\text{BC} = 96 \text{ chs. } 96 \text{ lks.} = 1.98661$$

$$\text{AB} = 82.34 \text{ chs.}$$

$$\text{BC} = 96.96 \text{ chs.}$$

$$\text{AC} = 83.12 \text{ chs.}$$

To find the side AC (by the second rule).

$$\text{As the sin. } \angle A \text{ sin. } \angle C. \left\{ \begin{array}{l} 9.97759 \\ 9.90657 \end{array} \right\} = 19.88419$$

$$\text{is to radius sin. } \angle B. \left\{ \begin{array}{l} 9.910692 \\ 10.000000 \end{array} \right\} = 19.91069$$

so is twice the area (6500)

**3-81291**

**23-72360**

19.88416

to  $x^2 = \overline{2|3.83994}$

$$AC = 83.12 \text{ chs.}$$

$$= x = 1.91972$$

*To find the side BC.*

As sin. B sin. C

$$\left. \begin{array}{l} 9.91069 \\ 9.90657 \end{array} \right\} = 19.81724$$

$$9.90657 \}$$

is to rad. sin. A

$$\begin{array}{r} 9.97759 \\ 10.00000 \end{array} \} = 19.97759$$

10.00000

so is twice the area (6500) sq. chs.

$$= 3.81291$$

23-79050

19.81724

to  $x^3 = 2|3.97324$ .

BC = 96 chs. 96 lks.

$$= x = 1.98662$$

the same as before.

**EXAMPLE 2.**—Given the bearing of a Gore of land, north 12 deg. east, and north 21 deg. west, and the base 156 chains, 50 links, bearing north 86 deg. east.

It is required to cut off 100 acres from the frustrum end of the Gore.

$$\text{As sin. } 33^\circ = 9.73611$$

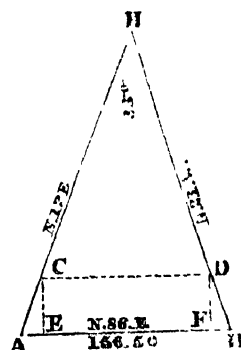
is to 156.50 chs.  $\equiv 2.19451$

so is  $\sin. 73^\circ = 9.98060$

12-17511

9.73611

to  $\Delta E = 2.43900$



perpendicular of whole triangle =  $AE \cdot \sin. A,$

$$= 9.98284$$

$$= 2.43900$$

$$\log. 264 \cdot 20 = 2 \cdot 42184$$

$$\begin{array}{rcl}
 \log. 264 \cdot 20 & & = 2 \cdot 42184 \\
 + \log. 156 \cdot 50 & & = 2 \cdot 19451 \\
 \hline
 2 \overline{) 41330} & & = 4 \cdot 61635
 \end{array}$$

Area of triangle =  $\frac{20665}{2}$  square chains.

Subtract  $\frac{1000}{2}$  sq. chs., the area of the frustrum to be cut off.  
 there results  $\frac{19665}{2}$  sq. chs. the portion to be left at the vertex end.

$$\begin{array}{rcl}
 \text{As } 20665 \text{ square chains} & & = 4 \cdot 31523 \\
 \text{is to } 19665 \text{ square chains} & & = 4 \cdot 29368 \\
 \text{so is } 274 \cdot 80 \overline{)}^3 & & \underline{4 \cdot 87800} \\
 & & 9 \cdot 17168 \\
 & & 4 \cdot 31523
 \end{array}$$

$$\begin{array}{rcl}
 274 \cdot 80 & & 2 \overline{) 4 \cdot 85645} \\
 278 \cdot 05 \text{ chains} & & = 2 \cdot 42823 \\
 \hline
 6 \cdot 75 \text{ chains} = AC
 \end{array}$$

As  $\sin. 74^\circ = BD$ ,  $\sin. 73^\circ$

$BD = AC$ ,  $\sin. 74^\circ$

$\sin. 73^\circ$

$$\log. 6 \cdot 75 = 0 \cdot 82930$$

$$\log. \sin. 74^\circ = 9 \cdot 98284$$

$$\underline{10 \cdot 81214}$$

$$-\log. \sin. 73^\circ = -9 \cdot 98060$$

$$BD, 6 \cdot 785 \text{ chs.} = 0 \cdot 83154$$

*To prove this work,*

$$\cos. 74^\circ. AC = AE$$

$$\cos. 73^\circ. BD = BF$$

$$\log. AC, 6 \cdot 75 \text{ chs.} = 0 \cdot 82930 \text{ and } EC = \sin. 74^\circ. AC$$

$$\log. \cos. 74^\circ = 9 \cdot 44034$$

$$\log. \sin. 74^\circ = 9 \cdot 98284$$

$$\log. AE, 1 \cdot 8605 = 0 \cdot 26964$$

$$\log. 6 \cdot 75 = 0 \cdot 82930$$

$$\log. 6 \cdot 488 = 0 \cdot 81214$$

$$\log. BD, 6 \cdot 78 \text{ chs.} = 0 \cdot 83154$$

$$\log. \cos. 73^\circ = 9 \cdot 46594$$

$$\underline{0 \cdot 29748}$$

$$1 \cdot 8605$$

$$1 \cdot 9837$$

$$2 \overline{) 3 \cdot 8442}$$

$$1 \cdot 9221$$

$$156 \cdot 50$$

$$1 \cdot 92$$

$$\log. 154 \cdot 58 = 2 \cdot 18915$$

$$\log. 6 \cdot 48 = 0 \cdot 81214$$

$$100 \ 1 \ 8 = \text{sq. chs. } 1003 = 3 \cdot 00129$$

R. P.

or 1 8 of excess in 100 acres.

sufficiently near for practical purposes, as if the perpendicular EC had been 6 chains 47, instead of 6 chains 48, the result would have been exactly 100 acres.

## CHAP. VII.

### POLE STAR.

THE  $\alpha$  of Ursa Minor, or as it is more commonly termed Polaris, is about  $1\frac{1}{2}^{\circ}$  from the true pole, and revolves round it in 23 hours 56 minutes. When it is at its greatest distance, east or west, it is said to be at its eastern, or western elongation.

The true bearing of the pole star, that is, the angle made at the centre of the earth, between the true pole and the pole star, is called the polar azimuth; this, which should be taken at the time of its greatest elongation, depends upon the latitude of the place, and the distance of the star from the pole. This distance is called the polar distance, it is subject to a small annual diminution, called precession, which is 19.3 seconds annually. In the year 1830 this distance was  $1^{\circ} 35' 50''$ ; by multiplying the number of years since by 19.3 seconds, and deducting the product, the actual polar distance can be obtained. Now the azimuth, or angle of variation of the pole star, can be always determined by the following proportion:—

As radius : sin. lat. :: sin. polar dist. : azimuth.

Having obtained the polar distance to any day and place, in order to ascertain the angle of variation, or polar azimuth, find the time in the Nautical Ephemeris, when the star is about its greatest eastern or western elongation, and, with a theodolite or circumferentor which is furnished with a telescope, observe, carefully,

when the star, having ceased to move in its first direction, begins to retrograde ; fix the telescope carefully in that place, and direct an assistant to take a stake, with a lighted candle upon it, and put it down in the same line, at some 8 or 10 chains distance. This is the line of elongation ; then say,—as radius is to the whole distance, which must be carefully measured, so is the tangent of the angle of variation, to the actual distance in feet, measured at right angles to the former ; the line connecting this new point, and the place where the instrument was, is the meridian line required. Take the bearing of this line, and the angle, found between this and the magnetic north, becomes the angle of variation of the compass ; should the needle point to the east of this meridian line, the variation is easterly ; should it point to the west, it is westerly.

## CHAP. VIII.

### ON LOCAL ATTRACTION.

THE needle being often, from various causes, diverted from its polarity, it becomes requisite, in running a line, to try the *backward* bearing at every station, and to see if it corresponds with the forward : if it does not, try the last forward station again, to see if any error may have been committed in taking it, should there be none, as the previous backward bearing, by assumption, must correspond with the preceding forward station, there can be no attraction there, and the attraction must be at the one where the backward bearing differed. Allow for the error or angle of attraction at the next station,

and proceed until this error be compensated. During the continuance of this angle of error, there is local attraction in the neighbourhood.


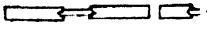
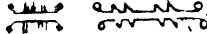
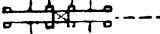





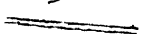


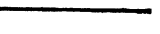



















As at the first station, in starting, there is no backward bearing to prove the non-existence of attraction, it is impossible to say whether the error be at the first or second station, should the backward bearing of the second station not correspond to the forward bearing of the first; by taking, however, a third station, and taking therefrom and thereto, the bearings of both the first and second stations, the error will be discoverable immediately, as the backward and forward bearings of (from 2 to 3 and from 1 to 3) cannot both agree. Where the disagreement is, there is the local attraction.

Attraction may, however, commence with the first station, and not be discovered until some stations afterwards; should this be suspected, it becomes necessary to test the first bearing relatively, by a line making any angle with it, (*which angle has been measured by the chain,*) to such a distance, as may be considered beyond the sphere of the supposed attraction. There are many other *local* checks beside this; lines are seldom run in space unconnectedly in the woods, and, by measuring the distance from the next base line, its correctness may soon be determined.



## CHAP. IX.

THIS page contains a small collection of those conventional signs that are most in use; the method of representation is such as is generally adopted.

CONVENTIONAL SIGNS.	
	River
	Canal
	Bridges
	Drawbridge
	Ford
	Horse Ferry
	Rope Ferry
ROADS.	
	Turnpike Road
	Highway
	Occ. Road
	Bridle Road
	Rail Road
	Cutting
	Embankment
MILLS.	
	Windmill
	Sawmill
	Watermill
	Coal
	Limekiln
	Stone Quarry
	Town
	Village
	with Church
	Post House
	Turnpike
	Smithy
	Telegraph
MILITARY.	
	Redoubt
	Fort
	Artillery Position
	Battery
	Mortar Battery

MINES.		BOUNDARIES.	
⊙	. . . Gold	-----	of a County
☽	. . . . Silver	-----	of a Parish
⚭	. . . . Tin	-----	of a Township
♀			
♂	. . . . Copper		
♂	. . . . Lead	Δ	Station Point
♂	. . . . Quicksilver		

## CHAP. X.

## ON PLOTING, SCALING, &amp;c.

BEFORE commencing to plot, it is always requisite to consider carefully the shape of the plan to be plotted, its size and character, and the most desirable position to place it upon the paper, so as to admit of the best vacant space for the insertion of the heading or title, with the usual specification, that should accompany it. It has generally been considered indispensable to place the plan, so as to have the north side of the plan on the top of the paper fronting you; but, I would recommend, that the position of the meridian line should be but a secondary consideration, and should, in every case, depend upon the size and shape of the plan; where these two desiderata can be combined, it is better, of course, to do so; though I certainly cannot deem it a matter of so much moment as it is often made.

Before commencing your plan, take care also to have the paper properly stretched upon a drawing board, if the size of the plan will admit of it, and finish the whole plan before taking the paper off the board. At the bottom of the paper, make a scale of the required proportion, carefully dividing it into tenths and hundredths as the case may be, and let all *long* lines upon the plan be measured off this scale. The *short* ones, that is, lines of offsets; lines of distances, less than tenths, may be taken off the ivory scale, from which the scale upon the paper was first obtained.

Paper contracts very much, so much so, that there is frequently a difference in the length of the same line between one year and another of two hundredths, when the line is 10 or 11 tenths long; a line one year may measure 20 or 30 chs., and the next be only 19 chs. 80 lks., or 29 chs. 75 lks., being a loss of a link or two in each chain. In distances, therefore, of less than a chain's length, there can be no perceptible error. These distances might always be taken by the common ivory scale of equal parts, and, in fact, should be, as these smaller subdivisions could never be so well divided upon a paper scale; and the divisions upon paper would, from constant using in the making of the plan, soon wear through one into the other. These remarks may appear superfluous to a novice, but experience will soon show him the value of them. The neglect of these considerations has been the source of many a day's loss of work to a beginner.

Having made the scale, lay down your base line very accurately, and draw it carefully in with lake, marking the various stations upon it and its total length. Then take, with the compasses, the various lengths of the

sides of the ~~several~~ triangles, of which the survey is composed, and lay off the different points of intersection, ~~testing~~ rigorously, as you proceed, the constructed, with the measured lengths of the respective *check-lines*.

Do all this *before* an offset is put in, unless the offset be afterwards used as the point of a more convenient base line for another triangle.

When this part of the plotting is found correct, draw the lines in very plainly with lake.

In marking off the several distances on the base line use one of the long scales, and, placing it close against the given line, prick off, with a fine needle, the proper distances, and round the points, as centres, draw a small circle, but on no account use any black lead pencil, however finely sharpened, in stations of this kind. Do this also, in determining the point of inclination of the sides of the triangle. And in cases where great accuracy is required, I should prefer striking the radii, not with a bow pen, but with the finely pointed compasses themselves.

This can be done so slightly, as accurately to show the intersecting arcs, without at all injuring the paper. The mere injury of the paper, however, should never be a subject of any consideration, in comparison with the accuracy of the drawing, as, in all cases, a correct and accurate office drawing must be made, as a plan of reference, in which accuracy is the only desideratum,—accuracy in the outline,—accuracy in the determining of the areas. In a finished plan it is totally different; a finished plan is seldom made, in fact, it ought never to be made, an authority for the extension or compilation of surveys, as accuracy of detail, and beauty of colouring, are seldom, and not easily, combined.

Having finished these subsidiary lines, as they may

with propriety be termed, proceed to the laying off the offset points.

The best method of doing this, is to place the long scale, above referred to, close against the given line, having the zero points of each coinciding, and get an assistant to read off the several distances thereon, whence offsets are taken, first going through the right offsets, then the left. An offset scale is now necessary, which is a small scale of about 3 inches, divided in the same way as the long one, but the zero points being either edge of the scale. This is placed against the long scale, and the lengths on the measured line are determined by the long line, while the distance of any point, or offset therefrom, is determined by the offset scale; this latter point is alone marked. Practice and care will ensure considerable rapidity, as well as accuracy, in this plan.

When the scale is 2 or 4 chains to the inch, any offset, less than 10 links, must be done by the hand.

With reference to the division of the scales—the scales used for horizontal delineations are generally 2 or 4 chains to the inch, or 20 chains, or 40 chains. The usual scale used in the plotting of the tithe commutation surveys, is that of 3 chains to the inch.

The division of the common and vertical sections, is that of 5 or 50 feet, 10 or 100 feet to the inch.

I have one caution to give beginners, in the purchasing of scales, never to purchase a scale having different divisions upon it, as it is productive of considerable and serious error.

The additional expense when you are purchasing—of purchasing one of a set—is soon made up by the saving of the frequent losses of time, that a difference of divisions on the same scale must inevitably occasion.





## CHAP. XI.

## TOPOGRAPHICAL SURVEYING.

IN surveying a tract of country, it is frequently not only necessary that a correct delineation should be obtained of the various outline of hill and dale, of river and forest, but that some method should be adopted of conveying upon the same plan a pretty fairly correct representation of the relative heights of its different parts. The positions of the various boundaries in the plan are, or always should be, the horizontal spaces they would occupy on the earth's surface, totally independent of their height; this height must, therefore, be obtained in some other way; straight lines drawn from the summit, diverging to the bottom, more or less close, as the hill is steep or otherwise, are used in the one case: this is called the vertical method; in the other, the irregularities of the earth are represented by waving horizontal lines, approaching or receding in proportion to the steepness of the ground, or its graduated ascent. The two plates, Nos. 6 and 7, are examples of the first method. There is a boldness of style, and a faithfulness of representation, which I am disposed to consider must give it the preference. This style of drawing is called military drawing, probably from the special demand for these



topographical features of country for military purposes. By this, an officer is enabled, at a glance, to ascertain whether a commanding point, he is desirous of occupying, is accessible to cavalry, or unapproachable by infantry; and he is thereby enabled to decide, whether he can venture to attempt the dislodgement of an enemy from one hill, or can permanently occupy another, in spite of odds that may be sent against him.

This kind of drawing has its advantages even to the civil Surveyor; if well done, the introduction of the features of the country is a great improvement to the plan, abstractedly; and, to the proprietor of an estate, the topographical details are often useful in assisting him in the laying of it out.

I need scarcely refer to the great assistance, that the engineers of the present day have derived from the faithfulness of the topographical delineations of this country in the ORDNANCE MAPS, in the selection of the best lines of routes for railways; every professional man must have experienced this himself.





# LEVELLING,

ITS NATURE AND OBJECTS.

## CHAP. I.

LEVELLING is the art of representing the inequalities of the earth's surface, and of determining the relative heights of any number of points above or below a line, equidistant, at every point, from the centre of the earth. This line is what is understood by the term—a level line; it is that line which is assumed by water when at rest.

The instrument, used for the purpose of levelling, is called a spirit level.

## THE Y SPIRIT LEVEL.

### *Its Description.*

This instrument is merely one portion of almost every other instrument, carried out to its greatest practical perfection. The bubble, which in most instruments forms only a subordinate part in the construction, is in this the chief, the only object of the instrument being to obtain a practical tangent to the earth's surface, or to place

the line of collimation of the telescope in a truly horizontal line. Hence it is termed **THE SPIRIT LEVEL—*par excellence***.

This instrument consists, like the others, of its parallel plates, with their two pairs of conjugate screws, of its telescopes, and its spirit level beneath. The telescope stands upon Y's, (I have selected the Y level in preference to the rest for description,) as in the case of the theodolite; and has, also, like that instrument, its milled-head adjusting-screw for the object-glass; and the moveable eye piece for neutralizing the parallax. The cross wires, however, are not always arranged the same way as in the theodolite; sometimes one horizontal and two perpendicular wires are used instead. By the latter contrivance you are enabled to secure the perpendicularity of the staff, in one direction, which you have no means of doing by the former.

The spirit level here, also, is furnished with its capstan-headed screws, for making it parallel to the axis of the telescope, vertically and laterally.

But in this instrument there is one contrivance, which the other instruments do not possess—of raising or depressing one of the Y's, or supports, on which the telescope rests, so as to have the axis of the telescope always at right angles to the axis of the instrument.

#### ADJUSTMENTS.

I need only repeat, that, for the line of collimation,\* the telescope must be turned round on its axis, that the intersection of its wires may always intersect one point, this adjustment will be found fully explained in

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\* The two first adjustments are similar to those of the theodolite.

the description of the theodolite. The adjustment of the second liability to error, in the non-parallelism of the level and the telescope, is obtained as before, by reversing the telescope in its Y's. These two adjustments must be completed first. Afterwards,

*To make the axis of the telescope always at right angles to the axis of the instrument; in other words, to secure the line of collimation being perfectly level in any portion of a complete revolution of the instrument.*

Set the telescope over any pair of conjugate screws, and make the bubble level; turn the instrument, till the telescope be over the conjugate pair: level it in this position; then turn it back to the first pair, and correct for any error that may have arisen from the last levelling, and continue till the bubble be central over the two pairs of conjugate screws; then turn the instrument one half revolution round, and, if the bubble still remain in the centre, the instrument is in adjustment; if not, the error can only be occasioned by the axis of the bubble, or, which is the same thing, the axis of the telescope not being truly perpendicular to the centering of the instrument.

To correct this error, raise or depress the moveable (Y) support by the milled-headed screw beneath, until the bubble be brought half-way to its proper position, and correct for the other half by the parallel screws. By repeating the correction two or three times, the greatest accuracy will be obtained.

It is necessary to examine the adjustment every morning before starting, and it should be seen to at every observation, though it will scarcely require re-adjusting the same day. I should observe, that there is, or ought to be, a cap over this adjusting (Y) screw which should be carefully kept on.

This level is a very delicate instrument, and soon liable to get out of adjustment without great care.

There are several other kinds of levels—Troughton's, Gravatt's, &c.: all good of their kind, and each, perhaps, more fitted than the rest for some peculiar kind of levelling.

In trial and check-levels, I would recommend Gravatt's or Troughton's, being calculated by their lightness and non-tendency of disarrangement, to get rapidly over the ground.

For the main sections, at every two chains, I should prefer the Y level; and for the putting down the rails, the formation of roads, and all work where accuracy, and not expedition, are required, I should decidedly give it the preference.

There is one fault I have found with most levels, that the tube of the telescope is not long enough to admit of reading off the staff within short distances, few reading within half a chain. Having had placed, for myself, in addition to the extending tube at the object end, another at the eye-glass to remedy this defect, I have been enabled to read within three yards; to this inner tube, of course, was attached the diaphragm of the cross wires and the lengthening eye-piece.

#### LEVELLING STAVES.

These are generally made twelve feet high, divided into feet, and again into tenths of a foot, and subdivided for facility of computation, into hundredths. The method of arranging this subdivision, constitutes the difference between the several staves in use. Some persons prefer one, some another, and I must leave the reader to select for himself. There are

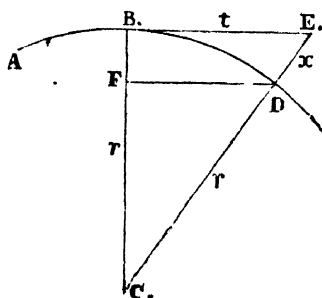
Sopwith's, Gravatt's, Cowper's, Stephenson's, &c. The one I myself prefer is, what I believe goes by the name of Stephenson's: and was first used on the London and Birmingham; the hundredths are obtained in the same way as in the common ivory protractor; the tenths of a foot through the whole length of the staff are bisected, making the two divisions "twentieths;" and these division lines extend the whole breadth across the staff; the opposite ends of these lines are connected by diagonal lines, each one with its preceding, viz.,—the left of No. 1 with the right of No. 2, the right of No. 2 with the left of No. 3, and so on. And five vertical lines are drawn along the whole of the staff, which thus divides each of these diagonal lines into five equal parts, each being the fifth part of the twentieth, or the hundredth of a foot.

The feet are distinguished by large red figures; the tenths in black, with a large full point at every .5.

The lines obtained by this means are only level lines for very short distances, being *tangents* to the earth at the several points of observation, and not their corresponding *arcs*, which are the true levels.

The value of this error and the practical methods adopted had better be at once explained.

Let ABD be a portion of the earth's circumference, whose centre is C. Let BE be any level distance of the instrument ( $t$ ), the true level line will be BD, and the error between the apparent and true level will be the versed sine BF. When the distance BE is great, this versed sine must be calculated by trigonometry.





nometry. But, for the usual distances of observation by the spirit level, ED, or sec.—rad., can be safely taken instead.

Now, CBC is a right-angled triangle, and therefore

$$\begin{aligned} r+x &= r^2+l^2 \\ r^2+2rx+x^2 &= r^2+l^2 \\ 2rx+x^2 &= l^2 \end{aligned}$$

throwing  $x^2$  away, as indefinitely small, in relation to  $2rx$ , we have

$$\begin{aligned} 2rx &= l^2 \\ \text{and } x &= \frac{l^2}{2r}, \text{ or} \end{aligned}$$

the error  $x$  = the square of the tangent, divided by twice the radius; and, as this divisor is a constant quantity, this error is proportioned to the squares of the distances.

The mean diameter of the earth is 7,916 miles; for one mile distance, therefore, we shall have  $x = \frac{1}{7916}$  miles, or 8,004 inches; for two miles distance, four times that quantity; for three miles, nine times; throwing away the  $\frac{1}{1000}$  of an inch as immaterial, the error of one mile's distance is 8 inches or  $\frac{2}{3}$  of a foot; for two miles,  $\frac{8}{3}$  feet; for three miles,  $\frac{27}{3}$  feet, &c.; or  $x$  in feet =  $\frac{1}{3}$  (distance in miles)<sup>2</sup>; which formula may be easily remembered. This value of  $x$  would not be sufficiently correct for many miles. Referring to the equation  $r+x = r^2+l^2$ ,

the value would then be  $x = \sqrt{r^2+l^2}-r$ .

*Example of applying this correction.*

Placed a spirit level at any point B, (see fig. page 255,) on the earth's surface, and found the point E, at 3 miles off, to be on an apparent level with the point B. What is the comparative height of the object E?

Now, BE is the apparent level, and BD the true level, B and D being points equidistant from the earth's centre. DE ( $x$ ) is the height of E above B, which, in feet, equals the distance (DE, in miles)  $3^2 \times \frac{1}{3} = 9 \times \frac{1}{3} = 6$  feet, the height of the object E above B.

The apparent height, therefore, of every distant object, as observed by a spirit level from any point, is always *less* than the true height, by this value of  $x$ , for the curvature of the earth.

## CHAP. II.

## REFRACTION.

THERE is also, in long observations, another correction necessary, arising from the effects of the density of the atmosphere, in refracting the rays from the object, which makes the apparent greater than the true height. The correction, therefore, must be subtracted from its apparent height.

Refraction increases the distance at which objects can be seen (*cæteris paribus*), in a proportion of 14 to 13, and raises the apparent height one-seventh of its correction for curvature.

Thus, in the last example, the object observed at a distance of three miles was apparently level with the instrument, but the correction for curvature being six feet, and the height (independently of the subsequent correction for refraction) was 6 feet,  $\frac{1}{7}$  of 6 =  $\frac{6}{7}$  feet = 10  $\frac{2}{7}$  inches, and therefore 5 feet 1  $\frac{2}{7}$  inches = the true height of the object, allowing for both corrections.

## EXAMPLES.

1st.—The observed heights of three objects, at a distance of 4, 6, and 8 miles, (calculated from observations taken by the theodolite,) were found to be respectively, 24, 25, and 28 feet. What are their true heights?

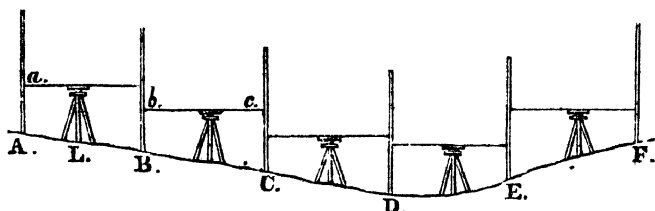
2nd.—Found the angle of elevation of the spire of a church, which was 420 chains 75 links off, to be  $1^{\circ} 10'$ . What is its real height above the point of observation?

## CHAP. III.

*To find the difference of levels between several points, or to trace a sectional line of the inequalities of the earth's surface.*

Let ABCDE be the line to be traced. Set the level (L) between the object, and read off the height Aa and that of Bb, the difference between Aa and Bb will be the number of feet that B is higher or lower than Aa; if Bb be greater than Aa, the point B will be

lower (by this difference) than  $Aa$ ; for the height, read off by the level staff, is the number of feet that each point observed is *beneath* the level of the line of collimation of the telescope—hence, where there is a number of points beneath the same level line, the greater the reading of the staff, the lower this point must be.



Then, because, in the first observation, the height at B (read by the level staff) is greater than that at A, the point B is lower than the point A. Again, in the second example, where, it must be observed, that another line of collimation is taken, because the height by the staff at C is greater than at B, the point C is lower than B. In the third observation, also, D is lower than C, and C being lower than B, and B than A, the ground falls thus far. At the fourth observation, however, because the height at D is greater than that at E, the point D is lower than E, and, therefore, E being higher than D, the ground rises to E, and as the reading at E is greater than at F, it goes on rising to F. The relative heights of the two ends of the line, at A and F, depend upon whether the ground falls, more or less, from A to D, than it rises from D to F.

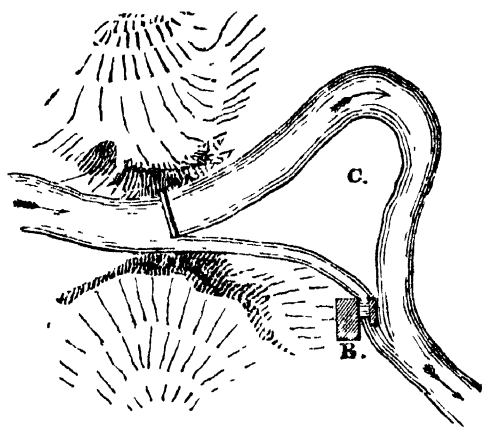
Now, the difference between the reading at B and A, in the first observation, added to the difference of readings at C and B, in the second observation, plus this difference between D and C, in the third, as there is a continued descent to the point D, will give the actual fall from A to D, or the number of feet, that the point D is lower than A. In the same way, the sum of the difference of readings of D and E, and of E and F, in their respective observations, will be the number of feet F is higher than D; if, therefore, the fall from A to D, be greater than the ascent from D to F, the difference will be the actual fall from A to F, or the number of feet that the point A is higher than the point F.

This is the principal object of levelling. It is very simple in theory, but in the carrying out of the practical operations, great care is

necessary. In this, as in most things which are of a simple character and which do not admit of checks in the course of the work, errors are very like to creep in imperceptibly. There are certain mechanical contrivances to guard against them, which will be subsequently explained.

This taking of several stations is called compound levelling. Simple levelling is merely the determining, by one observation, the relative heights of two given points.

Thus, supposing that it is required to ascertain, in the accompanying diagram, whether there was sufficient fall of water between that part of the river at A, and that, at B, to turn a grist and saw-mill at B. Placed the level at C, between the two stations, and found the first or back reading 0·45 feet, and the forward 11·52 feet; the back distance was 2 chains, and the forward distance 20 chains; required the fall.



The back reading is 0·45 feet, at the short distance of 2 chains, which will require no correction for curvation or refraction.

The forward reading, being at a distance of 18·50 chains, must be corrected.

Now,  $x$  (for curvature) =  $\frac{20^2}{7916} = 5 \text{ inches} = \cdot 416 \text{ feet}$ . This must be subtracted from the observed reading, and you obtain the true reading (independent of the refraction),

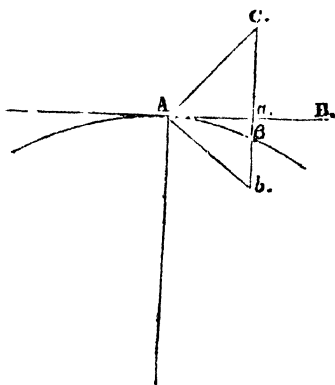
$$\begin{array}{r} 11\cdot 52 \text{ feet, observed reading} \\ 0\cdot 416 \\ \hline \end{array}$$

11·104 true distance beneath the first point.

Add, for refraction,  $\frac{1}{2}$  of  $\cdot 416$ , or  $\cdot 059$ , and the true height of the water, at A, above that at B, is  $11\cdot 163$  feet.

As in this case, to obtain the true reading, the correction for the curvation has been subtracted, and that for refraction added, which is the reverse of the preceding rule at page 257, we will investigate the subject a little more fully.

Let AB be a tangent line to the earth, at the point A, being the apparent level line; let Aa be the true level, and  $\beta$  the corresponding point to a. Let C be any object above the apparent level line, and b any object below the same level line. The object above would be observed by the theodolite, those beneath (for short distances and for the common purpose of levelling) by the level staves. Let aC be the distance above the positive height ( $+h$ ), and ab the distance beneath, which will then be ( $-h$ ). Now aC is the height above the line AB,  $\beta aC$  is the true height, therefore  $\beta a + aC$  is the true height of the point C, above A.



On the other hand, ab, the reading by the level staff, is the distance beneath the apparent level line AB, but the true line is A $\beta$ , and b $\beta$  is the true difference of levels between A and b, which is, therefore, equal to ( $-h + b$ ), or the difference between the two.

*Again, for the refraction.*

As refraction raises the apparent above the true height of an object, its correction must always lower the apparent height (it is therefore negative  $-R$ ).

If the object be above the apparent level, the error for refraction must be subtracted from the apparent height, thus ( $h + C - r$ ), where  $+C$  is the correction, for curvation, and  $r$ , is that for refraction, when these values are positive.

But if the object be below the apparent level, we have ( $-h + c - r$ ), that is, the curvation (which is the only positive value) must be subtracted, and the refraction must be added, to the negative value of  $h$ , which still remains negative, or beneath the apparent level.

Generally, therefore, let the reader bear in mind, that the correction for the curvature raises the object, and that for refraction depresses it, reducing or increasing its distance from the apparent level, according as the object is above or below it, *i.e.*, has been observed by the theodolite, or read off by the level staves.

*As the necessary calculations for curvature and refraction would be exceedingly tedious, in extensive operations, the following method renders them altogether unnecessary.*

Set the level in the centre of the object, as nearly as *the eye can tell*, and these corrections for both objects become equal and opposite, and therefore neutralize each other. The apparent level, being always parallel to the chord, that connects two objects, equidistant from the place of observation, must, therefore, have the same versed sine.

Before proceeding to the main level, we will briefly go over the method of conducting the trial and check levels, as they are termed, before taking the final level.

## CHAP. IV.

### TRIAL LEVELS.

HAVING determined upon the general line of route, the line is marked down upon the Ordnance map, and the several points, where the roads are crossed, are carefully measured from the scale, and determined upon the ground.

The portions of the line between these points are roughly picketed, *if time will permit*, or, if not, their general direction, by the compass, is ascertained, on the plan. The levels are then taken, as near as possible to this direction—the error of deviation being always confined to the intervals between the roads—the relative heights of these points of the roads being always ascertained, and bench-marks taken near. The inclination of the ground, on the right and the left of the line, is also, in the first trial level, carefully marked, so that the engineer may know on which side of the

levelled line to deviate, when he is in want of a piece of cutting, or an embankment.

The trial level, however, is, after all, but very rough work, and serves only as a general check upon the correctness of the subsequent levels.

The heights of the several points upon the roads are useful, as checking their heights by the next level, but the intervals are generally all *out of line*, and never to be depended upon; certainly not, when the direction of the line is not picketed. The only use, then, of this first trial level is, in addition to its furnishing the general face of the country, in a certain direction, its acting as a check upon the height of the roads, and upon the relative heights of the two ends of the line, so as to prevent any serious error in the final Section.

Such being the nature of the first trial level, it may easily be understood how easily errors may creep in in the parliamentary plans, when these level trials and the ORDNANCE MAP are often the only data, that the indecision of companies and the pressure of time, place within reach of the engineer.

#### CHECK LINES

Are levels taken for the purpose of determining, a second time, the heights between two distant points, which, agreeing with the first, becomes a strong proof of the correctness of the work in detail.

#### METHOD OF LEVELLING ON A LINE OF RAILWAY.

Having determined upon the point of commencement, select some fixed point, as near to it as possible, whose height being taken, becomes a mark of reference for any subsequent level. This is called a *bench mark*. Place your level about half way between this point and the next onward station, and fixing the legs

firmly in the ground, set the instrument level, and observe the height read off by the staff at the back station: this observation is called a back-sight, or back-set. Turn the instrument round to the bench mark and read off the height there. This is an intermediate, as not being a connecting-link in the consecutive series of backsets and foresets. Lastly, read off the height at the forward station, which is termed the foreset, taking care, before each of these observations, to see that the bubble is duly in the centre of the tube. Now take up the level, and place it, as before, between the next two stations, and so on, observing the back and forward readings in every case, and taking, in the way, such bench marks as may appear desirable for the purposes of reference hereafter. These bench marks should be of a permanent character, *near* to the line, and in conspicuous places.

These observations should, in all cases, be carefully taken, and due attention paid, on the part of the Surveyor, to the non-existence of parallax.

The staff-holder has, also, considerable responsibility reposed in him, as all the care on the Surveyor's part would be neutralized by inattention in the placing or moving of the staff. At every station the staff has to be read off twice, in opposite directions, and great care is requisite in turning it round. A peg, or board, or a penny, is sometimes used by the staff-holder for that purpose, so as to keep the staff always in the same point. Much error also frequently arises from the staff not being held perfectly upright. There is some difficulty in keeping it so, and men are apt to become tired and careless. The wind too will often disturb it.

Personally, I have found the greatest difficulty in getting men to keep it upright; they do not understand that it can be out of the perpendicular in two ways; they will often keep it straight, as to it not being too much to their left or right, while, frontwards, it will be considerably out.

This inconvenience is avoided in some level staves by a small plummet in the side of it, which enables the staff-holder to see whether



it is perpendicular in one direction ; and its lateral perpendicularity the Surveyor can judge of for himself.

*Method of keeping the Field Book.*

Divide the page into the several columns, as in the example, page 268, which are the field notes of a portion of a line of railway from Wandsworth to Croydon, commencing near the Wandsworth station, at a point which is carefully described, and can at any time be easily ascertained, being on the surface of the rail at  $\frac{1}{4}$  mile, or 20 chains, *below* the second bridge from the Wandsworth station.

This point, which was in a cutting, was assumed to be 140 feet above an imaginary level, to which the heights, at the several points on the line, bear reference, which is called a Datum Line.

This point might have been considered zero, but as whenever any other points were below this, their height would be negative, the constant changing of, from positive to negative, would be productive of considerable trouble and probable error, especially to those of my readers who might not be quite *au fait* at Algebraical calculation.

The height of the point of starting is put in the column of reduced levels, 140, and the 0·00, in the column of distance, shows that it is the starting point. The instrument is now placed between this point and the next favourable station (not exceeding, in any case, eight or ten chains from the level to the station) in this, from the deepness of the cutting, only 50 links between the *two* stations, and the back reading was found to be 11·04 feet, and the forward reading 1·49. These are placed in their respective columns. Now, as the foreset reads less than the backset, the ground

rises, and the difference between the two readings (9·55) is placed in the column of *Rises*. The height of the ground, where the instrument was placed, of course is not known. The instrument was then removed, to between the second and third station, and the staff turned round, when a second reading of each was taken, and placed in their proper columns. The last reading being less, the ground still rises, and the difference (11·47) is again placed in the column of *Rises*, and its distance (*always from the starting point*) is marked down, viz.:—86 links. Thus we proceed till, at 1 chain 66 links, we get to the top of the cutting, always rising.

Before continuing, we will see how the reduced levels are obtained. These reduced levels are the actual height of each point respectively above the assumed datum or standard line of height

The rises and falls are the differences at each point of its own and the preceding height, and denote whether the ground has risen or fallen between any two consecutive points. By adding, therefore, each rise to the preceding actual height, we obtain the actual height or reduced level of the point, and by deducting this difference where there is a fall, we also obtain the same result.

At the next station the ground falls, the foreset being greater than the intermediate, and 2·61 is placed in the column of falls, which, subtracted from 182·52, the reduced level at the preceding point, gives 179·91, the reduced level at this point. The height being that of the field immediately beyond the ditch on the top of the cutting, the distance was not wanted again, and it was, therefore, made an intermediate, the foreset at

the same station being at a distance of 10·89 from the starting point.

It must be observed generally that there should be no *foreset* without its distance. All B. M. of every kind—the heights of water,—in rivers and drains,—in fields,—on banks,—and every height taken for the purposes of illustration, should be made intermediate.

Having filled the page with the observed backsets, intermediates and foresets, add up the foresets and backsets, their difference will be the difference of height between the starting point and the last point on the page, and it will be equal to the difference between the sums of the rises and falls; it should be also equal to the difference between the assumed datum line and the last reduced height.

Having these three checks, there is no possibility of error when they all agree, and *in all cases every page must be so tested.*

Before calculating the reduced levels, as any error in the rises and falls must induce the same error in the reduced levels, observe first, whether the difference of the rises and falls is the same as that of the sums of the backsets and foresets, you can then safely proceed to the calculations of the reduced levels.

Observe, in placing the different readings, that, in the first line in the column of reduced levels, must be placed the assumed height of the starting point above the datum line; collateral with this must be 0·00 of distance, accompanied with a full and clear description of the exact locality of the starting point. (Consider that you are describing the place to a perfect stranger, who has never been on the spot before.)

On the next lower line, the first back reading must be placed, and, collaterally with it, *in the same line*, the next reading in the *intermediate* column, if it be that of a *bench mark* without a given distance, or of a station, which is not intended to be the backward station to the next observation. If it is, place it in the column of foresets, collaterally with the backset, in the same line. Observe, that every line in the reduced level must have a given height, and that to each of these heights, in the column of distances, there must be a distance given, or an observation of individuality.

The reason of placing the foreset in the same line as the backset is, that the height of each backset point is the height of the preceding foreset, and that it is the difference between the actual backset and foreset upon the same line, that gives the height of the latter.

In calculating the rises and falls, take the difference between the

two readings in the same line, when the one is a back reading, and the other, intermediate or foreset, and place the difference in its proper place, in the same line. Then, if the next reading be an intermediate, it will be placed under the preceding intermediate; the difference between these two intermediates must be placed in the same line as the latter, and so on throughout any number of intermediates. After these intermediates must, of course, come a foreset point, before the instrument can be removed, which foreset becomes a backset point of observation at the next placing of the level. The difference must be taken between this foreset and last preceding intermediate, and placed collaterally in the same line with the foreset.

This closes one portion of the line of section, and another is recommenced that would be totally unconnected with the former were it not that the last point in that becomes the first point in this,

Collaterally with the reduced levels, if these levels refer to benchmarks, write B. M. in large characters in the column of distances, and the nature of it under the head of remarks. If the reduced levels refer to points taken out of the line, which is frequently the case in trial levels to avoid obstacles, write "out of line;" also, in the column of distances, carry the distances on always from the starting point to 80 chains, and, if an observation be taken at that point, which should be done if possible, write 1 mile against it, and begin the chaining afresh,—continue to another 80 chains, marking this point 2 miles; begin here again also, and proceed as before.

## CHAP. V.

THE first page of the following notes has been completed, the rises and falls calculated, and the reduced levels put in. The other pages contain only the several readings at the different stations, and the respective distances or observations to each; the rest must be filled in by the student.

## TRIAL LEVELS

*On a proposed line of Railway between Wandsworth and Croydon.*

Backset	Inter.	Foreset	Rise	Fall	Reductn. Levels	Dist.	Remarks
					140.00	0.00	Datum line being 140 ft. below the level of the sea.
11.04		1.49	9.55		149.55	.50	Commencing at the surface of the
11.60		0.23	11.37		160.92	.86	rails opposite the centre of the
11.50		0.39	11.11		172.03	1.23	watch box, $\frac{1}{4}$ mile below the second bridge, from the Wandsworth station.
11.61		1.12	10.49		182.52	1.66	top of cutting of railway
6.04	8.65			2.61	179.91		in field
		4.79	3.86		183.77	10.89	
5.18	7.05			1.87	181.90		within field
		8.20		1.15	180.75	18.80	in field beyond
3.31		11.97		8.66	172.09	26.08	
1.23		9.95		8.72	163.37	29.84	
2.92	7.28			4.36	159.01		in field
		9.84		2.56	156.45	35.68	in field beyond
0.42		10.55		10.13	146.32	43.91	
0.55		6.94		6.39	139.93	52.39	
4.40		5.10		0.70	139.23	63.31	
3.66		2.97	0.69		139.92	B M	being hook of lower hinge of gate in lane
2.53	5.40			2.87	137.05	63.83	centre of road No. 1 BURNWOOD LANE
		2.42	2.98		140.03	64.33	in field beyond hedge and ditch 64.10 and 64.20
4.90	8.73			3.83	136.20		in gravel pit
		4.69	4.04		140.24	71.70	71.60 path crosses, 71.76 hedge crosses
5.40		4.84	1.56		141.80	0.15	1 MILE. 76.40 & 79.70 hedges cross
4.82		3.72	1.10		142.90	11.23	520 hedge, 11.03 ditch
1.00	2.35			1.35	141.55	B.M.	on rail by tree near shed
		6.27		3.92	137.63	18.70	18.78 and 21.74 hedge and fence
4.77		9.54		4.77	132.86	26.10	30.08 and 30.70 garden fence

## TRIAL LEVELS CONTINUED.

Backset	Inter.	Foreset	Dist.	Remarks
4.11		2.09	31.02	By brick wall, Occupation-road, same level
4.95	3.15		B. M.	on lower hinge of gate of blue rails
		3.22		out of line
5.20		2.45		out of line
7.37		4.18		out of line
3.48	6.03		47.53	Centre of road No.2, London and Tooting road
		4.50	B. M.	nut of lower hinge of white gate
5.06		2.51	57.48	52.53 ditch crosses
7.67		0.45		out of line 5914, and 6296, fence & ditch cross
8.83		0.68	75.00	69.70, and 76.30, ditch and fence cross
9.95		0.70	19.75	2 MILES
9.86		0.60	2.80	325 and 380, fences of road cross
8.97	11.78			Centre of road No. 3 Church-lane
		0.55	5.80	in field beyond
10.32		0.96	9.30	within field near fence of lane
2.01	5.92		9.80	Centre of road No. 4, Back-lane
		0.00	B. M.	on tree in Back-lane blazed (with nail)
8.73	0.21		B. M.	on top of post near drain on left of line & road
		0.25		out of line
4.34		1.60		out of line
0.40		11.77		out of line
0.40		9.65		out of line
2.72	6.10		B. M.	on top of post, on right, near corner of wall
		8.25		out of line
5.50		4.96		out of line
5.40		3.70		out of line
5.09	2.81		B. M.	on stump near gate at culvert
		2.15		out of line
5.00		1.00		out of line
8.05		7.02		out of line
1.30		11.35		out of line
1.86	2.08		B. M.	on top of further side of gate, other side of road
		6.85	48.30	Centre of road No. 5
5.56	1.48		48.12	4812 and 48.48 fences of road cross
	4.52		48.52	in field beyond
		6.55	49.82	
1.36		10.93	57.58	59.30, ditch crosses
1.57	6.41		59.63	60.70, ditch crosses
		6.19	61.03	
10.65	5.53		64.00	
		4.15	71.25	6900 fence, 71.86 ditch
5.70		4.03	79.18	
5.80	3.72		3.78	3 MILES, 390 and 430 fences
	4.30			Centre of road No. 6
	4.00		B. M.	top of near post of black gate on left of line

## CHAP. VI.

## PLOTTING OF THE MAIN SECTION LINE.

**DRAW** first the **DATUM LINE** in ink, carefully and finely; then lay off the several miles upon it, by means of a 12-foot ivory rule. The usual scale for sections of railways, deposited for Parliament, is 20 chains to the inch, or 4 inches to the mile. These distances must be carefully gone over once or twice to guard against error, and the number of miles must be marked against each. The method of chaining on a line from the beginning, and dividing the length into miles on the ground, confines every error of plotting within that distance, as the distances are always taken from the last mile point—so many miles + (plus) so many chains, till the next mile mark, when it begins again.

At every third mile, as well as at the beginning and ending, erect perpendiculars to the line, geometrically by the compasses, to about twice the height of the datum line, and draw a line through the distance parallel to the datum line; measure off upon this line the intermediate miles, and connect them with the corresponding points in the datum line.

Now, lay off, upon the datum line, the several distances on the column of distance for one mile, and draw very fine lines therefrom, parallel to the perpendiculars. These parallels, or supposed perpendicular lines, being confined within every mile, cannot be far wrong. In fact, the correctness of the section line depends upon their being truly perpendicular, or the distances in the section line, though correctly measured on the datum line, might be incorrectly projected.

Upon these perpendiculars measure carefully off, with a fine needle, on a scale of 100 feet to an inch, the several heights in the reduced levels corresponding to the distances, carefully distinguishing the roads and rivers, the position of hedges, &c., as in the accompanying plan, and connect their several heights; *put this in ink at once*, while the plotting is fresh in the memory, marking off in pencil the "reduced level" of the centre, against every road, the surface of the water of rivers, &c., and also the height of the ground at every half mile.

One mile being now finished, and not before, proceed to the next; finish that before you commence the following, and so on throughout.











## CHAP. VII.

## CROSS LEVELS.

*Cross Section on line of London and Southampton Railway.*

Backset	Inter.	Foreset	Dist.	Remarks
7·80			0·00	Commenced at surface of rails opposite the centre of the watch box, $\frac{1}{4}$ mile below the 2nd bridge, from Wandsworth Station.
	7·24		3·00	
	6·98		4·00	
	6·85		6·00	
	6·97		8·00	
		7·40	10·00	
4·74	5·03		12·00	
	5·30		13·00	
	5·49		16·00	
	5·75		18·00	
		6·05	20·00	Ground level with rail line hereabouts.
3·57	3·78		22·00	Being surface of rail at the chain opposite the intersection of a drain, with railway on the right side, downwards.
	3·92		24·00	
		4·18	26·00	

*Cross Section No. 1, Burntwood Lane*

Backset	Inter.	Foreset	Dist.
0·83			0 00
	1·84		1·00
	2·87		2·00
	3·27		3·00
	3·79		4·00
	4·82		6·00
	5·35		7·00
	6·04		8·00
	6·66		9·00
		7·55	*10·00
0·00	1·43		11·00
	2·60		12·00
	3·74		13·00
	5·74		15·00
	7·32		16·00
	9·30		17·00
	10·87		18·00
		12·00	19·00

*Cross Section No. 2, London to Tooting.*

Backset	Inter	Foreset	Dist.
0·15			0·00
	1·04		1·00
	1·86		2·00
	2·79		3·00
	5·87		4·00
	6·00		6·00
	6·83		7·00
	7·54		8·00
	8·15		9·00
	8·50		†10·00
		6·90	†B. M.
2·14	3·54		†11 00
	4 64		12·00
	4·50		13·00
	5·12		14·00
	6·13		16·00
	6·16		17·00
	6·42		18·00
	6·50		19·00
		5·92	20·00

\* Line crosses.  
Reduced level, 137 05.

† Line crosses. Reduced level,  
139·93.

‡ On nut of lower hinge of white gate.

## CROSS SECTIONS CONTINUED.

*No. 3, Church Lane.*

Backset	Inter.	Foreset	Dist.
0.56			0.00
	2.31		1.00
	7.23		3.00
		10.53	4.00
0.14	7.45		6.00
		11.45	7.00
0.72		10.51	9.00
0.43	4.36	* 9.82	
		9.97	11.00
0.12	7.68		13.00
		10.37	14.00
0.07	2.32		15.09
	6.27		17.00
	8.02		18.00
	9.22		19.90
		10.05	20.00

\* Line crosses 32 down hill from fence on left of line. Reduced level, 175.85

*No. 4, Back Lane.*

Backset	Inter.	Foreset	Dist.
1.07			0.00
	1.81		1.00
	2.80		2.00
	6.58		4.00
	8.76		5.00
0.91		10.55	6.00
	7.61		8.00
2.75		10.67	9.00
	6.51		10.00
	7.22		† 10.25
	1.30		‡ B.M.
0.53		9.48	11.00
0.41		8.24	13.00
0.56		8.32	15.00
1.53		7.84	17.00
	6.59		19.00
		7.95	20.00

† Line crosses. Reduced level, 192.30.

‡ On blazed tree

*Cross Section No. 5.*

Backset	Inter.	Foreset	Dist.
6.04			0.00
	5.70		1.00
	4.20		3.00
	3.10		4.00
		2.34	5.00
5.15	0.71		B. M.
	3.85		7.00
		4.21	8.00
4.95	0.42		B. M.
	6.80		11.00
	8.47		12.00
		10.27	13.00
1.21	5.77		14.00
		8.13	15.00
4.75	1.50		§ B. M.
		7.79	17.00
0.64		8.15	19.00
0.47		8.27	21.00
0.69		7.39	23.00
2.76		5.75	25.00

§ Line crosses at 16.63 chs. Reduced level, 184.39.

## CHAP. VIII.

LEVELLING AND PLOTTING OF THE CROSS  
SECTIONS.

HAVING, in the course of proceeding with the main section, carefully marked upon the ground the exact spot in the centre of the road, where the line crosses, and taken the height of the nearest bench mark to it, about ten chains on either side of this spot are measured, in chains' lengths, along the road, for the purpose of obtaining the height of the road at every chain.

In taking the cross levels, be careful always to level one way, *from left to right*, that is, measure always from the left side of the line, (looking in the direction the main section has been levelled, or towards the termination of the line,) to the centre, where the line crosses, onwards to the right side of the line. This will prevent error. Unless the number of chains levelled on either side, of where the line crosses, be always the same, which, depending upon the inequality of the section line of the road, is seldom the case, the consequences of neglecting this might be serious, as the line would cross in its wrong place, and the calculations for lowering or raising the surface, for the bridge across it, would be altogether incorrect.

Call these chains' lengths, therefore, in your level book (*beginning as far, on the left of the line, as from the nature of the ground may be necessary, and numbering to the centre*), 1 chain, 2 chains, and so on.

Place your level opposite to the centre stake, between 0 chains and where the line crosses, and take the several readings at every chain's full length, and also at the spot where the line crosses, and should the bench mark, before alluded to, be on that side, take that also; then remove your level to midway between the centre, where the line crosses, and the extreme point you intend levelling to, and take the several readings.

Enter these in your book, according to the examples given, making the reading at 0 chains your first back set, at 1 chain your first intermediate, placing it in the next lower line, so that you may be enabled to ascertain the height of the starting point, 0·00 in the column of distances. Mark all the subsequent readings, taken at the same place, *intermediate*, excepting that where the line crosses, which should be first *foreset*, saying in every case, in the column of remarks, line X. The B. M. will be an intermediate, as usual.

Having now obtained all the field data necessary, calculate carefully the rises and falls, using the ordinary checks to ensure accuracy. Then turn to the notes of your main section, and observe there the height of the B.M., and of that part of the road where the line crosses; enter these in the column of reduced levels, as the reduced levels of the same points, taken in the cross section, and, determining the subsequent portion of the section as usual, *reverse the calculations of the upper portion*. By this means, you obtain the several heights of the road, relatively to the whole section, and as the height of where the line crosses, obtained by the section levels, should be the same as that by the main section, you have a certain check upon the general accuracy of the levelling, as well as a special proof that the *correct* bench mark, and point of crossing of the road, have been taken.

Now plot the sections of the several cross roads, numbering them, carefully, No. 1, No. 2, &c., to agree to the corresponding roads in the main section, marking where the line crosses.

The scale, generally used for these cross sections, is 5 chains horizontal, and 50 feet vertical.

The section being now completed, the field book is no longer wanted, and the subsequent proceedings depend upon the skill and judgment of the engineer, to overcome, in the selection of his gradients, such local difficulties of hills or hollows—of roads and rivers—as the face of the section may present.

In determining the lines of gradients, practice and experience are the best masters, and I refer my readers to them.

## CHAP. IX.

*Method of determining the heights, that the several roads should be raised or lowered, and whether they should be passed over or under.*

When a road approach is *passed over* by the line, the usual distance allowed, from the surface of the road to the surface of the rails, is 20 feet; when the approach is passed under, the allowance is 19 feet.

This being premised, we will proceed to the calculations of roads, Nos. 1, 2, and 3, in the given section, and leave the remainder to the reader for practice.

The heights of the several gradients selected, were, 140 feet at starting, which running level for a quarter of a mile, then ascended at a rate of 1 in 354, to the centre of road No. 2, (at a distance of 1 mile, 47 chs. 53 links, from the starting point,) where it is 160 feet above the datum line; the gradient then changes into one of 1 in 413 to the 3 mile, where its height is 178 feet. The gradient line in the plan, it must be remembered, is *the line of the surface of the rails*, and not what is called the *balance* line, or ground line of the ballasting, which is so called, from its being the line adopted in the calculations, of both the cuttings and embankments; the earth, in the former, being dug out, and in the latter, filled up, as far as A; upon this, afterwards, is placed the ballasting, which, in the calculations of earthwork, is reckoned at 2 feet.

*The distance of the first road (cross section No. 1),* by turning to the main field book, will be found to be 63·83 chains, it falls, therefore, in the second division of gradients, being 140 feet at one end, and 160 at the other. Find the length of this gradient, which begins at 20 chains, and ends at 127·53 chains, being, therefore, 107·53 chains long. Knowing the length and excess of height at the end, the excess of height, at any point upon it, can be calculated by similar triangles, thus—

As the whole distance, 107·53 chains : 20 feet :: 23·83 chains,



the distance of the road No. 1, from the commencement of the gradient: its corresponding excess at that point, viz., 5·45 feet, which added to 140 feet, its height at starting, gives 145·45, the height of the surface of the rails, where the line crosses the centre of the road No. 1. By turning again to the field book (to the reduced levels), the height of the surface of the road is 137·05 feet, making the rail line 8·45 feet higher than the road line, or surface of approach.

If the road can be conveniently lowered, lower it, so as to make it 20 feet beneath the rail line; to effect this, it must be lowered 20 feet—8·15 feet or 8 feet 15 inches, and passed over.

In the same way, it may be found, that at road No. 3, the surface of rails is 165·75 feet, while that of the approach is 175·35, making the approach 9 feet 7 inches *higher* than the rail. The road, therefore, must be *raised* 19 feet—9 feet 7 inches, or 9 feet 5 inches, and passed *under*.

When, as in the case of road No. 2, the surface of the rails crosses a road, 20 feet, or more than 20 feet above it, the road remains the same, *the level is unaltered*.

#### EXAMPLES.

The reader is now recommended to calculate for himself, roads Nos. 2, 4, and 5, and see if he makes them agree with the plan.

At No. 2, the line is 20 feet higher than the road; at No. 4, the road is 25½ feet above the line; and at No. 5, the road is to be raised 8 feet 10 inches, and passed under.

## CHAP. X.

*Calculations of the superficial areas, occupied by the railroad, and of the solid contents of the cuttings and embankments.*

#### SUPERFICIAL AREAS.

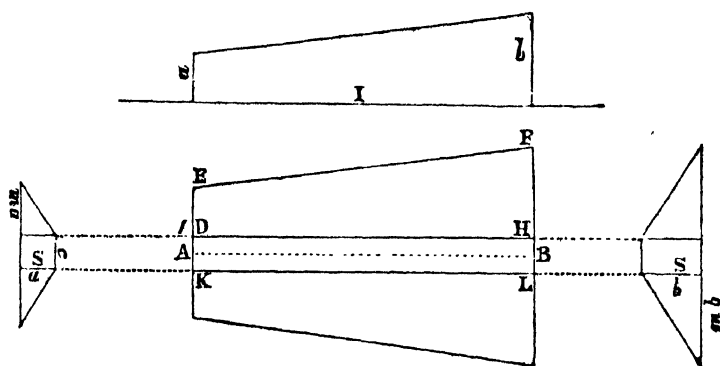
HAVING determined the effects of the line in the altering of the approaches, the next thing to be ascertained, is the superficial area that it will occupy, so as to estimate the expence of purchasing the ground. This will, of course, depend upon the heights or depths of the cuttings and embankments, upon the width of the line, and the proportion of the slopes.

In making these calculations, much depends upon the object in view and the pressure of time. In preparing for deposit, the difference of heights between the section line and the *balance* line are taken

by the compasses ; while, in preparing for the contracts, it is requisite they should be calculated from the field book, and the rates of inclination in the section. Again, for deposit, these differences are only taken at any apparent change of inclination in the ground line, while for the contracts they must be made for every two chains.

\* Being able to obtain these differences of heights, whether by compasses (from the section), or by calculations (from the field notes), as the case may require, we will take some general case, and explain the best method of calculating the areas of the several blocks.

Let  $a, b$ , be the several heights ; let  $c$  be the width of the railway ; and let the slopes be as  $m$  to 1 (by the term "*slopes*" is meant, technically, that the base of the slope is to the height of the slope as  $m : 1$ ) ; therefore, if the heights be  $a, b$ , the base of the slopes will be  $ma\ mb$ .



Let AB be any length of line, whose height at A is ( $a$ ), and at B is ( $b$ ). Then, if ADHB be taken as the half of the line, DE and HF will be the extent of the slopes on the one side, and DE and HF will be respectively equal to  $ma$  and  $mb$  ; and, therefore, the area of AEFB will be  $\frac{c + ma + mb}{2} \times \pi$ , where  $\pi$  equals the length, but AEFB is only half the area ; therefore the whole area of the block equals  $(c + ma + mb) \cdot \pi$ .

**EXAMPLE.**—Required the superficial area of a portion of railways, 20 chains long and 12 feet high at one end, and 10 feet at the other, the slopes being 2 to 1 (the width of line being 33 feet) ; now  $c = 33$ ,  $a = 12$ ,  $b = 10$ , and  $m = 2$ , and  $\pi = 20$  chs. ; therefore,

$$\frac{33 + 23 + 20}{66} \times 20 = 23.333 \text{ sq. chains} = \begin{matrix} \text{A. R. P.} \\ 2 & 1 & 13. \end{matrix}$$

**EXAMPLE.**—What will be the superficial area of a cutting of a railroad, whose depths at every chain are 0, 2, 3, 5, 7, 9, 8, 6, 4, 2, 1, 0 feet; the other data as in last example?

The formula for the area is  $(c+ma+mb)\pi$ ; in the two end areas, in the present example, where  $a=0$ , and  $b=0$ ,  $ma$  and  $mb$  would be 0; and the areas will be  $(c+mb)\pi$  in the one ease, and  $(c+ma)\pi$  in the other.

Now the whole area  $= \frac{\pi}{66} \left\{ (33+4) + (33+4+6) + \&c. \right\}$ ; or, by taking separately the whole of the *central area*, which will be  $(11 \text{ chains} \times \frac{33}{66})$ , and using the formula (given in the first part, for calculating the areas of offsets, page 75), for the areas of the *slopes*, we have for their area  $2\pi \frac{(4+6+10+14+18, \&c.)}{66}$  sq. chs.; therefore, the whole area  $= \frac{1}{66} \times \left\{ 33 \times 11 + 2\pi \frac{(4+6+10+\&c.)}{66} \right\}$   
 $= 8.34 \text{ square chains} = 0 \ 8 \ 13.$

**EXAMPLE.**—What will be the area of a mile of railway, whose heights of embankment for the first quarter of a mile, at every 5 chains, are 12,\* 25, 50, and 20 feet; for the next 25 chains, the line is level; and then, to the end, runs through a cutting, which rises gradually to the height of 40 feet, at half way, and descends to the end, having the other data as the preceding?

A. R. P.  
*Ans.* 15   3   15

\* In this case, area of each slope equals  $d \left( \frac{bg}{2} + ch + dl + \&c. \right)$  see page 76, where, in the given formula, the perpendicular at the starting point  $= 0$ . In the section of a cutting or embankment, the area of the slopes would then be  $d \left( \frac{bg}{2} + ch + dl + em + \frac{fn}{2} \right)$

## CHAP. XI.

## CUTTINGS AND EMBANKMENTS.

THERE are two or three methods adopted in practice, in the calculation of these quantities, viz., the Prismoidal Formula, Bidder's Tables, M'Niel's, &c.

*Prismoidal Formula.*

Let  $c$  be any cutting, having, at one end, the height  $= a$ , at the other  $= b$ ; length of cutting  $= \pi$ , slope 2 to 1, or generally  $m$  to 1.

Now the solid, thus cut off, assumes the form of an imperfect prism, which may properly be divided into three divisions; the *central* one, being a solid, generated by a plane of the given heights and length; moving along a plane, at right angles to it, to a distance, equal to the width of the railway; and the two slopes, equal to each other, being strictly frustra of pyramids; the height of the frustra being equal to the length of the cutting, and the sides of either end, being the sides of a right-angled triangle, whose base  $=$  the height of the cutting at that end; and perpendicular, the proportion of the given slope to it.

Having the same data as above, (*See Diagram, Page 277.*)

the central contents  $= \pi \frac{c}{2} (a+b)$ , where  $c$   $=$  width of railway

$$\begin{aligned} \text{slopes} &= \frac{\text{areas of the two ends} + \text{the mean area}}{3} \times \text{the length} \\ &= \frac{\pi}{3} (ma^2 + mb^2 + mab) \end{aligned}$$

$$\begin{aligned} \therefore \text{The whole contents} &= \frac{\pi}{2} c (a+b) + \frac{m}{3} (a^2 + b^2 + ab) \\ &= \frac{\pi}{6} (3ac + 3bc + 2a^2m + 2b^2m + 2abm) \\ &= \frac{\pi}{6} (ac + a^2m + bc + b^2m + 2ac + 2bc + m(a+b)^2) \\ &= \frac{\pi}{6} (\overline{c+ma} \cdot a + \overline{c+bm} \cdot b + 4 \left( \frac{a+b}{2} \right) \cdot (c + \frac{m}{2} \cdot \overline{a+b})) \end{aligned}$$

which is the *prismoidal formula*; where  $\overline{c+ma}$ ,  $a$  and  $\overline{c+mb}$ ,  $b$  are areas of the two ends, and  $4 \left( \frac{a+b}{2} \right) (c + \frac{m}{2} \cdot \overline{a+b})$  is 4 times the area of a section midway; the whole area being equal to the sum of these three into  $\frac{1}{6}$  the length, whether of cutting or embankment.

2. *Moseley's Formula.\**

$$\text{The central area} = \pi \frac{c}{2} (a+b) = \frac{\pi m}{6} \left( \frac{3c}{m} \cdot \overline{a+b} \right)$$

$$\text{slopes} = \frac{\pi}{3} m (a^2 + b^2 + ab) = \frac{\pi m}{6} (2a^2 + 2b^2 + 2ab)$$

$$\therefore \text{whole contents} = \frac{\pi m}{6} \left( \frac{3c}{m} (a+b) + 2a^2 + 2b^2 + 2ab \right)$$

$$\text{Now let } a = y_1 \text{ and } b = y_2, \text{ we have 1st contents} = \frac{\pi m}{6} \left( \frac{3c}{m} (y_1 + y_2) + 2y_1^2 + 2y_2^2 + 2y_1 y_2 \right).$$

Now, as the adjoining cutting must commence with the same height as this terminates, supposing the other height of the second cutting =  $y_3$ , and next height  $y_4$ , we have—

$$\text{area of 1st cutting} = \frac{\pi m}{6} \left( \frac{3c}{m} (y_1 + y_2) + 2y_1^2 + 2y_2^2 + 2y_1 y_2 \right)$$

$$\text{2nd cutting} = \frac{\pi m}{6} \left( \frac{3c}{m} (y_2 + y_3) + 2y_2^2 + 2y_3^2 + 2y_2 y_3 \right)$$

$$\text{and the series} = \frac{\pi m}{6} \left( \frac{3c}{m} (y_1 + 2y_2 + 2y_3 + \dots + 2y_{n-1} + y_n) + 2(y_1^2 + 2y_2^2 + 2y_3^2 + \dots + 2y_{n-1}^2 + y_n^2) + 2(y_1 y_2 + y_2 y_3 + y_3 y_4 + \dots + y_{n-1} y_n) \right) \text{ or B,}$$

$$\text{and } \therefore = \frac{\pi m}{6} \left( \frac{3c}{m} (2 \sum y_n \dots (y_1 + y_n) + 2(2 \sum y_n^2 - (y_1^2 + y_n^2) + 2(B)) \right).$$

Now, when  $y_1$  and  $y_n$  are = 0, which is the case in a complete cutting or embankment; we have—

$$\text{Solidity} = \frac{1}{6} \pi m \left( \frac{3c}{m} \sum y_n + 4 \sum y_n^2 + 2(B) \right),$$

when  $\sum y_n$  = the sum of the simple quantities,

and  $\sum y_n^2$  = the sum of their squares,

and B = the sum of the continued products of the simple quantities.

\* This formula, for which I am indebted to Professor Moseley, of King's College, I have had several opportunities of testing, and I have found it exceedingly useful. The calculations from the Prismoidal Formula, which is, geometrically deduced, are strictly accurate; but they are too tedious for general purposes. Bidder's tables, on the other hand, though convenient for common practice, are still, from being only calculated to full feet and limited to 50 feet, unfitted for contract estimates. Whereas Moseley's formula, being applicable to any height and any subdivision, and embracing any extent of cutting, whose heights are taken at any equal distances whatever, becomes especially useful for the final calculations; which are usually made at every two chains of distance, and to heights of one hundredth of a foot.

3. *Bidder's Formula,*

for computing the solid contents, is—

$$\left. \begin{array}{l} \text{for the slopes} = \frac{22}{27} ((a+b)^2 - ab) \\ \text{for the centre} = \frac{11}{9} (a+b) \end{array} \right\} \text{ in yards.}$$

This is for a chain in length, and in the centre for a foot wide ; the slopes being one to one.

Now, it has been previously shown that—

$$\frac{\pi c}{2} (a+b) = \text{contents of centre.}$$

Let  $c = 1$  foot,

and  $\pi =$  one chain or 66 feet,  $\therefore \frac{66}{2} (a+b)$  in cubic feet  $=$  centre.

$$\frac{66}{2} \text{ of } \frac{1}{27} \text{ or } \frac{11}{9} (a+b) \text{ in cubic yards} = \text{centre.}$$

$$\text{And the slopes} = \frac{\pi m}{3} (a^2 + b^2 + ab).$$

Let  $m = 1$ , and  $\pi = 66$  feet.

$$\therefore \text{slopes} = \frac{66}{3} \text{ feet, or } \frac{22}{27} (a^2 + b^2 + ab) \text{ in yards,}$$

$$\text{and } \therefore = \frac{22}{27} (a+b-ab) \text{ cubic yards.}$$

The central contents, thus obtained, would have to be multiplied by the length of the sections in chains, and the width of the line in feet; and the contents of the slopes, by the rates of  $m$  to 1; and by the length of the section in chains.

*M'Neil's Tables* are also for slopes of 1 to 1, and for base of 1 foot; but not for lengths of 1 chain.

The quantities obtained, therefore, in *M'Neil's tables*, would have to be multiplied, for the centre, by the width of railway in feet; and, for the slopes, by the length  $\pi$  (in feet).

These two tables, however, are only calculated for long sections, and for full feet, (for which they are invaluable,) principally, for computing the whole contents of a line previous to going to Parliament, to form the first estimate of the expense.

**EXAMPLE.**—Required the solid contents of a cutting or embankment, whose several heights at each chain's length are, 0, 5, 10, 15, 20, 25, 30, 30, 24, 18, 15, 10, 0—slopes 2 to 1, and the width of railway, 30 feet.

## BY BIDDER'S TABLES.

Lengths	Height at either end.	Central Contents	Contents of Slope
1'00	0	6'1	20
1'00	4	18'3	143
1'00	10	30'6	387
1'00	15	42'8	754
1'00	21	55'0	1243
1'00	26	67'2	1854
1'00	30	73'3	2200
1'00	36	66'0	1789
1'00	44	51'3	1085
1'00	54	40'3	667
1'00	66	30'6	387
1'00	80	12'2	82
		493'7	10611
		30	2
		14811'1	21222
			14811
			36033

493·7 being the number of cubic feet in the central column, for the width of one foot, which must, therefore, be multiplied by 30 for the whole width, and 10611 being the contents of the slopes for the ratio of one to one, which must, in this case, at the ratio of two to one, be doubled; the sum of the two, 36033, will be the number of cubic yards required.

## BY MOSELEY'S FORMULA.

The sum of the simple quantities =  $202 = \sum y_n$

the sum of their squares =  $4400 = \sum y_n^2$

the sum of their products

(of each with its preceding) =  $4222 = B$

Now the formula =  $\frac{\pi m}{6} \left\{ \frac{3c}{m} \sum y_n \right\} + 4 \sum y_n^2 + 2B \}$

$2 \sum y_n = 404$ ,  $\frac{3c}{m} = 45$ ;  $4 \sum y_n^2 = 176000$ ,  $2B = 8444$

$\therefore \frac{1}{3} \cdot 66 (404 \times 45 + 17600 + 8444) = \text{cubical contents.}$

$404 \times 45 = 18180$   
 $17600$   
 $8444$

$41224 \times 22 = 972928 \text{ cubic feet} = 36034 \text{ cubic yards,}$

differing by one cubic yard only, from the previous result by Bidder's tables. The calculation of this by the *Prismoidal Formula* would be too tedious an operation. Of the two methods given, Bidder's tables have the advantage where the numbers are high, while the accuracy of calculation required for contract work, when the heights at every chain may be decimals of a foot, can only be done by Moseley's, Bidder's being calculated to feet only.

**EXAMPLE.**—Calculate, by Moseley's formula and Bidder's tables, the cubical contents of an embankment, whose heights at every chain are 0, 2, 3, 5, 7, 9, 8, 6, 4, 2, 1, 0; slopes 2 to 1; width of centre, 33 feet.

*Ans.* 5178 cubic yards.

**EXAMPLE BY THE PRISMOIDAL FORMULA.**—What are the contents in cubic yards of a cutting, where the heights at every 2 chains are, 0, 3, 5, 7·5, 8, 9·5, 8, 6, 4, 0, the width of the line being 33 feet, and the slopes 2 to 1.

Now  $\frac{c + ma}{c + mb} \cdot a =$  area at one end.  
 $\frac{c + mb}{c + ma} \cdot b =$  area at other.

$$\frac{a + b}{2} \left( c + \frac{m}{2} a + b \right) = \text{area midway.}$$

$33 + 6 \times 3$ ,  $33 + 10 \times 5$ ,  $33 + 15 \times 7\cdot5$ ,  $33 + 16 \times 8$  &c., are the several end areas, and

$\frac{3}{2} (33 + 3)$ ,  $\frac{3 + 5}{2} (33 + 3 + 5)$ ,  $\frac{5 + 7\cdot5}{2} (33 + 5 + 7\cdot5)$ , &c., are the several areas midway.

Now the sum of the end areas = 2404 square feet, and that of the middle area = 2380.

Four times the middle area, *plus* twice the end areas (as the areas at every height serve for end areas to two sections), multiplied into  $\frac{1}{3}$  the common length, will be the contents in cubic feet.

$$\frac{2(2404) + 4(2380) \times 132}{3} = 315216 \text{ cubic feet} = 11675 \text{ cubic}$$

yards, the solid contents required.



Instead of the preceding, however, there are two methods in common use among contractors, which, though apparently correct, are really far from being so.

*The first* is by taking the MEAN OF THE TWO END AREAS, which make the results *too much*.

*The second*, by taking THE AREA OF THE MEAN HEIGHT, which, on the other hand, makes it *too little*.

Now, in the proof of Bidder's formula, it has been shown that the contents of slopes  $= \frac{\pi}{3} (m(a+b-ab))$ , and of the centre  $= \frac{\pi c}{2} \cdot a+b$

therefore the whole contents are  $= \pi (\frac{m}{3} (a+b-ab) + \frac{a+b}{2} \cdot c)$

and by assuming  $x$  and  $y$ , as the unknown values of the respective excess and deficiency in the two erroneous methods above, and equating them with the true formula, we can obtain their respective values.

Of these two incorrect formulæ

the first  $\pi (\frac{c+ma}{2} \cdot a+c + \frac{mb}{2} \cdot b) = \frac{3ac+3ma^2+3bc+3mb^2}{6}$

the second  $= \pi \cdot \frac{a+b}{2} (c+\frac{m}{2} \cdot a+b) = \frac{6ac+6bc+3ma^2+6ma+3mb^2}{6}$

Finding the differences between these and the correct formula, which are the values of  $x$  and  $y$  respectively, we obtain

an excess for the first of  $\frac{m}{6} \cdot a-b$

a deficiency for the second of  $\frac{m}{12} \cdot a-b$

where  $a$  and  $b$ , as usual, represent the heights at the ends, and  $m$  the ratio of the slopes.

These corrections have to be multiplied by  $\pi$ , the length of the section.

THE END.

# CORRECTED AND OMITTED ANSWERS.

Page.	Ex.	
24	2	49 acres, 0 roods, and 10 perches.
25	4	231384 square yards.
29		For AF read AD.
	4	For DE read AF.
	5	For BC, 6·50, read FC, 6·50.
		For DB read AC.
32		For 6 read 0.
34	2	57 acres, 1 rood, 10 perches.
	3	9 acres, 2 roods, 0 perches.
35	3	10 acres, 1 rood, 26 perches.
46	3	Comes under Case 2.
48	2	Base = 73·99 chains. $\left\{ \begin{array}{l} \angle \text{ at B} = 33^{\circ} 4' \\ \angle \text{ at C} = 29^{\circ} 6' \end{array} \right.$
	3	<i>In the question</i> , should be 70 chains and 50 chains.
120	4	14 acres, 1 rood, 6 perches.
122	2	16 acres, 2 roods, $12\frac{1}{10}$ perches.
	3	$79\cdot2\cdot7\frac{3}{100}$ acres Scotch = 100 English. By 11200 sq. ft.
123	1	41 acres, 3 roods, 9 perches.
	2	61 acres, 2 roods, $36\frac{1}{10}$ perches.
	3	1944 English acres. $7\frac{3}{8}$ miles.
	4	65 acres, 0 roods, 35 perches.
124	1	280 sq. gads + $72\frac{1}{2}$ sq. feet, or 0 acres, 2 roods, 23 perches statute.
	2	56162711.
146	1	DE, 13 chains 17 links; $\angle$ DEF, $103^{\circ} 48' 20''$ .
148	1	8 miles 3 furlongs and a half.
149	1	17 chains 62 links.
150	1	249 feet.
151	1	188 feet high; 90 feet wide.
152	1	Difference of height, 202 feet.
153	1	<i>In the question</i> , for $14^{\circ} 15'$ read $14^{\circ} 45'$ . 249 feet.
181	2	$42^{\circ} 3' 25''$ .
216	1	New Road should bear S. $24^{\circ} 38'$ W., 60 chains 82 links.
218	2	5 chains 42 links, and 6 chains 79 links.
227	1	12 acres, 3 roods, 9 perches.
	2	22 acres, 3 roods, 13 perches.
	2	7·04 chains.
230	2	$\left\{ \begin{array}{l} \text{A, 117 acres, and } 12\cdot72 \text{ chains frontage.} \\ \text{B, 67 acres, and } 7\cdot28 \text{ chains frontage.} \end{array} \right.$
257	1	$33\frac{1}{2}$ , $45\frac{1}{2}$ , $64\frac{1}{2}$ feet.
	2	$581\frac{1}{2}$ feet.

## ERRATA.

In page 40 for  $\frac{1}{\cos. A}$  read  $\frac{1}{-\cos. A}$   
for sec. A read —sec. A.

165 line 24, for cosec. DG read cosec. *d.* DG.

205 line 3, for them read these  
for is read as.

. . .

# APPENDIX.



TABLES

OF

**LOGARITHMS,**

OF

SINES, COSINES, AND TANGENTS.

AND

, A TRAVERSE TABLE,

SIX PLACES OF DECIMALS.



## TABLE OF LOGARITHMS.

THIS table extends to six places of logarithms, for all numbers from 1 to 10,000, which is amply sufficient for all ordinary practical purposes.

In looking for the logarithm of any number, it must be remembered, that,—as in natural numbers, the *position* of the decimal point alone determines the value of the numbers,—so, in logarithms, that value is represented by the number of digits in the index, *prefixed* to the logarithm, which is the logarithmic decimal point or characteristic.

Logarithms of natural numbers, therefore, are the same, whether those numbers be decimal or not; so that to find the logarithm of any given number, you must first find the logarithm of the figures independently of their integral value, and then prefix the proper Index, to represent the position of the decimal point.

### *To find the Logarithms.*

Look for the first three significant figures in the columns of numbers, on the left of the page; then follow, *laterally*, through the columns of logarithms, till you come to the one, which is headed by the next figure in the given number. The logarithm thus found, which is the logarithm, *collateral*, with the first three significant figures, and, *vertically*, under the next figure, will be the required logarithm of the given number.

EXAMPLE to find the logarithm of 4285. Look down the column of numbers for 428, then laterally to the column headed 5; then collaterally with 428, and, vertically, under 5, you will find the required logarithm 631951, which is the logarithm, whether the number be 4285 or 42·85 or ·0004285.

For the index, which represents the position of the decimal point, see chapter on logarithms, in the Introduction.

The method of finding the logarithms of numbers, *greater* than 10,000, or having more than four significant figures, will be found also fully explained there.

# LOGARITHM, OF NUMBERS

*To find the number, corresponding to a given Logarithm.*

Look in the columns of logarithms, for the nearest logarithm to the given one (omitting the index); then, looking, collaterally, to the column of numbers for the first three figures, and vertically, to the number on the top of the column for the fourth, you will obtain the corresponding number to four places of figures, which will be whole numbers, or decimals, according to the index of the logarithm.

For determining the position of the decimal point, as represented by this index, or for carrying out the number to more places of figures, see the chapter on the subject in the Introduction.

EXAMPLE—What is the natural number corresponding to the given logarithm 1·252130 ?

Looking in the columns of logarithms, you will find 252125 is the nearest logarithm; collaterally, in the column of numbers you will find 178, the first three figures; and vertically, at the head of the column 7, the fourth figure, which will give the required number 1787; having its integral value regulated by the given index 1, which will make 1787, all decimals, viz., ·1787.

## LOGARITHMS OF NUMBERS FROM 1 TO 10,000.

No.	Log	No.	Log.	No.	Log.	No.	Log.	No.	Log.
1	000000	21	322219	41	612781	61	783330	81	908185
2	301030	22	342423	42	623249	62	792392	82	913814
3	477121	23	361728	43	633468	63	799341	83	919078
4	602060	24	380211	44	643453	64	806180	84	924279
5	698970	25	397940	45	653213	65	812913	85	929419
6	778151	26	414973	46	662758	66	819544	86	934496
7	8450 8	27	431364	47	672098	67	826075	87	939319
8	903090	28	447158	48	681241	68	832509	88	944483
9	954243	29	462398	49	690196	69	838849	89	949390
10	000000	30	477121	50	698970	70	845098	90	954243
11	041393	31	491362	51	707570	71	851258	91	959041
12	079181	32	505150	52	716003	72	857383	92	963768
13	113943	33	518514	53	724276	73	863323	93	968483
14	146128	34	531479	54	732394	74	869292	94	973128
15	176091	35	544068	55	740363	75	875061	95	977724
16	204120	36	556303	56	748188	76	880814	96	982271
17	230449	37	568202	57	755875	77	886491	97	986772
18	255273	38	579784	58	763428	78	892093	98	991226
19	278754	39	591065	59	770852	79	897627	99	995685
20	301030	40	602060	60	778151	80	903096	100	000000

No	0	1	2	3	4	5	6	7	8	9
100	000000	000434	000858	001301	001734	002166	002598	003029	003461	003891
101	004321	004751	005181	005609	006038	006466	006894	007321	007748	008174
102	008500	009026	009451	009876	010300	010724	011147	011570	011993	012415
103	012837	013259	013680	014100	014521	014940	015360	015779	016197	016616
104	017033	017451	017868	018284	018700	019116	019532	019947	020361	020775
105	021189	021603	022016	022428	022841	023252	023664	024075	024486	024896
106	025306	025715	026125	026533	026942	027350	027757	028164	028571	028978
107	029384	029789	030193	030600	031001	031408	031812	032216	032619	033021
108	033424	033826	034227	034628	035029	035430	035830	036230	036629	037028
109	037426	037825	038223	038620	039017	039414	039811	040207	040602	040998
110	041393	041787	042182	042576	042969	043362	043755	044148	044540	044932
111	045323	045714	046105	046495	046885	047275	047664	048053	048442	048830
112	049218	049606	049993	050380	050766	051153	051538	051924	052309	052694
113	053078	053463	053846	054230	054613	054996	055378	055760	056142	056524
114	056905	057286	057666	058046	058426	058805	059185	059563	059942	060320
115	060698	061075	061452	061829	062206	062582	062958	063333	063709	064083
116	064458	064832	065206	065580	065953	066326	066699	067071	067443	067815
117	068186	068557	068928	069298	069668	070038	070407	070776	071145	071514
118	071882	072250	072617	072985	073352	073718	074083	074451	074816	075182
119	075547	075912	076276	076640	077004	077368	077731	078094	078457	078810
120	079181	079543	079904	080266	080626	080987	081347	081707	082067	082426
121	082785	083144	083503	083861	084219	084576	084934	085291	085647	086004
122	086360	086716	087071	087426	087781	088136	088490	088845	089198	089552
123	089905	090258	090611	090963	091315	091667	092018	092370	092721	093071
124	093422	093772	094122	094471	094820	095169	095518	095866	096215	096562
125	096910	097257	097604	097951	098298	098644	098990	099335	099681	100026
126	100371	100715	101059	101403	101747	102091	102434	102777	103119	103462
127	103804	104146	104487	104828	105169	105510	105851	106191	106531	106871
128	107210	107549	107888	108227	108565	108903	109241	109579	109916	110253
129	110590	110926	111263	111599	111934	112270	112605	112940	113275	113609
130	113943	114277	114611	114944	115278	115611	115943	116276	116608	116940
131	117271	117603	117934	118265	118595	118926	119256	119586	119915	120245
132	120574	120903	121231	121560	121888	122216	122544	122871	123198	123525
133	123852	124178	124504	124830	125156	125481	125806	126131	126456	126781
134	127105	127429	127753	128076	128399	128722	129045	129368	129690	130012
135	130334	130655	130977	131298	131619	131939	132260	132580	132900	133219
136	133538	133858	134177	134496	134814	135133	135451	135769	136086	136403
137	136721	137037	137354	137671	137987	138303	138618	138934	139249	139564
138	139879	140194	140508	140822	141136	141450	141763	142076	142389	142702
139	143015	143327	143639	143951	144263	144574	144885	145196	145507	145818
140	146128	146438	146748	147058	147367	147676	147985	148294	148603	148911
141	149219	149527	149835	150142	150449	150756	151063	151370	151676	151982
142	152288	152594	152900	153205	153510	153815	154120	154424	154728	155032
143	155338	155640	155943	156246	156549	156852	157154	157457	157759	158061
144	158369	158664	158965	159266	159567	159868	160168	160469	160769	161068
145	161368	161667	161967	162266	162564	162863	163161	163460	163758	164055
146	164353	164650	164947	165244	165541	165838	166134	166430	166726	167022
147	167317	167613	167908	168203	168497	168792	169086	169380	169674	169968
148	170262	170555	170848	171141	171434	171726	172019	172311	172603	172895
149	173186	173476	173769	174060	174351	174641	174932	175222	175512	175803



No	0	1	2	3	4	5	6	7	8	9
150	176091	176381	176670	176959	177248	177536	177825	178113	178401	178689
151	178977	179264	179552	179839	180126	180413	180699	180986	181272	181558
152	181844	182129	182415	182700	182985	183270	183555	183839	184123	184407
153	184691	184975	185259	185542	185825	186108	186391	186674	186956	187239
154	187521	187803	188084	188366	188647	188928	189209	189490	189771	190051
155	190332	190612	190892	191171	191451	191730	192010	192289	192567	192846
156	193125	193403	193681	193959	194237	194514	194792	195069	195346	195623
157	195699	195976	196253	196529	196805	197081	197356	197632	197907	198182
158	198637	198912	199186	199461	199735	200009	200283	200557	200830	201104
159	201397	201670	201943	202216	202488	202761	203033	203305	203577	203848
160	204120	204391	204663	204934	205204	205475	205746	206016	206286	206556
161	206826	207096	207365	207634	207904	208173	208441	208710	208979	209247
162	209515	209783	210051	210319	210586	210853	211121	211388	211654	211921
163	212188	212454	212720	212986	213252	213518	213783	214049	214314	214579
164	214841	215109	215373	215638	215902	216166	216430	216694	216957	217221
165	217481	217747	218010	218273	218536	218798	219060	219323	219585	219846
166	220108	220370	220631	220892	221153	221414	221675	221936	222196	222456
167	222716	222976	223236	223496	223755	224015	224274	224533	224792	225051
168	225309	225568	225826	226084	226342	226600	226858	227115	227372	227630
169	227887	228144	228400	228657	228913	229170	229426	229682	229938	230193
170	230149	230404	230660	230915	231170	231424	231679	231933	232188	232442
171	232696	232950	233204	233457	233711	233964	234217	234470	234723	234976
172	235228	235481	235733	235985	236237	236489	236741	236992	237244	237495
173	238016	238267	238518	238769	239019	239269	239519	239769	240019	240268
174	240519	240769	241018	241267	241516	241765	242014	242263	242511	242760
175	243018	243266	243514	243762	244010	244257	244505	244752	245000	245247
176	245513	245760	246006	246252	246499	246745	246991	247237	247482	247728
177	247973	248219	248464	248709	248954	249198	249443	249687	249932	250176
178	250120	250364	250608	250851	251095	251338	251581	251825	252068	252311
179	252553	252796	253038	253280	253522	253764	254006	254248	254490	254731
180	255273	255514	255755	255996	256237	256477	256718	256958	257198	257439
181	257679	257918	258158	258398	258637	258877	259116	259355	259594	259833
182	260071	260310	260548	260787	261025	261263	261501	261739	261976	262214
183	262451	262688	262925	263162	263399	263636	263873	264109	264346	264582
184	264818	265054	265290	265525	265761	265996	266232	266467	266702	266937
185	267172	267406	267641	267875	268110	268344	268578	268812	269046	269279
186	269513	269746	269980	270213	270446	270679	270912	271144	271377	271609
187	271842	272074	272306	272538	272770	273001	273233	273464	273696	273927
188	274158	274389	274620	274850	275081	275311	275542	275772	276002	276232
189	276462	276692	276921	277151	277380	277609	277838	278067	278296	278525
190	278754	278982	279211	279439	279667	279895	280123	280351	280578	280806
191	281033	281261	281488	281715	281942	282169	282396	282622	282849	283075
192	283308	283527	283745	283979	284205	284431	284656	284882	285107	285332
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603	780317	780389	780461	780533	780605	780677	780749	780821	780893	780965
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990	995635	995679	995723	995767	995811	995854	995898	995942	995986	996030
991	996074	996117	996161	996205	996249	996293	996337	996380	996424	996468
992	996512	996555	996599	996643	996687	996731	996774	996818	996862	996906
993	996949	996993	997037	997080	997124	997168	997212	997255	997299	997343
994	997386	997430	997474	997517	997561	997605	997648	997692	997736	997779
995	997823	997867	997910	997954	997998	998041	998085	998129	998172	998216
996	998259	998303	998347	998390	998434	998477	998521	998564	998608	998652
997	998695	998739	998782	998826	998869	998913	998956	998999	999043	999087
998	999131	999174	999218	999261	999305	999348	999392	999435	999477	999522
999	999565	999609	999652	999696	999739	999783	999826	999870	999913	999957

## TABLE II.

LOGARITHMIC SINES, COSINES, TANGENTS, AND  
COTANGENTS,

*Calculated to the natural radius of 10,000 millions,  
or the logarithmic radius of 10 index.*

THE secant and cosecant will be at once obtained from the tables, by the following formula, which will be found in the chapter on Trigonometry, page 38.

$$\text{sec.} = \frac{\text{rad.}^2}{\cos.} \quad \text{therefore, log. sec.} = 20 - \text{log. cos.}$$

$$\text{cosec.} = \frac{\text{rad.}^2}{\sin.} \quad \text{log. cosec.} = 20 - \text{log. sine.}$$

## PROBLEM I.

*To find the sine, tangent, &c.. of a given arc.*

If the degrees in the arc be *less than 45 degrees*, look at the top of the page for the degrees, reading downwards in the *left* hand column for the minutes, then, collaterally with the minutes, in the column headed sines, tangents, &c., at the *top* of the page, will be found the logarithmic sine or tangent required.

If the degrees be *more than 45°* look at the bottom of the page for the degrees, reading upwards, in the right hand column, for the minutes; then, collaterally, in the column, which has the proper heading, at the *bottom* of the page, will be found the logarithmic sine, or tangent required.

EXAMPLE 1.—It is required to find the sine of the arc of 29 degrees 45 minutes.

This arc being less than 45 degrees, look for 29 degrees at the top of the page, then reading downwards in the left hand, find 45 minutes; collaterally with this, in the column of sines, will be found 9.695671, the logarithmic sine required.

**EXAMPLE 2.**—It is required to find the tangent of the arc of 54 degrees 30 minutes.

This being above 45 degrees, look for 54 degrees, in the bottom of the page, and for 30 minutes, in the right hand column; then, in the column, headed tangents, at the *bottom* of the page, will be found, collaterally with the 30 minutes, the required tangent 10·146732.

*When the given angle is in seconds*, look out in the tables for the logarithmic sines, tangents, &c., of the next lower and next higher minutes; then say,—

As the difference between the next lower and next greater arc, in seconds: the difference between their corresponding logarithmic sines or tangents :: the excess, in seconds, of the given arc above the next lower in the tables: the logarithmic excess of the sine or tangent of the given angle, above that of the next lower angle in the tables.

Add this excess to the next lower, in the case of the sine, tangent, and secant, subtracting it for their complements, and you obtain the logarithmic sine, or tangent required.

Or, multiply the difference of the next lower and next higher logarithmic sines and tangents by the seconds in the given angle, and divide by 60, for the excess aforesaid.

**EXAMPLE** —What is the sine of the angle of  $15^{\circ} 45' 30''$ ?

$$\log. \sin. 15^{\circ} 46' = 9\cdot434122$$

$$\log. \sin. 15^{\circ} 45' = 9\cdot433675$$

$$\text{diff. } 1' = 447$$

$$\text{As } 1' \text{ or } 60'' : 30'' : 447 : x$$

$$447 \times \frac{30}{60} = 223 = \text{excess}$$

$$\log. \sin. 15^{\circ} 45' = 9\cdot433675$$

$$\text{excess} = 223$$

$$\log. \sin. 15^{\circ} 45' 30'' = 9\cdot433898$$

## PROBLEM II.

*To find the number of degrees and minutes of an arc, when the logarithmic sine or tangent is given.*

If the given logarithmic sine or tangent be found, or nearly so, in the tables, observe, whether the column, in which it is found, is headed sine or tangent, at the bottom or the top of the page; if at the *top*, look there for the number of degrees, and downwards in the left

hand column, for the minutes; if, on the contrary, at the *bottom*, the degrees will be found there, and, reading upwards in the right hand column, will be found the number of minutes required, placed collaterally with the given logarithmic sine or tangent.

**EXAMPLE**—What are the degrees and minutes of an arc, whose sine is 9.937895?

This will be found in the second column of logarithms, being headed sines, at the *bottom* of the page, where will be found 60 degrees; then tracing, collaterally, to the right hand column of minutes, will be found 5, making the required angle 60 degrees 5 minutes.

When the given logarithmic sine is not to be found exactly in the tables, and *the calculations are carried out to seconds* say,—As the difference between the next lower and next higher logarithmic sine or tangent, : the difference between their corresponding angles in the tables, in seconds, :: the difference between the next lower logarithmic sine or tangent and the given one, : excess in seconds, of the required angle.

Add this, in the case of the sine, tangent, or secant, and *vice versa*, to the next lower angle in the table, and you obtain the required angle in seconds.

Or, multiply the logarithmic excess by 60 seconds, and divide by the logarithmic difference for the actual excess in seconds.

**EXAMPLE 2.**—How many degrees, minutes, and seconds, are there in the arc, whose tangent is 10.246305?

next higher tan. 10.246174 = log. 60° 27'. given tan. 10.246305

next lower tan. 10.246180 = log. 60° 26'. = 10.246180

diff.  $\frac{294}{60''}$  log. excess = 125

As 294 : 60'' : 125 : x = actual excess

$x = \frac{125 \times 60}{294} = 25$  seconds which added to

the lower, makes the required angle = 60° 26' 25."

0°				1°			
Sine	Cosine	Tan.	Cotan.	Sine	Cosine	Tan.	Cotan.
0	10.000000			8.211855	9.999934	8.241921	11.758079
1	6.463726	10.000000	6.463726	13.536274	8.249033	9.999932	8.249102
2	6.764756	10.000000	6.764756	13.253544	8.256094	9.999929	8.256165
3	6.940847	10.000000	6.940847	13.059153	8.263042	9.999927	8.263115
4	7.065786	10.000000	7.065786	12.934214	8.269881	9.999925	8.269956
5	7.162696	10.000000	7.162696	12.837304	8.276614	9.999922	8.276691
6	7.241877	9.999999	7.241878	12.758122	8.283243	9.999920	8.283323
7	7.308824	9.999999	7.308825	12.691175	8.289773	9.999918	8.289856
8	7.366816	9.999999	7.366817	12.633133	8.296207	9.999915	8.296292
9	7.417968	9.999999	7.417970	12.582030	8.302546	9.999913	8.302634
10	7.463726	9.999998	7.463727	12.536273	8.308794	9.999910	8.308884
11	7.505118	9.999998	7.505120	12.494880	8.314954	9.999907	8.315046
12	7.542906	9.999997	7.542909	12.457091	8.321027	9.999905	8.321122
13	7.577668	9.999997	7.577672	12.422328	8.327016	9.999902	8.327114
14	7.609853	9.999996	7.609857	12.390143	8.332924	9.999899	8.333025
15	7.639816	9.999996	7.639820	12.360180	8.338753	9.999897	8.338856
16	7.667845	9.999995	7.667849	12.332151	8.344504	9.999894	8.344610
17	7.694173	9.999995	7.694179	12.306821	8.350181	9.999897	8.350289
18	7.718997	9.999994	7.719003	12.280907	8.355783	9.999888	8.355895
19	7.742478	9.999993	7.742484	12.257516	8.361315	9.999885	8.361430
20	7.764754	9.999993	7.764761	12.235239	8.366777	9.999882	8.366895
21	7.785943	9.999992	7.785951	12.214049	8.372171	9.999879	8.372232
22	7.806146	9.999991	7.806155	12.193845	8.377499	9.999876	8.377622
23	7.825451	9.999990	7.825460	12.174540	8.382762	9.999873	8.382893
24	7.843934	9.999989	7.843944	12.156056	8.387962	9.999870	8.388632
25	7.861662	9.999989	7.861674	12.138326	8.393101	9.999867	8.393234
26	7.878695	9.999988	7.878708	12.121292	8.398179	9.999864	8.398315
27	7.895085	9.999987	7.895099	12.104901	8.403199	9.999861	8.403338
28	7.910879	9.999986	7.910894	12.089106	8.408161	9.999858	8.408304
29	7.926119	9.999985	7.926134	12.073866	8.413068	9.999854	8.413213
30	7.940842	9.999983	7.940858	12.059142	8.417919	9.999851	8.418068
31	7.955082	9.999982	7.955100	12.044900	8.422717	9.999848	8.422869
32	7.968870	9.999981	7.968889	12.031111	8.427402	9.999845	8.427618
33	7.982233	9.999980	7.982253	12.017747	8.432156	9.999841	8.432315
34	7.995198	9.999979	7.995219	12.004881	8.436860	9.999838	8.436962
35	8.007787	9.999977	8.007809	11.992191	8.441394	9.999834	8.441560
36	8.020021	9.999976	8.020044	11.979956	8.445941	9.999831	8.446110
37	8.031919	9.999975	8.031945	11.968056	8.450440	9.999827	8.450613
38	8.043501	9.999973	8.043527	11.956473	8.454893	9.999824	8.455070
39	8.054781	9.999972	8.054809	11.945191	8.459301	9.999820	8.459481
40	8.065776	9.999971	8.065806	11.934194	8.463665	9.999816	8.463849
41	8.076500	9.999969	8.076531	11.923469	8.467985	9.999813	8.468172
42	8.086965	9.999968	8.086997	11.913003	8.472263	9.999809	8.472454
43	8.097183	9.999966	8.097217	11.902783	8.476498	9.999805	8.476693
44	8.107167	9.999964	8.107203	11.892797	8.480693	9.999801	8.480892
45	8.116926	9.999963	8.116963	11.883037	8.484848	9.999797	8.485050
46	8.126471	9.999961	8.126510	11.873490	8.488963	9.999794	8.489170
47	8.135810	9.999959	8.135851	11.864149	8.493040	9.999790	8.493250
48	8.144953	9.999958	8.144996	11.855004	8.497078	9.999786	8.497293
49	8.153907	9.999956	8.153952	11.846048	8.501080	9.999782	8.501298
50	8.162661	9.999954	8.162727	11.837273	8.505054	9.999778	8.505267
51	8.171230	9.999952	8.171328	11.828672	8.508974	9.999774	8.509200
52	8.179713	9.999950	8.179763	11.820237	8.512867	9.999769	8.513098
53	8.187985	9.999948	8.188036	11.812964	8.516726	9.999765	8.516961
54	8.196103	9.999946	8.196156	11.805844	8.520551	9.999761	8.520790
55	8.204070	9.999944	8.204126	11.798874	8.524343	9.999757	8.524586
56	8.211895	9.999942	8.211953	11.788047	8.528102	9.999753	8.528349
57	8.219582	9.999940	8.219641	11.780359	8.531828	9.999748	8.532080
58	8.227134	9.999938	8.227195	11.772805	8.535523	9.999744	8.535779
59	8.234557	9.999936	8.234621	11.765379	8.539186	9.999740	8.539447
60	8.241856	9.999934	8.241921	11.758079	8.542819	9.999735	8.543084
Cosine	Sine	Cotan.	Tan.	Cosine	Sine	Cotan.	Tan.

2°					3°				
	Sine	Cosine	Tan.	Cotan.	Sine	Cosine	Tan.	Cotan.	
0	8°542819	0°999735	8°543084	11°456916	8°718800	0°999404	8°719396	11°250604	60
1	8°546422	0°999731	8°550691	11°453309	8°721204	0°999398	8°721806	11°278194	59
2	8°550094	0°999726	8°550208	11°459732	8°724595	0°999391	8°724204	11°275796	58
3	8°553539	0°999722	8°553517	11°461813	8°725972	0°999384	8°726588	11°273412	57
4	8°557054	0°999717	8°557336	11°442664	8°728337	0°999378	8°728959	11°271041	56
5	8°560540	0°999713	8°560828	11°439479	8°730688	0°999371	8°731317	11°268683	55
6	8°563999	0°999708	8°564291	11°435709	8°733027	0°999364	8°733663	11°266337	54
7	8°567434	0°999704	8°567727	11°432273	8°735351	0°999357	8°735996	11°264004	53
8	8°570836	0°999699	8°571137	11°428463	8°737667	0°999350	8°738317	11°261683	52
9	8°574214	0°999694	8°574520	11°425486	8°739969	0°999343	8°740626	11°259374	51
10	8°577606	0°999689	8°577877	11°422123	8°742250	0°999336	8°742922	11°257078	50
11	8°580992	0°999685	8°581208	11°418792	8°744536	0°999329	8°745207	11°254793	49
12	8°584193	0°999680	8°584514	11°415486	8°746802	0°999322	8°747479	11°252521	48
13	8°587469	0°999675	8°587795	11°412205	8°749055	0°999315	8°749740	11°250260	47
14	8°590721	0°999670	8°591051	11°408949	8°751297	0°999308	8°751989	11°248011	46
15	8°593948	0°999665	8°594233	11°405717	8°753528	0°999301	8°754227	11°245773	45
16	8°597152	0°999660	8°597492	11°402508	8°755747	0°999294	8°756453	11°243547	44
17	8°600332	0°999655	8°600677	11°399323	8°757955	0°999287	8°758668	11°241332	43
18	8°603489	0°999650	8°603839	11°396161	8°760151	0°999279	8°760872	11°239128	42
19	8°606623	0°999645	8°606978	11°393022	8°762337	0°999272	8°763065	11°236935	41
20	8°609734	0°999640	8°610094	11°389909	8°764511	0°999265	8°765246	11°234754	40
21	8°612823	0°999635	8°613189	11°386811	8°766675	0°999257	8°767417	11°232583	39
22	8°615891	0°999630	8°616262	11°383738	8°768828	0°999250	8°769578	11°230422	38
23	8°618937	0°999624	8°619313	11°380687	8°770970	0°999242	8°771727	11°228273	37
24	8°621962	0°999619	8°622343	11°377657	8°773101	0°999235	8°773866	11°226131	36
25	8°624965	0°999614	8°625352	11°374648	8°775223	0°999227	8°775995	11°224005	35
26	8°627948	0°999608	8°628340	11°371660	8°777331	0°999220	8°778114	11°221886	34
27	8°630911	0°999603	8°631308	11°368692	8°779431	0°999212	8°780222	11°219778	33
28	8°633854	0°999597	8°634256	11°365741	8°781524	0°999205	8°782320	11°217680	32
29	8°636776	0°999592	8°637184	11°362816	8°783605	0°999197	8°784408	11°215592	31
30	8°639680	0°999586	8°640093	11°359907	8°785675	0°999189	8°786486	11°213574	30
31	8°642563	0°999581	8°642982	11°357018	8°787736	0°999181	8°788554	11°211446	29
32	8°645428	0°999575	8°645853	11°354147	8°789787	0°999174	8°790613	11°209387	28
33	8°648274	0°999570	8°648704	11°351296	8°791828	0°999166	8°792602	11°207338	27
34	8°651102	0°999564	8°651537	11°348463	8°793859	0°999158	8°794701	11°205299	26
35	8°653911	0°999558	8°654352	11°345648	8°795881	0°999150	8°796731	11°203269	25
36	8°656702	0°999553	8°657149	11°342851	8°797894	0°999142	8°798752	11°201248	24
37	8°659475	0°999547	8°659928	11°340072	8°799897	0°999134	8°800763	11°199237	23
38	8°662230	0°999541	8°662689	11°337311	8°801892	0°999126	8°802765	11°197235	22
39	8°664968	0°999535	8°665433	11°334567	8°803876	0°999118	8°804758	11°195242	21
40	8°667689	0°999529	8°668160	11°331840	8°805852	0°999110	8°806742	11°193258	20
41	8°670393	0°999524	8°670870	11°329130	8°807819	0°999102	8°808717	11°191283	19
42	8°673080	0°999518	8°673563	11°326437	8°809777	0°999094	8°810683	11°189317	18
43	8°675751	0°999512	8°676239	11°323761	8°811726	0°999086	8°812641	11°187359	17
44	8°678405	0°999506	8°678900	11°321100	8°813667	0°999077	8°814589	11°185411	16
45	8°681043	0°999500	8°681544	11°318456	8°815599	0°999069	8°816529	11°183471	15
46	8°683665	0°999493	8°684172	11°315828	8°817522	0°999061	8°818461	11°181539	14
47	8°686273	0°999487	8°686784	11°313216	8°819436	0°999053	8°820384	11°179616	13
48	8°688869	0°999481	8°689381	11°310619	8°821343	0°999044	8°822298	11°177702	12
49	8°691438	0°999475	8°691963	11°308037	8°823240	0°999036	8°824205	11°175795	11
50	8°693996	0°999469	8°694529	11°305471	8°825130	0°999027	8°826103	11°173897	10
51	8°696543	0°999463	8°697081	11°302919	8°827011	0°999019	8°827992	11°172008	9
52	8°699078	0°999456	8°699617	11°300383	8°828884	0°999010	8°829874	11°170126	8
53	8°701589	0°999449	8°702139	11°297861	8°830749	0°999002	8°831745	11°168252	7
54	8°704090	0°999443	8°704640	11°295354	8°832607	0°998993	8°833613	11°166387	6
55	8°706577	0°999437	8°707140	11°292860	8°834456	0°998984	8°835471	11°164529	5
56	8°709049	0°999431	8°709618	11°290382	8°836297	0°998976	8°837321	11°162679	4
57	8°711507	0°999424	8°712083	11°287917	8°838130	0°998967	8°839103	11°160837	3
58	8°713952	0°999418	8°714534	11°285466	8°839956	0°998958	8°840996	11°159002	2
59	8°716383	0°999411	8°716972	11°283028	8°841774	0°998950	8°842825	11°157175	1
60	8°718800	0°999404	8°719396	11°280604	8°843585	0°998941	8°844644	11°155356	0
	Cosine	Sine	Cotan.	Tan.	Cosine	Sine	Cotan.	Tan.	

4°					5°				
	Sine	Cosine	Tan.	Cotan.	Sine	Cosine	Tan.	Cotan.	
0	8°43555	9°998941	8°844644	11°153356	8°940296	9°998344	8°941952	11°058048	00
1	8°845387	9°998932	8°846455	11°153545	8°941738	9°998333	8°943404	11°056596	59
2	8°847183	9°998923	8°848260	11°151746	8°943174	9°998322	8°944852	11°055148	58
3	8°848971	9°998914	8°850057	11°149943	8°944606	9°998311	8°946295	11°053705	57
4	8°850751	9°998905	8°851846	11°148154	8°946034	9°998300	8°947734	11°052266	56
5	8°852525	9°998896	8°853628	11°146372	8°947456	9°998289	8°949168	11°050832	55
6	8°854291	9°998887	8°855403	11°144597	8°948874	9°998277	8°950597	11°049403	54
7	8°856049	9°998878	8°857171	11°142829	8°950287	9°998266	8°952021	11°047979	53
8	8°857801	9°998869	8°858932	11°141068	8°951696	9°998255	8°953441	11°046559	52
9	8°859546	9°998860	8°860696	11°139314	8°953100	9°998243	8°954856	11°045144	51
10	8°861283	9°998851	8°862433	11°137567	8°954519	9°998232	8°956267	11°043733	50
11	8°863014	9°998841	8°864173	11°135827	8°955934	9°998220	8°957651	11°042326	49
12	8°864738	9°998832	8°865906	11°134094	8°957281	9°998209	8°959075	11°040925	48
13	8°866455	9°998823	8°867632	11°132368	8°958670	9°998197	8°960473	11°039527	47
14	8°868165	9°998813	8°869351	11°130649	8°960052	9°998186	8°961866	11°038134	46
15	8°869868	9°998804	8°871064	11°128936	8°961429	9°998174	8°963255	11°036745	45
16	8°871565	9°998795	8°872720	11°127230	8°962801	9°998163	8°964639	11°035361	44
17	8°873255	9°998785	8°874469	11°125531	8°964170	9°998152	8°966019	11°033981	43
18	8°874938	9°998776	8°876162	11°123838	8°965534	9°998139	8°967394	11°032606	42
19	8°876615	9°998766	8°877849	11°122151	8°966898	9°998128	8°968766	11°031234	41
20	8°878285	9°998757	8°879529	11°120471	8°968249	9°998116	8°970133	11°029867	40
21	8°879949	9°998747	8°881202	11°118798	8°969600	9°998104	8°971496	11°028504	39
22	8°881607	9°998738	8°882869	11°117131	8°970947	9°998092	8°972855	11°027145	38
23	8°883258	9°998728	8°884530	11°115470	8°972289	9°998080	8°974209	11°025791	37
24	8°884903	9°998718	8°886185	11°113815	8°973628	9°998068	8°975560	11°024440	36
25	8°886542	9°998708	8°887833	11°112167	8°974962	9°998056	8°976906	11°023094	35
26	8°888174	9°998699	8°889476	11°110524	8°976293	9°998044	8°978248	11°021752	34
27	8°889801	9°998689	8°891112	11°108888	8°977619	9°998032	8°979586	11°020414	33
28	8°891421	9°998679	8°892742	11°107258	8°978941	9°998020	8°980921	11°019079	32
29	8°893035	9°998669	8°894366	11°105634	8°980259	9°998008	8°982251	11°017749	31
30	8°894643	9°998659	8°895984	11°104016	8°981573	9°998996	8°983577	11°016423	30
31	8°896246	9°998649	8°897396	11°102404	8°982883	9°998984	8°984899	11°015101	29
32	8°897842	9°998639	8°899020	11°100797	8°984189	9°998972	8°986217	11°013783	28
33	8°899432	9°998629	8°900803	11°099197	8°985491	9°998959	8°987532	11°012468	27
34	8°901017	9°998619	8°902395	11°097602	8°986789	9°998947	8°988842	11°011158	26
35	8°902596	9°998609	8°903977	11°096013	8°988083	9°998935	8°990149	11°009851	25
36	8°904169	9°998599	8°905593	11°094430	8°989374	9°998922	8°991551	11°008549	24
37	8°905736	9°998589	8°907147	11°092853	8°990660	9°998910	8°992750	11°007250	23
38	8°907297	9°998578	8°908719	11°091281	8°991913	9°998897	8°994045	11°005955	22
39	8°908853	9°998568	8°910285	11°089715	8°993222	9°998885	8°995337	11°004663	21
40	8°910404	9°998558	8°911846	11°088154	8°994497	9°998872	8°996624	11°003376	20
41	8°911949	9°998548	8°913401	11°086599	8°995768	9°998860	8°997908	11°002092	19
42	8°913488	9°998537	8°914951	11°085049	8°997036	9°998847	8°999188	11°000812	18
43	8°915022	9°998527	8°916495	11°083505	8°998299	9°998835	8°100465	11°099535	17
44	8°916550	9°998516	8°918034	11°081966	8°999560	9°998822	8°101738	11°098262	16
45	8°918073	9°998506	8°919568	11°080422	8°100816	9°998809	8°103007	11°096993	15
46	8°919591	9°998495	8°921096	11°078904	8°102069	9°998797	8°104272	11°095728	14
47	8°921103	9°998485	8°922619	11°077381	8°103318	9°998784	8°105534	11°094466	13
48	8°922610	9°998474	8°924136	11°075864	8°104563	9°998771	8°106792	11°093208	12
49	8°924112	9°998464	8°925649	11°074351	8°105805	9°998758	8°108047	11°091953	11
50	8°925609	9°998453	8°927156	11°072841	8°107044	9°998745	8°109298	11°090702	10
51	8°927100	9°998442	8°928658	11°071342	8°108278	9°998732	8°110546	11°089454	9
52	8°928587	9°998431	8°930155	11°069845	8°109510	9°998719	8°111790	11°088210	8
53	8°930068	9°998421	8°931647	11°068353	8°110737	9°998706	8°113031	11°086969	7
54	8°931544	9°998410	8°933131	11°066866	8°111962	9°998693	8°114268	11°085732	6
55	8°933015	9°998399	8°934616	11°065384	8°113182	9°998680	8°115502	11°084498	5
56	8°934481	9°998388	8°936093	11°063907	8°114400	9°998667	8°116732	11°083268	4
57	8°935942	9°998377	8°937565	11°062435	8°115613	9°998654	8°117959	11°082041	3
58	8°937398	9°998366	8°939032	11°060968	8°116824	9°998641	8°119183	11°080817	2
59	8°938850	9°998355	8°940491	11°059506	8°118031	9°998628	8°120403	11°079597	1
60	8°940296	9°998344	8°941952	11°058048	8°119235	9°998614	8°121620	11°078380	0
	Cosine	Sine	Cotan.	Tan.	Cosine	Sine	Cotan.	Tan.	

0°					7°				
	Sine	Cosine	Tan.	Cotan.	Sine	Cosine	Tan.	Cotan.	
0	0.019235	0.997614	0.021620	10.978380	0.085894	0.996751	0.089144	10.910856	60
1	0.020435	0.997601	0.022834	10.977166	0.086922	0.996735	0.090187	10.909813	59
2	0.021632	0.997588	0.024044	10.975956	0.087947	0.996720	0.091228	10.908772	58
3	0.022825	0.997574	0.025251	10.974749	0.088970	0.996704	0.092266	10.907734	57
4	0.024016	0.997561	0.026455	10.973545	0.089990	0.996688	0.093302	10.906698	56
5	0.025203	0.997547	0.027655	10.972345	0.091008	0.996673	0.094336	10.905664	55
6	0.026386	0.997534	0.028852	10.971148	0.092024	0.996657	0.095367	10.904633	54
7	0.027567	0.997520	0.030046	10.969954	0.093037	0.996641	0.096395	10.903605	53
8	0.028744	0.997507	0.031237	10.968763	0.094047	0.996625	0.097422	10.902578	52
9	0.029918	0.997493	0.032425	10.967575	0.095056	0.996610	0.098446	10.901554	51
10	0.031089	0.997480	0.033609	10.966391	0.096062	0.996594	0.099468	10.900532	50
11	0.032257	0.997466	0.034791	10.965209	0.097065	0.996578	0.100487	10.899513	49
12	0.033421	0.997452	0.035969	10.964031	0.098066	0.996562	0.101504	10.898496	48
13	0.034582	0.997439	0.037144	10.962856	0.099065	0.996546	0.102519	10.897481	47
14	0.035741	0.997425	0.038316	10.961684	0.100062	0.996530	0.103532	10.896468	46
15	0.036896	0.997411	0.039485	10.960515	0.101056	0.996514	0.104542	10.895458	45
16	0.038048	0.997397	0.040651	10.959349	0.102048	0.996498	0.105550	10.894450	44
17	0.039197	0.997383	0.041813	10.958187	0.103037	0.996482	0.106556	10.893444	43
18	0.040342	0.997369	0.042973	10.957087	0.104025	0.996465	0.107559	10.892441	42
19	0.041485	0.997355	0.044130	10.955870	0.105010	0.996449	0.108550	10.891440	41
20	0.042625	0.997341	0.045284	10.954716	0.105992	0.996433	0.109559	10.890441	40
21	0.043762	0.997327	0.046434	10.953566	0.106973	0.996417	0.110556	10.889444	39
22	0.044895	0.997313	0.047582	10.952418	0.107951	0.996400	0.111551	10.888449	38
23	0.046026	0.997299	0.048727	10.951273	0.108927	0.996384	0.112543	10.887457	37
24	0.047154	0.997285	0.049869	10.950131	0.109901	0.996368	0.113533	10.886467	36
25	0.048277	0.997271	0.051008	10.948992	0.110873	0.996351	0.114521	10.885479	35
26	0.049400	0.997257	0.052144	10.947856	0.111842	0.996335	0.115507	10.884493	34
27	0.050519	0.997242	0.053277	10.946723	0.112809	0.996318	0.116491	10.883509	33
28	0.051635	0.997228	0.054407	10.945593	0.113774	0.996302	0.117472	10.882526	32
29	0.052749	0.997214	0.055535	10.944465	0.114737	0.996285	0.118452	10.881548	31
30	0.053859	0.997199	0.056659	10.943341	0.115698	0.996269	0.119429	10.880571	30
31	0.054966	0.997185	0.057781	10.942219	0.116656	0.996252	0.120404	10.879596	29
32	0.056071	0.997170	0.058900	10.941100	0.117613	0.996235	0.121377	10.878623	28
33	0.057172	0.997156	0.060016	10.939984	0.118567	0.996219	0.122348	10.877652	27
34	0.058271	0.997141	0.061130	10.938870	0.119519	0.996202	0.123317	10.876683	26
35	0.059367	0.997127	0.062244	10.937760	0.120469	0.996185	0.124284	10.875716	25
36	0.060460	0.997112	0.063348	10.936652	0.121417	0.996168	0.125249	10.874751	24
37	0.061551	0.997098	0.064453	10.935547	0.122362	0.996151	0.126211	10.873789	23
38	0.062639	0.997083	0.065556	10.934444	0.123306	0.996134	0.127172	10.872828	22
39	0.063724	0.997068	0.066655	10.933345	0.124248	0.996117	0.128130	10.871870	21
40	0.064806	0.997053	0.067752	10.932248	0.125187	0.996100	0.129087	10.870913	20
41	0.065885	0.997039	0.068846	10.931154	0.126125	0.996083	0.130041	10.869959	19
42	0.066962	0.997024	0.069938	10.930062	0.127060	0.996066	0.130994	10.869006	18
43	0.068036	0.997009	0.071027	10.928973	0.127993	0.996049	0.131944	10.868056	17
44	0.069107	0.996994	0.072113	10.927887	0.128925	0.996032	0.132893	10.867107	16
45	0.070176	0.996979	0.073197	10.926803	0.129854	0.996015	0.133839	10.866161	15
46	0.071242	0.996964	0.074278	10.925722	0.130781	0.995998	0.134784	10.865216	14
47	0.072306	0.996949	0.075356	10.924644	0.131706	0.995980	0.135726	10.864274	13
48	0.073366	0.996934	0.076432	10.923566	0.132630	0.995963	0.136667	10.863333	12
49	0.074424	0.996919	0.077505	10.922495	0.133551	0.995946	0.137605	10.862395	11
50	0.075480	0.996904	0.078576	10.921424	0.134470	0.995928	0.138542	10.861458	10
51	0.076533	0.996889	0.079644	10.920355	0.135387	0.995911	0.139478	10.860524	9
52	0.077583	0.996874	0.080710	10.919290	0.136303	0.995894	0.140409	10.859591	8
53	0.078631	0.996858	0.081773	10.918227	0.137216	0.995876	0.141343	10.858660	7
54	0.079676	0.996843	0.082833	10.917167	0.138128	0.995859	0.142269	10.857731	6
55	0.080719	0.996828	0.083891	10.916109	0.139037	0.995841	0.143196	10.856804	5
56	0.081759	0.996812	0.084947	10.915053	0.139944	0.995823	0.144121	10.855879	4
57	0.082797	0.996797	0.086000	10.914000	0.140850	0.995806	0.145044	10.854956	3
58	0.083832	0.996782	0.087050	10.912950	0.141754	0.995788	0.145966	10.854034	2
59	0.084864	0.996766	0.088098	10.911902	0.142655	0.995771	0.146888	10.853115	1
60	0.085894	0.996751	0.089144	10.910856	0.143555	0.995753	0.147803	10.852197	0
	Cosine	Sine	Cotan.	Tan.	Cosine	Sine	Cotan.	Tan.	



8°				9°			
Sine	Cosine	Tan.	Cotan.	Sine	Cosine	Tan.	Cotan.
0 9°143555	9°995753	9°147803	10°852197	9°194332	9°994020	9°199713	10°800287
1 9°144453	9°995735	9°148718	10°851282	9°195129	9°994000	9°200519	10°799471
2 9°145349	9°995717	9°149632	10°850368	9°195925	9°994580	9°201345	10°798655
3 9°146243	9°995699	9°150544	10°849456	9°196719	9°994560	9°202159	10°797841
4 9°147136	9°995681	9°151454	10°848546	9°197511	9°994540	9°202971	10°797029
5 9°148026	9°995664	9°152363	10°847637	9°198302	9°994519	9°203782	10°796218
6 9°148915	9°995646	9°153269	10°846731	9°199099	9°994499	9°204592	10°795408
7 9°149802	9°995628	9°154174	10°845826	9°199897	9°994479	9°205400	10°794400
8 9°150686	9°995610	9°155077	10°844923	9°200696	9°994459	9°206207	10°793793
9 9°151569	9°995591	9°155978	10°844022	9°201451	9°994438	9°207013	10°792987
10 9°152451	9°995573	9°156877	10°843123	9°202234	9°994418	9°207817	10°792183
11 9°153330	9°995555	9°157775	10°842225	9°203017	9°994398	9°208619	10°791381
12 9°154206	9°995537	9°158671	10°841329	9°203797	9°994377	9°209420	10°790580
13 9°155083	9°995519	9°159565	10°840435	9°204577	9°994357	9°210220	10°789780
14 9°155957	9°995501	9°160457	10°839543	9°205354	9°994336	9°211018	10°788982
15 9°156830	9°995482	9°161347	10°838653	9°206131	9°994316	9°211815	10°788185
16 9°157700	9°995464	9°162236	10°837764	9°206906	9°994295	9°212611	10°787389
17 9°158569	9°995446	9°163123	10°836877	9°207679	9°994274	9°213406	10°786595
18 9°159435	9°995427	9°164008	10°835992	9°208452	9°994254	9°214198	10°785802
19 9°160301	9°995409	9°164892	10°835108	9°209222	9°994233	9°214989	10°785011
20 9°161164	9°995390	9°165774	10°834226	9°210000	9°994212	9°215780	10°784220
21 9°162025	9°995372	9°166654	10°833346	9°210760	9°994191	9°216568	10°783432
22 9°162885	9°995353	9°167532	10°832466	9°211526	9°994171	9°217356	10°782644
23 9°163743	9°995334	9°168409	10°831591	9°212291	9°994150	9°218142	10°781858
24 9°164600	9°995316	9°169284	10°830716	9°213055	9°994129	9°218926	10°781074
25 9°165454	9°995297	9°170157	10°829843	9°213818	9°994108	9°219710	10°780290
26 9°166317	9°995278	9°171029	10°828971	9°214579	9°994087	9°220492	10°779508
27 9°167159	9°995260	9°171899	10°828101	9°215338	9°994066	9°221272	10°778728
28 9°168008	9°995241	9°172767	10°827233	9°216097	9°994045	9°222052	10°777948
29 9°168856	9°995222	9°173634	10°826366	9°216854	9°994024	9°222830	10°777170
30 9°169702	9°995203	9°174499	10°825501	9°217609	9°994003	9°223607	10°776393
31 9°170547	9°995184	9°175362	10°824638	9°218363	9°993982	9°224382	10°775618
32 9°171389	9°995165	9°176224	10°823776	9°219116	9°993960	9°225156	10°774844
33 9°172230	9°995146	9°177084	10°822916	9°219868	9°993939	9°225929	10°774071
34 9°173070	9°995127	9°177942	10°822058	9°220618	9°993918	9°226700	10°773300
35 9°173908	9°995098	9°178799	10°821201	9°221367	9°993897	9°227471	10°772529
36 9°174744	9°995079	9°179655	10°820345	9°222115	9°993875	9°228239	10°771761
37 9°175578	9°995070	9°180508	10°819492	9°222861	9°993854	9°229007	10°770993
38 9°176411	9°995051	9°181360	10°818646	9°223606	9°993832	9°229773	10°770227
39 9°177242	9°995032	9°182211	10°817789	9°224349	9°993811	9°230539	10°769461
40 9°178072	9°995013	9°183059	10°816941	9°225092	9°993789	9°231302	10°768698
41 9°178900	9°994993	9°183907	10°816093	9°225833	9°993768	9°232065	10°767935
42 9°179726	9°994974	9°184752	10°815248	9°226573	9°993746	9°232826	10°767174
43 9°180551	9°994955	9°185597	10°814403	9°227311	9°993725	9°233586	10°766414
44 9°181374	9°994935	9°186439	10°813561	9°228048	9°993703	9°234345	10°765655
45 9°182196	9°994916	9°187280	10°812720	9°228784	9°993681	9°235103	10°764897
46 9°183016	9°994896	9°188120	10°811880	9°229518	9°993660	9°235859	10°764141
47 9°183834	9°994877	9°188958	10°811042	9°230252	9°993638	9°236614	10°763386
48 9°184651	9°994857	9°189794	10°810206	9°230984	9°993616	9°237368	10°762632
49 9°185466	9°994838	9°190629	10°809371	9°231715	9°993594	9°238120	10°761880
50 9°186280	9°994818	9°191462	10°808533	9°232444	9°993572	9°238872	10°761128
51 9°187092	9°994798	9°192294	10°807706	9°233172	9°993550	9°239622	10°760378
52 9°187903	9°994779	9°193124	10°806876	9°233899	9°993528	9°240371	10°759629
53 9°188712	9°994759	9°193953	10°806047	9°234625	9°993506	9°241118	10°758882
54 9°189519	9°994739	9°194780	10°805220	9°235349	9°993484	9°241866	10°758135
55 9°190325	9°994720	9°195606	10°804394	9°236073	9°993462	9°242610	10°757389
56 9°191130	9°994700	9°196430	10°803570	9°236795	9°993440	9°243354	10°756646
57 9°191933	9°994680	9°197253	10°802747	9°237515	9°993418	9°244097	10°755903
58 9°192734	9°994660	9°198074	10°801926	9°238235	9°993396	9°244839	10°755161
59 9°193534	9°994640	9°198894	10°801106	9°238953	9°993374	9°245579	10°754421
60 9°194332	9°994620	9°199713	10°800287	9°239670	9°993351	9°246319	10°753681
Cosine	Sine	Cotan.	Tan.	Cosine	Sine	Cotan.	Tan.

10°					11°				
	Sine	Cosine	Tan.	Cotan.	Sine	Cosine	Tan.	Cotan.	
0	9°239070	9°993351	9°246319	10°753081	9°280509	9°991947	9°288652	10°711348	60
1	9°240356	9°993329	9°247057	10°752943	9°281248	9°991922	9°289326	10°710674	59
2	9°241101	9°993307	9°247704	10°752806	9°281987	9°991897	9°290009	10°710001	58
3	9°241814	9°993284	9°248350	10°751470	9°282544	9°991878	9°290671	10°709329	57
4	9°242520	9°993262	9°249004	10°750786	9°283190	9°991848	9°291342	10°708658	56
5	9°243237	9°993240	9°249698	10°750002	9°283836	9°991823	9°292013	10°707987	55
6	9°243947	9°993217	9°250370	10°749270	9°284480	9°991799	9°292682	10°707318	54
7	9°244650	9°993195	9°251461	10°748589	9°285124	9°991774	9°293350	10°706650	53
8	9°245363	9°993172	9°252191	10°747809	9°285766	9°991749	9°294017	10°705983	52
9	9°246069	9°993149	9°252920	10°747080	9°286408	9°991724	9°294684	10°705316	51
10	9°246775	9°993127	9°253648	10°746352	9°287048	9°991699	9°295349	10°704651	50
11	9°247478	9°993104	9°254374	10°745626	9°287688	9°991674	9°296013	10°703987	49
12	9°248181	9°993081	9°255100	10°744900	9°288326	9°991640	9°296677	10°703323	48
13	9°248883	9°993059	9°255824	10°744176	9°288964	9°991624	9°297339	10°702661	47
14	9°249583	9°993036	9°256547	10°743453	9°289600	9°991599	9°298001	10°701999	46
15	9°250282	9°993013	9°257269	10°742731	9°290236	9°991574	9°298662	10°701338	45
16	9°250980	9°992990	9°257990	10°742010	9°290870	9°991549	9°299322	10°700678	44
17	9°251677	9°992967	9°258710	10°741290	9°291504	9°991524	9°299980	10°700020	43
18	9°252373	9°992944	9°259429	10°740571	9°292137	9°991498	9°300638	10°699362	42
19	9°253067	9°992921	9°260146	10°739854	9°292768	9°991473	9°301295	10°698705	41
20	9°253761	9°992898	9°260863	10°739137	9°293399	9°991448	9°301951	10°698049	40
21	9°254453	9°992875	9°261578	10°738422	9°294029	9°991422	9°302607	10°697393	39
22	9°255144	9°992852	9°262292	10°737706	9°294658	9°991397	9°303261	10°696737	38
23	9°255834	9°992829	9°263006	10°736995	9°295286	9°991372	9°303914	10°696086	37
24	9°256523	9°992806	9°263717	10°736283	9°295913	9°991346	9°304567	10°695433	36
25	9°257211	9°992783	9°264428	10°735572	9°296539	9°991321	9°305218	10°694782	35
26	9°257898	9°992759	9°265138	10°734862	9°297164	9°991295	9°305869	10°694131	34
27	9°258583	9°992736	9°265847	10°734153	9°297788	9°991270	9°306519	10°693481	33
28	9°259268	9°992713	9°266555	10°733445	9°298412	9°991244	9°307168	10°692832	32
29	9°259951	9°992690	9°267261	10°732739	9°299034	9°991218	9°307816	10°692184	31
30	9°260633	9°992666	9°267967	10°732033	9°299655	9°991193	9°308463	10°691537	30
31	9°261314	9°992643	9°268671	10°731329	9°300276	9°991167	9°309109	10°690891	29
32	9°261994	9°992619	9°269375	10°730625	9°300895	9°991141	9°309754	10°690246	28
33	9°262673	9°992596	9°270077	10°729923	9°301514	9°991115	9°310403	10°689601	27
34	9°263351	9°992572	9°270779	10°729221	9°302132	9°991090	9°311042	10°688958	26
35	9°264027	9°992549	9°271479	10°728521	9°302748	9°991064	9°311685	10°688315	25
36	9°264703	9°992525	9°272178	10°727822	9°303364	9°991038	9°312327	10°687673	24
37	9°265377	9°992501	9°272876	10°727124	9°303979	9°991012	9°312968	10°687032	23
38	9°266051	9°992478	9°273573	10°726427	9°304593	9°990986	9°313608	10°686392	22
39	9°266723	9°992454	9°274269	10°725731	9°305207	9°990960	9°314247	10°685753	21
40	9°267395	9°992430	9°274964	10°725036	9°305819	9°990934	9°314885	10°685115	20
41	9°268065	9°992406	9°275658	10°724342	9°306430	9°990908	9°315523	10°684477	19
42	9°268734	9°992382	9°276351	10°723649	9°307041	9°990882	9°316159	10°683841	18
43	9°269402	9°992359	9°277043	10°722957	9°307650	9°990855	9°316795	10°683205	17
44	9°270069	9°992335	9°277734	10°722266	9°308259	9°990829	9°317430	10°682570	16
45	9°270735	9°992311	9°278424	10°721576	9°308867	9°990803	9°318064	10°681936	15
46	9°271400	9°992287	9°279113	10°720887	9°309474	9°990777	9°318697	10°681303	14
47	9°272064	9°992263	9°279801	10°720199	9°310080	9°990750	9°319330	10°680670	13
48	9°272726	9°992239	9°280488	10°719512	9°310685	9°990724	9°319961	10°680039	12
49	9°273388	9°992214	9°281174	10°718826	9°311289	9°990697	9°320592	10°679408	11
50	9°274049	9°992190	9°281858	10°718142	9°311893	9°990671	9°321222	10°678778	10
51	9°274708	9°992166	9°282542	10°717458	9°312495	9°990645	9°321851	10°678149	9
52	9°275367	9°992142	9°283225	10°716775	9°313097	9°990618	9°322479	10°677521	8
53	9°276025	9°992118	9°283907	10°716093	9°313698	9°990591	9°323106	10°676894	7
54	9°276681	9°992093	9°284588	10°715412	9°314297	9°990565	9°323733	10°676267	6
55	9°277337	9°992069	9°285268	10°714732	9°314897	9°990538	9°324358	10°675642	5
56	9°277991	9°992044	9°285947	10°714053	9°315495	9°990511	9°324983	10°675017	4
57	9°278645	9°992020	9°286624	10°713376	9°316092	9°990485	9°325607	10°674393	3
58	9°279297	9°991996	9°287301	10°712699	9°316689	9°990458	9°326231	10°673769	2
59	9°279948	9°991971	9°287977	10°712023	9°317284	9°990431	9°326853	10°673147	1
60	9°280599	9°991947	9°288652	10°711348	9°317879	9°990404	9°327475	10°672525	0
	Cosine	Sine	Cotan.	Tan.	Cosine	Sine	Cotan.	Tan.	

12°				13°			
Sine	Cosine	Tan.	Cotan.	Sine	Cosine	Tan.	Cotan.
0 9°317879	9°990404	9°327475	10°672525	9°352088	9°988724	9°363361	10°636636
1 9°318473	9°990378	9°328095	10°671905	9°352635	9°988695	9°363940	10°636060
2 9°319066	9°990351	9°328715	10°671285	9°353181	9°988666	9°364516	10°635485
3 9°319658	9°990324	9°329334	10°670666	9°353726	9°988636	9°365090	10°634910
4 9°320249	9°990297	9°329953	10°670047	9°354271	9°988607	9°365664	10°634336
5 9°320840	9°990270	9°330570	10°669430	9°354815	9°988578	9°366237	10°633763
6 9°321430	9°990243	9°331187	10°668813	9°355358	9°988548	9°366810	10°633190
7 9°322019	9°990215	9°331803	10°668197	9°355901	9°988519	9°367382	10°632618
8 9°322607	9°990185	9°332418	10°667582	9°356443	9°988489	9°367953	10°632047
9 9°323194	9°990161	9°333033	10°666967	9°356984	9°988460	9°368524	10°631476
10 9°323780	9°990134	9°333646	10°666354	9°357524	9°988430	9°369094	10°630906
11 9°324366	9°990107	9°334259	10°665741	9°358064	9°988401	9°369663	10°630337
12 9°324950	9°990079	9°334871	10°665129	9°358603	9°988371	9°370232	10°629768
13 9°325534	9°990052	9°335482	10°664516	9°359141	9°988342	9°370799	10°629201
14 9°326117	9°990025	9°336093	10°663907	9°359678	9°988312	9°371367	10°628633
15 9°326700	9°989997	9°336702	10°663298	9°360215	9°988282	9°371933	10°628067
16 9°327281	9°989970	9°337311	10°662689	9°360752	9°988252	9°372499	10°627501
17 9°327862	9°989942	9°337919	10°662081	9°361287	9°988223	9°373064	10°626936
18 9°328442	9°989915	9°338527	10°661473	9°361822	9°988193	9°373629	10°626371
19 9°329021	9°989887	9°339133	10°660860	9°362356	9°988163	9°374193	10°625807
20 9°329599	9°989860	9°339739	10°660261	9°362889	9°988133	9°374756	10°625244
21 9°330176	9°989832	9°340344	10°659656	9°363422	9°988103	9°375319	10°624681
22 9°330753	9°989804	9°340948	10°659052	9°363954	9°988073	9°375881	10°624119
23 9°331329	9°989777	9°341552	10°658448	9°364485	9°988043	9°376442	10°623558
24 9°331903	9°989749	9°342155	10°657845	9°365016	9°988013	9°377003	10°622997
25 9°332478	9°989721	9°342757	10°657243	9°365546	9°987983	9°377563	10°622437
26 9°333051	9°989693	9°343358	10°656642	9°366075	9°987953	9°378122	10°621878
27 9°333624	9°989665	9°343958	10°656040	9°366604	9°987922	9°378681	10°621319
28 9°334195	9°989637	9°344558	10°655442	9°367131	9°987892	9°379239	10°620761
29 9°334767	9°989610	9°345157	10°654843	9°367659	9°987862	9°379797	10°620203
30 9°335337	9°989582	9°345755	10°654245	9°368185	9°987832	9°380354	10°619646
31 9°335906	9°989553	9°346353	10°653647	9°368711	9°987801	9°380910	10°619090
32 9°336475	9°989525	9°346949	10°653051	9°369236	9°987771	9°381466	10°618534
33 9°337043	9°989497	9°347545	10°652455	9°369761	9°987740	9°382020	10°617980
34 9°337610	9°989469	9°348141	10°651859	9°370285	9°987710	9°382575	10°617425
35 9°338176	9°989441	9°348735	10°651265	9°370808	9°987679	9°383129	10°616871
36 9°338742	9°989413	9°349329	10°650671	9°371330	9°987649	9°383682	10°616318
37 9°339307	9°989385	9°349922	10°650078	9°371852	9°987618	9°384234	10°615766
38 9°339871	9°989356	9°350514	10°649486	9°372373	9°987588	9°384786	10°615214
39 9°340434	9°989328	9°351106	10°648894	9°372894	9°987557	9°385337	10°614663
40 9°340996	9°989300	9°351697	10°648303	9°373414	9°987526	9°385886	10°614112
41 9°341558	9°989271	9°352287	10°647713	9°373933	9°987496	9°386438	10°613562
42 9°342119	9°989243	9°352876	10°647124	9°374452	9°987466	9°386987	10°613013
43 9°342679	9°989214	9°353465	10°646535	9°374970	9°987434	9°387536	10°612464
44 9°343239	9°989186	9°354053	10°645947	9°375487	9°987403	9°388084	10°611916
45 9°343797	9°989157	9°354640	10°645360	9°376003	9°987372	9°388631	10°611369
46 9°344355	9°989128	9°355227	10°644773	9°376519	9°987341	9°389178	10°610822
47 9°344912	9°989100	9°355813	10°644187	9°377035	9°987310	9°389724	10°610276
48 9°345469	9°989071	9°356398	10°643602	9°377549	9°987279	9°390270	10°609730
49 9°346024	9°989042	9°356982	10°643018	9°378063	9°987248	9°390815	10°609185
50 9°346579	9°989014	9°357566	10°642434	9°378577	9°987217	9°391360	10°608640
51 9°347134	9°988985	9°358140	10°641851	9°379090	9°987186	9°391903	10°608097
52 9°347687	9°988956	9°358731	10°641269	9°379601	9°987155	9°392447	10°607553
53 9°348240	9°988927	9°359313	10°640687	9°380113	9°987124	9°392989	10°607011
54 9°348792	9°988898	9°359893	10°640107	9°380624	9°987092	9°393531	10°606469
55 9°349343	9°988869	9°360474	10°639526	9°381134	9°987061	9°394073	10°605927
56 9°349893	9°988840	9°361053	10°638947	9°381642	9°987030	9°394614	10°605386
57 9°350443	9°988811	9°361632	10°638368	9°382152	9°986998	9°395154	10°604846
58 9°350992	9°988782	9°362210	10°637790	9°382661	9°986967	9°395694	10°604306
59 9°351540	9°988753	9°362787	10°637213	9°383168	9°986936	9°396233	10°603767
60 9°352088	9°988724	9°363364	10°636636	9°383675	9°986904	9°396771	10°603229
Cosine	Sine	Cotan.	Tan.	Cosine	Sine	Cotan.	Tan.

14°				15°			
Sine	Cosine	Tan.	Cotan.	Sine	Cosine	Tan.	Cotan.
0 9°383675	9°586994	9°396771	10°603229	9°412936	9°984941	9°428052	10°571948
1 9°384182	9°586873	9°397309	10°602691	9°413467	9°984910	9°428558	10°571442
2 9°384687	9°586841	9°397846	10°602154	9°413938	9°984876	9°429062	10°570938
3 9°385192	9°586909	9°398383	10°601617	9°414408	9°984842	9°429566	10°570434
4 9°385697	9°586778	9°398919	10°601081	9°414878	9°984808	9°430070	10°569930
5 9°386201	9°586746	9°399455	10°600545	9°415347	9°984774	9°430573	10°569427
6 9°386704	9°586714	9°399990	10°600010	9°415815	9°984740	9°431075	10°568925
7 9°387207	9°586683	9°400524	10°599476	9°416283	9°984706	9°431577	10°568423
8 9°387709	9°586651	9°401058	10°598942	9°416751	9°984672	9°432079	10°567921
9 9°388210	9°586619	9°401591	10°598409	9°417217	9°984638	9°432580	10°567420
10 9°388711	9°586587	9°402124	10°597876	9°417684	9°984603	9°433080	10°566920
11 9°389211	9°586555	9°402656	10°597341	9°418150	9°984569	9°433584	10°566420
12 9°389711	9°586523	9°403187	10°596813	9°418615	9°984535	9°434080	10°565920
13 9°390210	9°586491	9°403718	10°596282	9°419079	9°984500	9°434579	10°565421
14 9°390708	9°586459	9°404249	10°595751	9°419544	9°984466	9°435078	10°564922
15 9°391206	9°586427	9°404778	10°595222	9°420007	9°984432	9°435576	10°564424
16 9°391703	9°586395	9°405308	10°594692	9°420470	9°984397	9°436073	10°563927
17 9°392199	9°586363	9°405836	10°594164	9°420933	9°984363	9°436570	10°563430
18 9°392695	9°586331	9°406364	10°593636	9°421395	9°984328	9°437067	10°562933
19 9°393191	9°586299	9°406892	10°593108	9°421857	9°984294	9°437563	10°562437
20 9°393685	9°586266	9°407419	10°592581	9°422318	9°984259	9°438059	10°561941
21 9°394179	9°586234	9°407945	10°592055	9°422778	9°984224	9°438554	10°561446
22 9°394673	9°586202	9°408471	10°591529	9°423238	9°984190	9°439048	10°560952
23 9°395166	9°586169	9°408996	10°591001	9°423697	9°984155	9°439543	10°560457
24 9°395658	9°586137	9°409521	10°590479	9°424156	9°984120	9°440036	10°559964
25 9°396150	9°586104	9°410045	10°589955	9°424615	9°984085	9°440529	10°559471
26 9°396641	9°586072	9°410569	10°589431	9°425073	9°984050	9°441022	10°558978
27 9°397132	9°586039	9°411092	10°588908	9°425530	9°984015	9°441514	10°558486
28 9°397621	9°586007	9°411615	10°588385	9°425987	9°983981	9°442006	10°557994
29 9°398111	9°585974	9°412137	10°587863	9°426443	9°983946	9°442497	10°557503
30 9°398600	9°585942	9°412658	10°587342	9°426899	9°983911	9°442988	10°557012
31 9°399088	9°585909	9°413179	10°586821	9°427354	9°983875	9°443479	10°556521
32 9°399575	9°585876	9°413699	10°586301	9°427809	9°983840	9°443968	10°556032
33 9°400062	9°585843	9°414219	10°585781	9°428263	9°983805	9°444458	10°555542
34 9°400549	9°585811	9°414738	10°585262	9°428717	9°983770	9°444947	10°555053
35 9°401035	9°585778	9°415257	10°584743	9°429170	9°983735	9°445435	10°554565
36 9°401520	9°585745	9°415775	10°584225	9°429623	9°983700	9°445923	10°554077
37 9°402005	9°585712	9°416293	10°583707	9°430075	9°983665	9°446411	10°553589
38 9°402489	9°585679	9°416810	10°583190	9°430527	9°983629	9°446898	10°553102
39 9°402972	9°585646	9°417326	10°582674	9°430978	9°983594	9°447384	10°552616
40 9°403455	9°585613	9°417842	10°582158	9°431429	9°983558	9°447870	10°552130
41 9°403938	9°585580	9°418358	10°581642	9°431879	9°983523	9°448356	10°551644
42 9°404420	9°585547	9°418873	10°581127	9°432329	9°983487	9°448841	10°551159
43 9°404901	9°585514	9°419387	10°580613	9°432777	9°983452	9°449326	10°550674
44 9°405382	9°585480	9°419901	10°580099	9°433226	9°983416	9°449810	10°550190
45 9°405862	9°585447	9°420415	10°579585	9°433675	9°983381	9°450294	10°549706
46 9°406341	9°585414	9°420927	10°579073	9°434122	9°983345	9°450777	10°549223
47 9°406820	9°585381	9°421440	10°578560	9°434569	9°983309	9°451260	10°548740
48 9°407299	9°585347	9°421952	10°578048	9°435016	9°983273	9°451743	10°548257
49 9°407777	9°585314	9°422463	10°577537	9°435462	9°983238	9°452225	10°547775
50 9°408254	9°585280	9°422974	10°577026	9°435908	9°983202	9°452706	10°547294
51 9°408731	9°585247	9°423484	10°576516	9°436353	9°983166	9°453187	10°546813
52 9°409207	9°585213	9°423993	10°576007	9°436798	9°983130	9°453668	10°546332
53 9°409682	9°585180	9°424503	10°575497	9°437242	9°983094	9°454148	10°545852
54 9°410157	9°585146	9°425011	10°574989	9°437686	9°983058	9°454628	10°545372
55 9°410632	9°585113	9°425519	10°574481	9°438129	9°983022	9°455107	10°544893
56 9°411106	9°585079	9°426027	10°573973	9°438572	9°982986	9°455586	10°544414
57 9°411579	9°585045	9°426534	10°573469	9°439014	9°982950	9°456061	10°543936
58 9°412052	9°585011	9°427041	10°572959	9°439456	9°982914	9°456542	10°543458
59 9°412524	9°584978	9°427547	10°572453	9°439897	9°982878	9°457019	10°542981
60 9°412996	9°584944	9°428052	10°571948	9°440338	9°982842	9°457496	10°542504
Cosine	Sine	Cotan.	Tan.	Cosine	Sine	Cotan.	Tan.

16°				17°			
Sine	Cosine	Tan.	Cotan.	Sine	Cosine	Tan.	Cotan.
0 9°440338	9°982842	9°457496	10°542504	9°465935	9°980596	9°485339	10°514661
1 9°440778	9°982805	9°457973	10°542027	9°466348	9°980558	9°485791	10°514209
2 9°441218	9°982769	9°458448	10°541551	9°466761	9°980519	9°486242	10°513758
3 9°441658	9°982733	9°458925	10°541075	9°467173	9°980480	9°486693	10°513307
4 9°442096	9°982696	9°459400	10°540600	9°467585	9°980442	9°487143	10°512857
5 9°442535	9°982660	9°459875	10°540125	9°467996	9°980403	9°487593	10°512407
6 9°442973	9°982624	9°460349	10°539651	9°468407	9°980364	9°488043	10°511957
7 9°443410	9°982587	9°460823	10°539177	9°468817	9°980325	9°488492	10°511508
8 9°443847	9°982551	9°461297	10°538703	9°469227	9°980286	9°488941	10°511059
9 9°444284	9°982514	9°461770	10°538230	9°469637	9°980247	9°489390	10°510610
10 9°444720	9°982477	9°462242	10°537758	9°470046	9°980208	9°489838	10°510162
11 9°445155	9°982441	9°462715	10°537285	9°470455	9°980169	9°490286	10°509714
12 9°445590	9°982404	9°463186	10°536814	9°470863	9°980130	9°490733	10°509267
13 9°446025	9°982367	9°463658	10°536342	9°471271	9°980091	9°491180	10°508820
14 9°446459	9°982331	9°464128	10°535872	9°471679	9°980052	9°491627	10°508373
15 9°446893	9°982294	9°464599	10°535401	9°472086	9°980012	9°492073	10°507927
16 9°447326	9°982257	9°465069	10°534931	9°472492	9°979973	9°492519	10°507481
17 9°447759	9°982220	9°465539	10°534461	9°472898	9°979934	9°492965	10°507035
18 9°448191	9°982183	9°466008	10°533992	9°473304	9°979895	9°493410	10°506590
19 9°448623	9°982146	9°466477	10°533523	9°473710	9°979855	9°493854	10°506146
20 9°449054	9°982109	9°466945	10°533055	9°474115	9°979816	9°494299	10°505701
21 9°449485	9°982072	9°467413	10°532587	9°474519	9°979776	9°494743	10°505257
22 9°449915	9°982035	9°467880	10°532120	9°474923	9°979737	9°495186	10°504814
23 9°450345	9°981998	9°468347	10°531653	9°475327	9°979697	9°495630	10°504370
24 9°450775	9°981961	9°468814	10°531186	9°475730	9°979658	9°496073	10°503927
25 9°451204	9°981924	9°469280	10°530720	9°476133	9°979618	9°496515	10°503485
26 9°451632	9°981886	9°469746	10°530254	9°476536	9°979579	9°496957	10°503043
27 9°452060	9°981849	9°470211	10°529789	9°476938	9°979539	9°497399	10°502601
28 9°452488	9°981812	9°470676	10°529324	9°477340	9°979499	9°497841	10°502153
29 9°452915	9°981774	9°471141	10°528859	9°477741	9°979459	9°498282	10°501718
30 9°453342	9°981737	9°471605	10°528395	9°478142	9°979420	9°498722	10°501278
31 9°453768	9°981700	9°472069	10°527931	9°478542	9°979380	9°499163	10°500837
32 9°454194	9°981662	9°472532	10°527468	9°478942	9°979340	9°499603	10°500397
33 9°454619	9°981625	9°472995	10°527005	9°479342	9°979300	9°500042	10°499958
34 9°455044	9°981587	9°473457	10°526543	9°479741	9°979260	9°500481	10°499519
35 9°455469	9°981549	9°473919	10°526081	9°480140	9°979220	9°500920	10°499080
36 9°455893	9°981512	9°474381	10°525619	9°480539	9°979180	9°501359	10°498641
37 9°456316	9°981474	9°474842	10°525156	9°480937	9°979140	9°501797	10°498203
38 9°456739	9°981436	9°475303	10°524697	9°481334	9°979100	9°502235	10°497765
39 9°457162	9°981399	9°475763	10°524237	9°481731	9°979059	9°502672	10°497328
40 9°457584	9°981361	9°476223	10°523777	9°482128	9°979019	9°503109	10°496891
41 9°458006	9°981323	9°476683	10°523317	9°482525	9°978979	9°503546	10°496451
42 9°458427	9°981285	9°477142	10°522858	9°482921	9°978939	9°503982	10°496018
43 9°458848	9°981247	9°477601	10°522399	9°483316	9°978898	9°504418	10°495582
44 9°459268	9°981209	9°478059	10°521941	9°483712	9°978858	9°504854	10°495146
45 9°459688	9°981171	9°478517	10°521483	9°484107	9°978817	9°505289	10°494711
46 9°460108	9°981133	9°478975	10°521025	9°484501	9°978777	9°505724	10°494276
47 9°460527	9°981095	9°479432	10°520568	9°484895	9°978737	9°506159	10°493841
48 9°460946	9°981057	9°479880	10°520111	9°485289	9°978696	9°506593	10°493407
49 9°461364	9°981019	9°480345	10°519655	9°485682	9°978655	9°507027	10°492973
50 9°461782	9°980981	9°480801	10°519199	9°486075	9°978615	9°507460	10°492540
51 9°462199	9°980942	9°481257	10°518743	9°486467	9°978574	9°507893	10°492107
52 9°462616	9°980904	9°481712	10°518288	9°486860	9°978533	9°508326	10°491674
53 9°463032	9°980866	9°482167	10°517833	9°487251	9°978493	9°508759	10°491241
54 9°463448	9°980827	9°482621	10°517379	9°487643	9°978452	9°509191	10°490809
55 9°463864	9°980789	9°483075	10°516925	9°488034	9°978411	9°509622	10°490378
56 9°464279	9°980750	9°483529	10°516471	9°488424	9°978370	9°510054	10°489946
57 9°464694	9°980712	9°483982	10°516018	9°488814	9°978329	9°510485	10°489515
58 9°465108	9°980673	9°484435	10°515565	9°489204	9°978288	9°510916	10°489084
59 9°465522	9°980635	9°484887	10°515113	9°489593	9°978247	9°511346	10°488654
60 9°465935	9°980596	9°485339	10°514661	9°489982	9°978206	9°511776	10°488224
Cosine	Sine	Cotan.	Tan.	Cosine	Sine	Cotan.	Tan.

18°					19°				
	Sine	Cosine	Tan.	Cotan.		Sine	Cosine	Tan.	Cotan.
0	9.489982	9.978206	9.511776	10.488224	9.512642	9.975670	9.536972	10.463028	60
1	9.490371	9.978165	9.512206	10.487794	9.513009	9.975627	9.537382	10.462618	59
2	9.490759	9.978124	9.512635	10.487365	9.513375	9.975583	9.537792	10.462208	58
3	9.491147	9.978083	9.513064	10.486936	9.513741	9.975539	9.538202	10.461798	57
4	9.491535	9.978042	9.513493	10.486507	9.514107	9.975496	9.538611	10.461389	56
5	9.491922	9.978001	9.513921	10.486079	9.514472	9.975452	9.539020	10.460980	55
6	9.490308	9.977959	9.514349	10.485651	9.514837	9.975408	9.539429	10.460571	54
7	9.492695	9.977918	9.514777	10.485223	9.515202	9.975365	9.539837	10.460163	53
8	9.493081	9.977877	9.515204	10.484796	9.515568	9.975321	9.540245	10.459755	52
9	9.493466	9.977835	9.515631	10.484369	9.515930	9.975277	9.540653	10.459347	51
10	9.493851	9.977794	9.516057	10.483943	9.516294	9.975233	9.541061	10.458939	50
11	9.494236	9.977752	9.516484	10.483516	9.516657	9.975189	9.541468	10.458532	49
12	9.494621	9.977711	9.516910	10.483090	9.517020	9.975140	9.541875	10.458125	48
13	9.495005	9.977669	9.517335	10.482665	9.517382	9.975101	9.542282	10.457719	47
14	9.495388	9.977628	9.517761	10.482239	9.517745	9.975057	9.542688	10.457312	46
15	9.495772	9.977586	9.518186	10.481814	9.518107	9.975013	9.543094	10.456906	45
16	9.496154	9.977544	9.518610	10.481390	9.518468	9.974969	9.543499	10.456501	44
17	9.496537	9.977503	9.519034	10.480966	9.518829	9.974925	9.543905	10.456095	43
18	9.496919	9.977461	9.519458	10.480542	9.519190	9.974880	9.544310	10.455690	42
19	9.497301	9.977419	9.519882	10.480118	9.519551	9.974836	9.544715	10.455285	41
20	9.497682	9.977377	9.520305	10.479695	9.519911	9.974792	9.545119	10.454881	40
21	9.498064	9.977335	9.520728	10.479272	9.520271	9.974748	9.545524	10.454476	39
22	9.498444	9.977293	9.521151	10.478849	9.520631	9.974703	9.545928	10.454072	38
23	9.498825	9.977251	9.521573	10.478427	9.520990	9.974659	9.546331	10.453669	37
24	9.499204	9.977209	9.521995	10.478005	9.521349	9.974614	9.546735	10.453265	36
25	9.499584	9.977167	9.522417	10.477583	9.521707	9.974570	9.547138	10.452862	35
26	9.499963	9.977125	9.522838	10.477162	9.522066	9.974525	9.547540	10.452460	34
27	9.500342	9.977083	9.523259	10.476741	9.522424	9.974481	9.547943	10.452057	33
28	9.500721	9.977041	9.523680	10.476320	9.522781	9.974436	9.548345	10.451655	32
29	9.501099	9.976999	9.524100	10.475900	9.523138	9.974391	9.548747	10.451253	31
30	9.501476	9.976957	9.524520	10.475480	9.523495	9.974347	9.549149	10.450851	30
31	9.501854	9.976914	9.524940	10.475060	9.523852	9.974302	9.549550	10.450450	29
32	9.502231	9.976872	9.525360	10.474641	9.524208	9.974257	9.549951	10.450049	28
33	9.502607	9.976830	9.525778	10.474222	9.524564	9.974212	9.550352	10.449648	27
34	9.502984	9.976787	9.526197	10.473803	9.524920	9.974167	9.550752	10.449248	26
35	9.503360	9.976745	9.526615	10.473385	9.525275	9.974122	9.551153	10.448847	25
36	9.503735	9.976702	9.527033	10.472967	9.525630	9.974077	9.551552	10.448440	24
37	9.504110	9.976660	9.527451	10.472549	9.525984	9.974032	9.551952	10.448048	23
38	9.504485	9.976617	9.527868	10.472132	9.526339	9.973987	9.552351	10.447649	22
39	9.504860	9.976574	9.528285	10.471715	9.526693	9.973942	9.552750	10.447250	21
40	9.505234	9.976532	9.528702	10.471298	9.527046	9.973897	9.553149	10.446851	20
41	9.505608	9.976489	9.529119	10.470881	9.527400	9.973852	9.553548	10.446452	19
42	9.505981	9.976446	9.529535	10.470465	9.527753	9.973807	9.553946	10.446054	18
43	9.506354	9.976404	9.529951	10.470049	9.528105	9.973761	9.554344	10.445656	17
44	9.506727	9.976361	9.530366	10.469634	9.528458	9.973716	9.554741	10.445259	16
45	9.507099	9.976318	9.530781	10.469219	9.528810	9.973671	9.555139	10.444861	15
46	9.507471	9.976275	9.531196	10.468804	9.529161	9.973625	9.555536	10.444464	14
47	9.507843	9.976232	9.531611	10.468389	9.529513	9.973580	9.555933	10.444067	13
48	9.508214	9.976189	9.532025	10.467975	9.529864	9.973535	9.556329	10.443671	12
49	9.508585	9.976146	9.532439	10.467561	9.530215	9.973489	9.556725	10.443275	11
50	9.508956	9.976103	9.532853	10.467147	9.530565	9.973444	9.557121	10.442879	10
51	9.509326	9.976060	9.533266	10.466734	9.530915	9.973398	9.557517	10.442483	9
52	9.509696	9.976017	9.533679	10.466321	9.531265	9.973352	9.557913	10.442087	8
53	9.510065	9.975974	9.534092	10.465905	9.531614	9.973307	9.558308	10.441692	7
54	9.510434	9.975930	9.534504	10.465496	9.531963	9.973261	9.558703	10.441297	6
55	9.510803	9.975887	9.534916	10.465084	9.532312	9.973215	9.559097	10.440903	5
56	9.511172	9.975844	9.535328	10.464672	9.532661	9.973169	9.559491	10.440509	4
57	9.511540	9.975800	9.535739	10.464261	9.533009	9.973124	9.559885	10.440115	3
58	9.511907	9.975757	9.536150	10.463850	9.533357	9.973078	9.560279	10.439721	2
59	9.512275	9.975714	9.536561	10.463439	9.533704	9.973032	9.560673	10.439327	1
60	9.512642	9.975670	9.536972	10.463028	9.534052	9.972986	9.561066	10.438934	0
	Cosine	Sine	Cotan.	Tan.		Cosine	Sine	Cotan.	Tan.

20°				21°			
Sine	Cosine	Tan.	Cotan.	Sine	Cosine	Tan.	Cotan.
0 9.534082	9.972986	9.561066	10.438934	9.554329	9.970152	9.584177	10.415823
1 9.534399	9.972940	9.561459	10.438541	9.554658	9.970103	9.584555	10.415445
2 9.534745	9.972894	9.561851	10.438149	9.554987	9.970055	9.584932	10.415068
3 9.535092	9.972848	9.562244	10.437756	9.555315	9.970006	9.585309	10.414691
4 9.535438	9.972802	9.562636	10.437364	9.555643	9.969957	9.585686	10.414314
5 9.535783	9.972755	9.563028	10.436972	9.555971	9.969909	9.586062	10.413938
6 9.536129	9.972709	9.563419	10.436581	9.556299	9.969860	9.586439	10.413561
7 9.536474	9.972663	9.563811	10.436189	9.556626	9.969811	9.586815	10.413185
8 9.536818	9.972617	9.564202	10.435798	9.556953	9.969762	9.587190	10.412819
9 9.537163	9.972570	9.564593	10.435407	9.557280	9.969714	9.587566	10.412434
10 9.537507	9.972524	9.564983	10.435017	9.557606	9.969665	9.587941	10.412059
11 9.537851	9.972478	9.565373	10.434627	9.557932	9.969616	9.588316	10.411684
12 9.538194	9.972431	9.565763	10.434237	9.558258	9.969567	9.588691	10.411309
13 9.538538	9.972385	9.566153	10.433847	9.558583	9.969518	9.589066	10.410934
14 9.538880	9.972338	9.566542	10.433458	9.558909	9.969469	9.589440	10.410560
15 9.539223	9.972291	9.566932	10.433063	9.559234	9.969420	9.589814	10.410186
16 9.539565	9.972245	9.567320	10.432680	9.559558	9.969370	9.590188	10.409812
17 9.539907	9.972198	9.567709	10.432291	9.559883	9.969321	9.590562	10.409438
18 9.540249	9.972151	9.568098	10.431902	9.560207	9.969272	9.590935	10.409065
19 9.540590	9.972105	9.568486	10.431514	9.560531	9.969223	9.591308	10.408692
20 9.540931	9.972058	9.568873	10.431127	9.560855	9.969173	9.591681	10.408319
21 9.541272	9.972011	9.569261	10.430739	9.561178	9.969124	9.592054	10.407946
22 9.541613	9.971964	9.569648	10.430352	9.561501	9.969075	9.592426	10.407574
23 9.541953	9.971917	9.570035	10.429965	9.561824	9.969025	9.592799	10.407201
24 9.542293	9.971870	9.570422	10.429578	9.562146	9.968976	9.593171	10.406829
25 9.542632	9.971823	9.570809	10.429191	9.562468	9.968926	9.593542	10.406458
26 9.542971	9.971776	9.571195	10.428805	9.562790	9.968877	9.593914	10.406086
27 9.543310	9.971729	9.571581	10.428419	9.563112	9.968827	9.594285	10.405715
28 9.543649	9.971682	9.571967	10.428033	9.563433	9.968777	9.594656	10.405344
29 9.543987	9.971635	9.572352	10.427648	9.563755	9.968728	9.595027	10.404973
30 9.544325	9.971588	9.572738	10.427262	9.564075	9.968678	9.595398	10.404602
31 9.544663	9.971540	9.573123	10.426877	9.564396	9.968628	9.595768	10.404232
32 9.545000	9.971493	9.573507	10.426493	9.564716	9.968578	9.596138	10.403862
33 9.545338	9.971446	9.573892	10.426108	9.565038	9.968528	9.596508	10.403492
34 9.545674	9.971398	9.574276	10.425724	9.565356	9.968479	9.596878	10.403122
35 9.546011	9.971351	9.574660	10.425340	9.565676	9.968429	9.597247	10.402753
36 9.546347	9.971303	9.575044	10.424956	9.565995	9.968379	9.597616	10.402384
37 9.546683	9.971256	9.575427	10.424573	9.566314	9.968329	9.597985	10.402015
38 9.547019	9.971208	9.575810	10.424190	9.566632	9.968278	9.598354	10.401646
39 9.547354	9.971161	9.576193	10.423807	9.566951	9.968228	9.598722	10.401278
40 9.547689	9.971113	9.576576	10.423424	9.567269	9.968178	9.599091	10.400909
41 9.548024	9.971066	9.576959	10.423041	9.567587	9.968128	9.599459	10.400541
42 9.548359	9.971018	9.577341	10.422659	9.567904	9.968078	9.599827	10.400173
43 9.548693	9.970970	9.577723	10.422277	9.568222	9.968027	9.600194	10.399806
44 9.549027	9.970922	9.578104	10.421896	9.568539	9.967977	9.600562	10.399438
45 9.549360	9.970874	9.578486	10.421514	9.568856	9.967927	9.600929	10.399071
46 9.549693	9.970827	9.578867	10.421133	9.569172	9.967876	9.601296	10.398704
47 9.550026	9.970779	9.579248	10.420752	9.569488	9.967826	9.601663	10.398337
48 9.550359	9.970731	9.579629	10.420371	9.569804	9.967775	9.602029	10.397971
49 9.550692	9.970683	9.580009	10.419991	9.570120	9.967725	9.602395	10.397605
50 9.551024	9.970635	9.580389	10.419611	9.570435	9.967674	9.602761	10.397239
51 9.551356	9.970587	9.580769	10.419231	9.570751	9.967624	9.603127	10.396873
52 9.551687	9.970538	9.581149	10.418851	9.571066	9.967573	9.603493	10.396507
53 9.552018	9.970490	9.581528	10.418472	9.571380	9.967522	9.603858	10.396142
54 9.552349	9.970442	9.581907	10.418093	9.571695	9.967471	9.604223	10.395777
55 9.552680	9.970394	9.582286	10.417714	9.572009	9.967421	9.604588	10.395412
56 9.553010	9.970345	9.582664	10.417335	9.572323	9.967370	9.604953	10.395047
57 9.553341	9.970297	9.583044	10.416956	9.572636	9.967319	9.605317	10.394683
58 9.553670	9.970249	9.583422	10.416578	9.572950	9.967268	9.605682	10.394318
59 9.554000	9.970200	9.583800	10.416200	9.573263	9.967217	9.606046	10.393954
60 9.554329	9.970152	9.584177	10.415823	9.573575	9.967166	9.606410	10.393590
Cosine	Sine	Cotan.	Tan.	Cosine	Sine	Cotan.	Tan.

22°					23°				
	Sine	Cosine	Tan.	Cotan.		Sine	Cosine	Tan.	Cotan.
0	9.578575	9.967166	9.606410	10.393590	9.591878	9.964026	9.627827	10.372148	60
1	9.578888	9.967115	9.606773	10.393227	9.592176	9.963972	9.628203	10.371797	59
2	9.574200	9.967064	9.607137	10.392863	9.592473	9.963919	9.628554	10.371446	58
3	9.574512	9.967013	9.607500	10.392500	9.592770	9.963865	9.628905	10.371095	57
4	9.574824	9.966961	9.607863	10.392137	9.593067	9.963811	9.629255	10.370745	56
5	9.575136	9.966910	9.608225	10.391775	9.593363	9.963757	9.629606	10.370394	55
6	9.575447	9.966859	9.608588	10.391412	9.593659	9.963704	9.629956	10.370044	54
7	9.575758	9.966808	9.608950	10.391050	9.593955	9.963650	9.630306	10.369694	53
8	9.576069	9.966756	9.609312	10.390688	9.594251	9.963596	9.630656	10.369344	52
9	9.576379	9.966705	9.609674	10.390326	9.594547	9.963542	9.631005	10.368995	51
10	9.576689	9.966653	9.610036	10.389964	9.594842	9.963488	9.631355	10.368645	50
11	9.576999	9.966602	9.610397	10.389603	9.595137	9.963434	9.631704	10.368296	49
12	9.577309	9.966550	9.610759	10.389241	9.595432	9.963379	9.632053	10.367947	48
13	9.577618	9.966499	9.611120	10.388880	9.595727	9.963325	9.632402	10.367598	47
14	9.577927	9.966447	9.611480	10.388520	9.596021	9.963271	9.632750	10.367250	46
15	9.578236	9.966395	9.611841	10.388159	9.596315	9.963217	9.633099	10.366901	45
16	9.578545	9.966344	9.612201	10.387799	9.596609	9.963163	9.633447	10.366553	44
17	9.578853	9.966292	9.612561	10.387439	9.596903	9.963108	9.633795	10.366205	43
18	9.579162	9.966240	9.612921	10.387079	9.597196	9.963054	9.634143	10.365857	42
19	9.579470	9.966188	9.613281	10.386719	9.597490	9.962999	9.634490	10.365509	41
20	9.579777	9.966136	9.613641	10.386359	9.597783	9.962945	9.634838	10.365162	40
21	9.580085	9.966085	9.614000	10.386000	9.598075	9.962890	9.635185	10.364815	39
22	9.580392	9.966033	9.614359	10.385641	9.598368	9.962836	9.635532	10.364468	38
23	9.580699	9.965981	9.614718	10.385282	9.598660	9.962781	9.635879	10.364121	37
24	9.581005	9.965929	9.615077	10.384923	9.598952	9.962727	9.636226	10.363774	36
25	9.581812	9.965876	9.615435	10.384565	9.599244	9.962672	9.636572	10.363428	35
26	9.581618	9.965824	9.615793	10.384207	9.599536	9.962617	9.636919	10.363081	34
27	9.581924	9.965772	9.616151	10.383849	9.599827	9.962562	9.637265	10.362735	33
28	9.582229	9.965720	9.616509	10.383491	9.600118	9.962508	9.637611	10.362389	32
29	9.582535	9.965668	9.616867	10.383133	9.600409	9.962453	9.637956	10.362044	31
30	9.582840	9.965615	9.617224	10.382776	9.600700	9.962398	9.638302	10.361698	30
31	9.583145	9.965563	9.617582	10.382418	9.600990	9.962343	9.638647	10.361355	29
32	9.583449	9.965511	9.617939	10.382061	9.601280	9.962288	9.638992	10.361008	28
33	9.583754	9.965458	9.618295	10.381705	9.601570	9.962233	9.639337	10.360663	27
34	9.584058	9.965406	9.618652	10.381348	9.601860	9.962178	9.639682	10.360318	26
35	9.584361	9.965353	9.619008	10.380992	9.602150	9.962123	9.640027	10.359973	25
36	9.584665	9.965301	9.619364	10.380636	9.602439	9.962067	9.640371	10.359629	24
37	9.584968	9.965248	9.619720	10.380280	9.602728	9.962012	9.640716	10.359284	23
38	9.585272	9.965195	9.620076	10.379924	9.603017	9.961957	9.641060	10.358940	22
39	9.585574	9.965143	9.620432	10.379568	9.603305	9.961902	9.641404	10.358596	21
40	9.585877	9.965090	9.620787	10.379213	9.603594	9.961846	9.641747	10.358253	20
41	9.586179	9.965037	9.621142	10.378858	9.603882	9.961791	9.642091	10.357909	19
42	9.586482	9.964984	9.621497	10.378503	9.604170	9.961735	9.642434	10.357566	18
43	9.586783	9.964931	9.621852	10.378148	9.604457	9.961680	9.642777	10.357223	17
44	9.587085	9.964879	9.622207	10.377793	9.604745	9.961624	9.643120	10.356880	16
45	9.587386	9.964826	9.622561	10.377439	9.605032	9.961569	9.643463	10.356537	15
46	9.587688	9.964773	9.622915	10.377085	9.605319	9.961513	9.643806	10.356194	14
47	9.587989	9.964720	9.623269	10.376731	9.605606	9.961458	9.644148	10.355852	13
48	9.588289	9.964666	9.623623	10.376377	9.605892	9.961402	9.644490	10.355510	12
49	9.588590	9.964613	9.623976	10.376024	9.606179	9.961346	9.644832	10.355168	11
50	9.588890	9.964560	9.624330	10.375670	9.606465	9.961290	9.645174	10.354826	10
51	9.589190	9.964507	9.624683	10.375317	9.606751	9.961235	9.645516	10.354484	9
52	9.589489	9.964454	9.625036	10.374964	9.607036	9.961179	9.645857	10.354143	8
53	9.589789	9.964400	9.625388	10.374612	9.607322	9.961123	9.646199	10.353801	7
54	9.590088	9.964347	9.625741	10.374259	9.607607	9.961067	9.646540	10.353460	6
55	9.590387	9.964294	9.626093	10.373907	9.607892	9.961011	9.646881	10.353119	5
56	9.590686	9.964240	9.626445	10.373555	9.608177	9.960955	9.647222	10.352778	4
57	9.590984	9.964187	9.626797	10.373203	9.608461	9.960899	9.647562	10.352438	3
58	9.591282	9.964133	9.627149	10.372851	9.608745	9.960843	9.647903	10.352097	2
59	9.591580	9.964080	9.627501	10.372499	9.609029	9.960786	9.648243	10.351757	1
60	9.591878	9.964026	9.627852	10.372148	9.609313	9.960730	9.648583	10.351417	0
	Cosine	Sine	Cotan.	Tan.	Cosine	Sine	Cotan.	Tan.	
67°					66°				



24°					25°				
	Sine	Cosine	Tan.	Cotan.		Sine	Cosine	Tan.	Cotan.
0	9°609313	9°960730	9°648583	10°351417	9°625948	9°957276	9°668673	10°331327	60
1	9°609597	9°960674	9°648923	10°351077	9°626219	9°957217	9°669002	10°330998	59
2	9°609880	9°960618	9°649900	10°350737	9°626490	9°957158	9°669332	10°330663	58
3	9°610164	9°960561	9°649692	10°350398	9°626760	9°957099	9°669661	10°330339	57
4	9°610447	9°960505	9°649942	10°350058	9°627030	9°957040	9°669991	10°330009	56
5	9°610729	9°960448	9°650281	10°349719	9°627300	9°956981	9°670320	10°329680	55
6	9°611012	9°960392	9°650620	10°349380	9°627570	9°956921	9°670649	10°329351	54
7	9°611294	9°960335	9°650959	10°349041	9°627840	9°956862	9°670977	10°329023	53
8	9°611576	9°960279	9°651297	10°348703	9°628109	9°956803	9°671306	10°328694	52
9	9°611858	9°960222	9°651636	10°348364	9°628378	9°956744	9°671635	10°328365	51
10	9°612140	9°960165	9°651974	10°348026	9°628647	9°956684	9°671963	10°328037	50
11	9°612421	9°960109	9°652312	10°347688	9°628916	9°956625	9°672291	10°327709	49
12	9°612702	9°960052	9°652650	10°347350	9°629185	9°956566	9°672619	10°327381	48
13	9°612983	9°959995	9°652988	10°347012	9°629453	9°956506	9°672947	10°327053	47
14	9°613264	9°959938	9°653326	10°346674	9°629721	9°956447	9°673274	10°326726	46
15	9°613545	9°959882	9°653663	10°346337	9°629989	9°956387	9°673602	10°326398	45
16	9°613825	9°959825	9°654000	10°346000	9°630257	9°956327	9°673929	10°326071	44
17	9°614105	9°959768	9°654337	10°345663	9°630524	9°956268	9°674257	10°325743	43
18	9°614385	9°959711	9°654674	10°345326	9°630792	9°956208	9°674584	10°325416	42
19	9°614665	9°959654	9°655011	10°344989	9°631059	9°956148	9°674911	10°325089	41
20	9°614944	9°959596	9°655348	10°344652	9°631326	9°956089	9°675237	10°324763	40
21	9°615223	9°959539	9°655684	10°344316	9°631593	9°956029	9°675564	10°324436	39
22	9°615502	9°959489	9°656020	10°343980	9°631859	9°955969	9°675890	10°324110	38
23	9°615781	9°959425	9°656356	10°343644	9°632125	9°955909	9°676217	10°323783	37
24	9°616060	9°959368	9°656692	10°343308	9°632392	9°955849	9°676543	10°323457	36
25	9°616338	9°959310	9°657028	10°342972	9°632658	9°955789	9°676869	10°323131	35
26	9°616616	9°959253	9°657364	10°342636	9°632923	9°955729	9°677194	10°322806	34
27	9°616894	9°959195	9°657699	10°342301	9°633189	9°955669	9°677520	10°322480	33
28	9°617172	9°959138	9°658034	10°341966	9°633454	9°955609	9°677846	10°322154	32
29	9°617450	9°959080	9°658369	10°341631	9°633719	9°955548	9°678171	10°321829	31
30	9°617727	9°959023	9°658704	10°341296	9°633984	9°955488	9°678496	10°321503	30
31	9°618004	9°958965	9°659039	10°340961	9°634249	9°955428	9°678821	10°321179	29
32	9°618281	9°958908	9°659373	10°340627	9°634514	9°955368	9°679146	10°320854	28
33	9°618558	9°958850	9°659708	10°340292	9°634778	9°955307	9°679471	10°320529	27
34	9°618834	9°958792	9°660042	10°339958	9°635042	9°955247	9°679795	10°320205	26
35	9°619110	9°958734	9°660376	10°339624	9°635306	9°955186	9°680120	10°319880	25
36	9°619386	9°958677	9°660710	10°339296	9°635570	9°955126	9°680444	10°319556	24
37	9°619662	9°958619	9°661043	10°338957	9°635834	9°955065	9°680768	10°319232	23
38	9°619938	9°958561	9°661377	10°338623	9°636097	9°955005	9°681092	10°318908	22
39	9°620213	9°958503	9°661710	10°338290	9°636360	9°954944	9°681416	10°318584	21
40	9°620488	9°958445	9°662043	10°337957	9°636623	9°954883	9°681740	10°318260	20
41	9°620763	9°958387	9°662376	10°337624	9°636886	9°954823	9°682063	10°317937	19
42	9°621038	9°958329	9°662709	10°337291	9°637148	9°954762	9°682387	10°317613	18
43	9°621313	9°958271	9°663042	10°336958	9°637411	9°954701	9°682710	10°317290	17
44	9°621587	9°958213	9°663375	10°336625	9°637673	9°954640	9°683033	10°316967	16
45	9°621861	9°958155	9°663707	10°336293	9°637935	9°954579	9°683356	10°316644	15
46	9°622135	9°958096	9°664039	10°335961	9°638197	9°954518	9°683679	10°316321	14
47	9°622409	9°958038	9°664371	10°335629	9°638458	9°954457	9°684001	10°315999	13
48	9°622682	9°957979	9°664703	10°335297	9°638720	9°954396	9°684324	10°315676	12
49	9°622956	9°957921	9°665035	10°334965	9°638981	9°954334	9°684646	10°315354	11
50	9°623229	9°957863	9°665366	10°334634	9°639242	9°954273	9°684968	10°315032	10
51	9°623502	9°957804	9°665698	10°334302	9°639503	9°954213	9°685290	10°314710	9
52	9°623774	9°957746	9°666029	10°333971	9°639764	9°954152	9°685612	10°314388	8
53	9°624047	9°957687	9°666360	10°333640	9°640024	9°954090	9°685934	10°314066	7
54	9°624319	9°957628	9°666691	10°333309	9°640284	9°954029	9°686255	10°313745	6
55	9°624591	9°957570	9°667021	10°332979	9°640544	9°953968	9°686577	10°313423	5
56	9°624863	9°957511	9°667352	10°332648	9°640804	9°953906	9°686898	10°313102	4
57	9°625135	9°957452	9°667682	10°332318	9°641064	9°953845	9°687219	10°312781	3
58	9°625406	9°957393	9°668013	10°331987	9°641324	9°953783	9°687540	10°312460	2
59	9°625677	9°957335	9°668343	10°331657	9°641583	9°953722	9°687861	10°312139	1
60	9°625948	9°957276	9°668673	10°331327	9°641842	9°953660	9°688182	10°311818	0
	Cosine	Sine	Cotan.	Tan.		Cosine	Sine	Cotan.	Tan.

26°				27°			
Sine	Cosine	Tan.	Cotan.	Sine	Cosine	Tan.	Cotan.
0 9°641842	9°953660	9°688182	10°311818	9°657047	9°949881	9°707166	10°292834
1 9°642101	9°953599	9°688202	10°311498	9°657295	9°949816	9°707478	10°292522
2 9°642360	9°953537	9°688223	10°311177	9°657542	9°949752	9°707790	10°292210
3 9°642618	9°953475	9°689143	10°310857	9°657790	9°949688	9°708102	10°291898
4 9°642877	9°953413	9°689463	10°310537	9°658037	9°949623	9°708414	10°291586
5 9°643135	9°953352	9°689783	10°310217	9°658284	9°949558	9°708726	10°291274
6 9°643393	9°953290	9°690103	10°309897	9°658531	9°949494	9°709037	10°290963
7 9°643650	9°953228	9°690423	10°309577	9°658778	9°949429	9°709349	10°290651
8 9°643908	9°953166	9°690742	10°309258	9°659025	9°949364	9°709660	10°290340
9 9°644165	9°953104	9°691062	10°308938	9°659271	9°949300	9°709971	10°290029
10 9°644423	9°953042	9°691381	10°308619	9°659517	9°949235	9°710282	10°289718
11 9°644680	9°952980	9°691700	10°308300	9°659763	9°949170	9°710593	10°289407
12 9°644936	9°952918	9°692019	10°307981	9°660009	9°949105	9°710904	10°289096
13 9°645193	9°952855	9°692338	10°307662	9°660255	9°949040	9°711215	10°288785
14 9°645450	9°952793	9°692656	10°307344	9°660501	9°948975	9°711525	10°288475
15 9°645706	9°952731	9°692975	10°307025	9°660746	9°948910	9°711836	10°288164
16 9°645962	9°952669	9°693293	10°306707	9°660991	9°948845	9°712146	10°287854
17 9°646218	9°952606	9°693612	10°306388	9°661236	9°948780	9°712456	10°287544
18 9°646474	9°952544	9°693930	10°306070	9°661481	9°948715	9°712766	10°287234
19 9°646729	9°952481	9°694248	10°305752	9°661726	9°948650	9°713076	10°286924
20 9°646984	9°952419	9°694566	10°305434	9°661970	9°948584	9°713386	10°286614
21 9°647240	9°952356	9°694883	10°305115	9°662214	9°948519	9°713696	10°286304
22 9°647494	9°952294	9°695201	10°304799	9°662459	9°948454	9°714005	10°285995
23 9°647749	9°952231	9°695518	10°304482	9°662703	9°948388	9°714314	10°285686
24 9°648004	9°952168	9°695836	10°304164	9°662946	9°948323	9°714624	10°285376
25 9°648258	9°952106	9°696153	10°303847	9°663190	9°948257	9°714933	10°285066
26 9°648512	9°952043	9°696470	10°303530	9°663433	9°948192	9°715242	10°284756
27 9°648766	9°951980	9°696787	10°303213	9°663677	9°948126	9°715551	10°284446
28 9°649020	9°951917	9°697103	10°302897	9°663920	9°948060	9°715860	10°284136
29 9°649274	9°951854	9°697420	10°302580	9°664163	9°947995	9°716168	10°283826
30 9°649527	9°951791	9°697736	10°302264	9°664406	9°947929	9°716477	10°283516
31 9°649781	9°951728	9°698053	10°301947	9°664648	9°947863	9°716785	10°283206
32 9°650034	9°951665	9°698369	10°301631	9°664891	9°947797	9°717093	10°282907
33 9°650287	9°951602	9°698685	10°301315	9°665133	9°947731	9°717401	10°282597
34 9°650539	9°951539	9°699001	10°300999	9°665375	9°947665	9°717709	10°282291
35 9°650792	9°951476	9°699316	10°300684	9°665617	9°947600	9°718017	10°281983
36 9°651044	9°951412	9°699632	10°300368	9°665859	9°947533	9°718325	10°281675
37 9°651297	9°951349	9°699947	10°300053	9°666100	9°947467	9°718633	10°281367
38 9°651549	9°951286	9°700263	10°299737	9°666342	9°947401	9°718940	10°281060
39 9°651800	9°951222	9°700578	10°299422	9°666583	9°947335	9°719248	10°280752
40 9°652052	9°951159	9°700893	10°299107	9°666824	9°947269	9°719555	10°280445
41 9°652304	9°951096	9°701208	10°298792	9°667065	9°947203	9°719862	10°280138
42 9°652555	9°951032	9°701523	10°298477	9°667305	9°947136	9°720169	10°279831
43 9°652806	9°950968	9°701837	10°298163	9°667546	9°947070	9°720476	10°279524
44 9°653057	9°950905	9°702152	10°297848	9°667786	9°947004	9°720783	10°279217
45 9°653308	9°950841	9°702466	10°297534	9°668027	9°946937	9°721089	10°278911
46 9°653558	9°950778	9°702781	10°297219	9°668267	9°946871	9°721396	10°278604
47 9°653808	9°950714	9°703095	10°296905	9°668506	9°946804	9°721702	10°278298
48 9°654059	9°950650	9°703409	10°296591	9°668746	9°946738	9°722009	10°277991
49 9°654309	9°950586	9°703722	10°296278	9°668986	9°946671	9°722315	10°277685
50 9°654558	9°950522	9°704036	10°295964	9°669225	9°946604	9°722621	10°277379
51 9°654808	9°950458	9°704350	10°295650	9°669464	9°946538	9°722927	10°277073
52 9°655058	9°950394	9°704663	10°295337	9°669703	9°946471	9°723232	10°276768
53 9°655307	9°950330	9°704976	10°295024	9°669942	9°946404	9°723538	10°276462
54 9°655556	9°950266	9°705290	10°294710	9°670181	9°946337	9°723844	10°276156
55 9°655805	9°950202	9°705603	10°294397	9°670419	9°946270	9°724149	10°275851
56 9°656054	9°950138	9°705916	10°294084	9°670658	9°946203	9°724454	10°275546
57 9°656302	9°950074	9°706228	10°293772	9°670896	9°946136	9°724760	10°275240
58 9°656551	9°950010	9°706541	10°293459	9°671134	9°946069	9°725065	10°274935
59 9°656799	9°949945	9°706854	10°293146	9°671372	9°946002	9°725370	10°274630
60 9°657047	9°949881	9°707166	10°292834	9°671609	9°945935	9°725674	10°274326
Cosine	Sine	Cotan.	Tan.	Cosine	Sine	Cotan.	Tan.

28°					29°				
	Sine	Cosine	Tan.	Cotan.	Sine	Cosine	Tan.	Cotan.	
0	9°671609	9°945935	9°725074	10°274326	9°685571	9°941819	9°743732	10°256248	60
1	9°671847	9°945868	9°725979	10°274021	9°686799	9°941749	9°744050	10°255950	59
2	9°672084	9°945800	9°726284	10°273716	9°686027	9°941679	9°744348	10°255652	58
3	9°672321	9°945733	9°726588	10°273412	9°686254	9°941609	9°744646	10°255355	57
4	9°672558	9°945666	9°726892	10°273108	9°686482	9°941539	9°744943	10°255057	56
5	9°672795	9°945598	9°727197	10°272803	9°686709	9°941469	9°745240	10°254760	55
6	9°673032	9°945531	9°727501	10°272499	9°686936	9°941398	9°745538	10°254462	54
7	9°673265	9°945464	9°727805	10°272195	9°687163	9°941328	9°745835	10°254165	53
8	9°673502	9°945396	9°728109	10°271891	9°687389	9°941258	9°746132	10°253868	52
9	9°673741	9°945328	9°728412	10°271588	9°687616	9°941187	9°746429	10°253571	51
10	9°673977	9°945261	9°728716	10°271284	9°687843	9°941117	9°746726	10°253274	50
11	9°674213	9°945193	9°729020	10°270980	9°688069	9°941046	9°747023	10°252977	49
12	9°674448	9°945125	9°729323	10°270677	9°688295	9°940975	9°747319	10°252681	48
13	9°674684	9°945058	9°729626	10°270374	9°688521	9°940905	9°747616	10°252384	47
14	9°674919	9°944990	9°729929	10°270071	9°688747	9°940834	9°747913	10°252087	46
15	9°675155	9°944922	9°730233	10°269767	9°688972	9°940763	9°748209	10°251791	45
16	9°675390	9°944854	9°730535	10°269465	9°689198	9°940693	9°748505	10°251495	44
17	9°675624	9°944786	9°730838	10°269162	9°689423	9°940622	9°748801	10°251199	43
18	9°675859	9°944718	9°731141	10°268859	9°689648	9°940551	9°749097	10°250902	42
19	9°676094	9°944650	9°731444	10°268556	9°689873	9°940480	9°749393	10°250607	41
20	9°676328	9°944582	9°731746	10°268254	9°690098	9°940409	9°749689	10°250311	40
21	9°676562	9°944514	9°732048	10°267952	9°690323	9°940338	9°749985	10°250015	39
22	9°676796	9°944446	9°732351	10°267649	9°690548	9°940267	9°750281	10°249719	38
23	9°677030	9°944377	9°732653	10°267347	9°690772	9°940196	9°750576	10°249424	37
24	9°677264	9°944309	9°732955	10°267045	9°690996	9°940125	9°750872	10°249128	36
25	9°677498	9°944241	9°733257	10°266743	9°691220	9°940054	9°751167	10°248833	35
26	9°677731	9°944172	9°733558	10°266442	9°691444	9°939982	9°751462	10°248538	34
27	9°677964	9°944104	9°733860	10°266140	9°691668	9°939911	9°751757	10°248243	33
28	9°678197	9°944036	9°734162	10°265838	9°691892	9°939840	9°752052	10°247948	32
29	9°678430	9°943967	9°734463	10°265535	9°692115	9°939768	9°752347	10°247653	31
30	9°678663	9°943899	9°734764	10°265236	9°692339	9°939697	9°752642	10°247358	30
31	9°678895	9°943830	9°735066	10°264934	9°692562	9°939625	9°752937	10°247063	29
32	9°679128	9°943761	9°735367	10°264633	9°692785	9°939554	9°753231	10°246769	28
33	9°679360	9°943693	9°735668	10°264332	9°693008	9°939482	9°753526	10°246474	27
34	9°679592	9°943624	9°735969	10°264031	9°693231	9°939410	9°753820	10°246180	26
35	9°679824	9°943555	9°736269	10°263731	9°693455	9°939339	9°754115	10°245885	25
36	9°680056	9°943486	9°736570	10°263430	9°693678	9°939267	9°754409	10°245591	24
37	9°680288	9°943417	9°736870	10°263130	9°693898	9°939195	9°754703	10°245297	23
38	9°680519	9°943348	9°737171	10°262829	9°694120	9°939123	9°754997	10°245003	22
39	9°680750	9°943279	9°737471	10°262529	9°694342	9°939052	9°755291	10°244709	21
40	9°680982	9°943210	9°737771	10°262229	9°694564	9°938980	9°755585	10°244415	20
41	9°681213	9°943141	9°738071	10°261929	9°694786	9°938908	9°755878	10°244122	19
42	9°681443	9°943072	9°738371	10°261629	9°695007	9°938836	9°756172	10°243828	18
43	9°681674	9°943003	9°738671	10°261329	9°695229	9°938763	9°756465	10°243535	17
44	9°681905	9°942934	9°738971	10°261029	9°695450	9°938691	9°756759	10°243241	16
45	9°682135	9°942864	9°739271	10°260729	9°695671	9°938619	9°757052	10°242948	15
46	9°682365	9°942795	9°739570	10°260430	9°695892	9°938547	9°757345	10°242655	14
47	9°682595	9°942726	9°739870	10°260130	9°696113	9°938475	9°757638	10°242362	13
48	9°682825	9°942656	9°740169	10°259831	9°696334	9°938402	9°757931	10°242069	12
49	9°683055	9°942587	9°740468	10°259532	9°696554	9°938330	9°758224	10°241776	11
50	9°683284	9°942517	9°740767	10°259233	9°696775	9°938258	9°758517	10°241483	10
51	9°683514	9°942448	9°741066	10°258934	9°696995	9°938185	9°758810	10°241190	9
52	9°683743	9°942378	9°741365	10°258635	9°697215	9°938113	9°759102	10°240898	8
53	9°683972	9°942308	9°741664	10°258336	9°697435	9°938040	9°759395	10°240605	7
54	9°684201	9°942239	9°741962	10°258038	9°697654	9°937967	9°759687	10°240313	6
55	9°684430	9°942169	9°742261	10°257739	9°697874	9°937895	9°759979	10°240021	5
56	9°684658	9°942099	9°742559	10°257441	9°698094	9°937822	9°760272	10°239728	4
57	9°684887	9°942029	9°742858	10°257142	9°698313	9°937749	9°760564	10°239436	3
58	9°685115	9°941959	9°743156	10°256844	9°698532	9°937676	9°760856	10°239144	2
59	9°685343	9°941889	9°743454	10°256546	9°698751	9°937604	9°761148	10°238852	1
60	9°685571	9°941819	9°743752	10°256248	9°698970	9°937531	9°761439	10°238561	0
	Cosine	Sine	Cotan.	Tan.	Cosine	Sine	Cotan.	Tan.	

30°					31°				
	Sine	Cosine	Tan.	Cotan.	Sine	Cosine	Tan.	Cotan.	
0	9°698970	9°937531	9°761439	10°238561	9°711839	9°933066	9°778774	10°221226	60
1	9°699189	9°937458	9°761731	10°238269	9°712050	9°932990	9°779060	10°220940	59
2	9°699407	9°937385	9°762023	10°237977	9°712260	9°932914	9°779346	10°220654	58
3	9°699626	9°937312	9°762314	10°237686	9°712469	9°932838	9°779632	10°220368	57
4	9°699844	9°937238	9°762606	10°237394	9°712679	9°932762	9°779918	10°220082	56
5	9°700062	9°937165	9°762897	10°237103	9°712889	9°932685	9°780203	10°219797	55
6	9°700280	9°937092	9°763188	10°236812	9°713098	9°932609	9°780489	10°219511	54
7	9°700498	9°937019	9°763479	10°236521	9°713308	9°932533	9°780775	10°219225	53
8	9°700716	9°936946	9°763770	10°236230	9°713517	9°932457	9°781060	10°218940	52
9	9°700933	9°936872	9°764061	10°235939	9°713726	9°932380	9°781346	10°218654	51
10	9°701151	9°936799	9°764352	10°235648	9°713935	9°932304	9°781631	10°218369	50
11	9°701368	9°936725	9°764643	10°235357	9°714144	9°932228	9°781916	10°218084	49
12	9°701585	9°936652	9°764933	10°235067	9°714352	9°932151	9°782201	10°217799	48
13	9°701802	9°936578	9°765224	10°234776	9°714561	9°932075	9°782486	10°217514	47
14	9°702019	9°936505	9°765514	10°234486	9°714769	9°931998	9°782771	10°217229	46
15	9°702236	9°936431	9°765805	10°234195	9°714978	9°931921	9°783056	10°216944	45
16	9°702452	9°936357	9°766095	10°233905	9°715186	9°931845	9°783341	10°216659	44
17	9°702669	9°936284	9°766386	10°233615	9°715394	9°931768	9°783626	10°216374	43
18	9°702885	9°936210	9°766676	10°233325	9°715602	9°931691	9°783910	10°216089	42
19	9°703101	9°936136	9°766965	10°233035	9°715809	9°931614	9°784195	10°215805	41
20	9°703317	9°936062	9°767255	10°232745	9°716017	9°931537	9°784479	10°215521	40
21	9°703533	9°935988	9°767545	10°232455	9°716224	9°931460	9°784764	10°215236	39
22	9°703749	9°935914	9°767834	10°232166	9°716432	9°931383	9°785048	10°214952	38
23	9°703964	9°935840	9°768124	10°231876	9°716639	9°931306	9°785332	10°214668	37
24	9°704179	9°935766	9°768414	10°231586	9°716846	9°931229	9°785616	10°214384	36
25	9°704395	9°935692	9°768703	10°231297	9°717053	9°931152	9°785900	10°214100	35
26	9°704610	9°935618	9°768992	10°231008	9°717259	9°931075	9°786184	10°213816	34
27	9°704825	9°935543	9°769281	10°230719	9°717466	9°930998	9°786468	10°213532	33
28	9°705040	9°935469	9°769571	10°230429	9°717673	9°930921	9°786752	10°213248	32
29	9°705254	9°935395	9°769860	10°230140	9°717879	9°930843	9°787036	10°212964	31
30	9°705469	9°935320	9°770148	10°229852	9°718085	9°930766	9°787319	10°212680	30
31	9°705683	9°935246	9°770437	10°229563	9°718291	9°930688	9°787603	10°212397	29
32	9°705898	9°935171	9°770726	10°229274	9°718497	9°930611	9°787886	10°212114	28
33	9°706112	9°935097	9°771015	10°228985	9°718703	9°930533	9°788170	10°211830	27
34	9°706326	9°935022	9°771303	10°228697	9°718909	9°930456	9°788453	10°211547	26
35	9°706539	9°934948	9°771592	10°228408	9°719114	9°930378	9°788736	10°211264	25
36	9°706753	9°934873	9°771880	10°228120	9°719320	9°930300	9°789019	10°210981	24
37	9°706967	9°934798	9°772168	10°227832	9°719525	9°930223	9°789302	10°210698	23
38	9°707180	9°934723	9°772457	10°227543	9°719730	9°930145	9°789585	10°210415	22
39	9°707393	9°934649	9°772745	10°227255	9°719935	9°930067	9°789868	10°210132	21
40	9°707606	9°934574	9°773033	10°226967	9°720140	9°929989	9°790151	10°209849	20
41	9°707819	9°934499	9°773321	10°226679	9°720345	9°929911	9°790434	10°209566	19
42	9°708032	9°934424	9°773608	10°226392	9°720549	9°929833	9°790716	10°209284	18
43	9°708245	9°934349	9°773896	10°226104	9°720754	9°929755	9°790999	10°209001	17
44	9°708458	9°934274	9°774184	10°225816	9°720958	9°929677	9°791281	10°208719	16
45	9°708670	9°934199	9°774471	10°225529	9°721162	9°929599	9°791563	10°208437	15
46	9°708882	9°934123	9°774759	10°225241	9°721366	9°929521	9°791846	10°208154	14
47	9°709094	9°934048	9°775046	10°224954	9°721570	9°929442	9°792128	10°207872	13
48	9°709306	9°933973	9°775333	10°224667	9°721774	9°929364	9°792410	10°207590	12
49	9°709518	9°933898	9°775621	10°224379	9°721978	9°929286	9°792692	10°207308	11
50	9°709730	9°933822	9°775908	10°224092	9°722181	9°929207	9°792974	10°207026	10
51	9°709941	9°933747	9°776195	10°223805	9°722385	9°929129	9°793256	10°206744	9
52	9°710153	9°933671	9°776482	10°223518	9°722588	9°929050	9°793538	10°206462	8
53	9°710364	9°933596	9°776769	10°223232	9°722791	9°928972	9°793819	10°206181	7
54	9°710575	9°933520	9°777055	10°222945	9°722994	9°928893	9°794101	10°205899	6
55	9°710786	9°933445	9°777342	10°222658	9°723197	9°928815	9°794383	10°205617	5
56	9°710997	9°933369	9°777628	10°222372	9°723400	9°928736	9°794664	10°205336	4
57	9°711208	9°933293	9°777915	10°222085	9°723603	9°928657	9°794946	10°205054	3
58	9°711419	9°933217	9°778201	10°221799	9°723805	9°928578	9°795227	10°204773	2
59	9°711629	9°933141	9°778488	10°221512	9°724007	9°928499	9°795508	10°204492	1
60	9°711839	9°933066	9°778774	10°221226	9°724210	9°928420	9°795789	10°204211	0
	Cosine	Sine	Cotan.	Tan.	Cosine	Sine	Cotan.	Tan.	

32°				33°			
Sine	Cosine	Tan.	Cotan.	Sine	Cosine	Tan.	Cotan.
0 9724210	9928420	9795789	10204211	9736109	9923591	9812517	10187483
1 9724412	9928312	9796070	10203930	9736303	9923509	9812794	10187206
2 9724614	9928263	9796351	10203649	9736498	9923427	9813070	10186930
3 9724816	9928183	9796632	10203368	9736692	9923345	9813347	10186653
4 9725017	9928104	9796913	10203087	9736886	9923263	9813623	10186377
5 9725219	9928025	9797194	10202806	9737080	9923181	9813899	10186101
6 9725420	9927946	9797474	10202526	9737274	9923098	9814176	10185824
7 9725622	9927867	9797755	10202245	9737467	9923016	9814452	10185548
8 9725823	9927787	9798036	10201964	9737661	9922933	9814728	10185272
9 9726024	9927708	9798316	10201684	9737855	9922851	9815004	10184996
10 9726225	9927629	9798596	10201404	9738048	9922768	9815280	10184720
11 9726426	9927549	9798877	10201123	9738241	9922686	9815555	10184445
12 9726626	9927470	9799157	10200843	9738434	9922603	9815831	10184169
13 9726827	9927390	9799437	10200563	9738627	9922520	9816107	10183893
14 9727027	9927310	9799717	10200283	9738820	9922438	9816382	10183618
15 9727228	9927231	9799997	10200003	9739013	9922355	9816658	10183342
16 9727428	9927151	9800277	10199723	9739206	9922272	9816933	10183067
17 9727628	9927071	9800557	10199443	9739398	9922189	9817209	10182791
18 9727828	9926991	9800836	10199164	9739590	9922106	9817484	10182516
19 9728027	9926911	9801116	10198884	9739783	9922023	9817759	10182241
20 9728227	9926831	9801396	10198604	9739975	9921940	9818035	10181965
21 9728427	9926751	9801675	10198325	9740167	9921857	9818310	10181690
22 9728626	9926671	9801955	10198045	9740359	9921774	9818585	10181415
23 9728825	9926591	9802234	10197766	9740550	9921691	9818860	10181140
24 9729024	9926511	9802513	10197487	9740742	9921607	9819135	10180865
25 9729223	9926431	9802792	10197208	9740934	9921524	9819410	10180590
26 9729422	9926351	9803073	10196928	9741125	9921441	9819684	10180316
27 9729621	9926270	9803351	10196649	9741316	9921357	9819959	10180041
28 9729820	9926190	9803630	10196370	9741508	9921274	9820234	10179766
29 9730018	9926110	9803909	10196091	9741699	9921190	9820508	10179492
30 9730217	9926029	9804187	10195813	9741889	9921107	9820783	10179217
31 9730415	9925949	9804466	10195534	9742080	9921023	9821057	10178943
32 9730613	9925868	9804745	10195255	9742274	9920939	9821332	10178668
33 9730811	9925788	9805023	10194977	9742462	9920856	9821606	10178394
34 9731009	9925707	9805302	10194698	9742652	9920772	9821880	10178120
35 9731206	9925626	9805580	10194420	9742842	9920688	9822154	10177846
36 9731404	9925545	9805859	10194141	9743033	9920604	9822429	10177571
37 9731602	9925465	9806137	10193863	9743223	9920520	9822703	10177297
38 9731799	9925384	9806415	10193585	9743413	9920436	9822977	10177023
39 9731996	9925303	9806693	10193307	9743602	9920352	9823251	10176749
40 9732193	9925222	9806971	10193029	9743792	9920268	9823524	10176476
41 9732390	9925141	9807249	10192751	9743982	9920184	9823798	10176202
42 9732587	9925060	9807527	10192473	9744171	9920099	9824072	10175928
43 9732784	9924979	9807805	10192195	9744361	9920015	9824345	10175655
44 9732980	9924897	9808083	10191917	9744550	9919931	9824619	10175381
45 9733177	9924816	9808361	10191639	9744739	9919846	9824893	10175107
46 9733373	9924735	9808639	10191362	9744928	9919762	9825166	10174834
47 9733569	9924654	9808916	10191084	9745117	9919677	9825439	10174561
48 9733765	9924572	9809193	10190807	9745306	9919593	9825713	10174287
49 9733961	9924491	9809471	10190529	9745494	9919508	9825986	10174014
50 9734157	9924409	9809748	10190252	9745683	9919424	9826259	10173741
51 9734353	9924328	9810025	10189975	9745871	9919339	9826532	10173468
52 9734549	9924246	9810302	10189698	9746060	9919254	9826805	10173195
53 9734744	9924164	9810580	10189420	9746248	9919169	9827078	10172922
54 9734939	9924083	9810857	10189143	9746436	9919085	9827351	10172649
55 9735135	9924001	9811134	10188866	9746624	9919000	9827624	10172376
56 9735330	9923919	9811410	10188589	9746812	9918915	9827897	10172103
57 9735525	9923837	9811687	10188313	9746999	9918830	9828170	10171830
58 9735719	9923755	9811964	10188036	9747187	9918745	9828442	10171558
59 9735914	9923673	9812241	10187759	9747374	9918659	9828715	10171285
60 9736109	9923591	9812517	10187483	9747562	9918574	9828987	10171013
Cosine	Sine	Cotan.	Tan.	Cosine	Sine	Cotan.	Tan.

LOGARITHMIC SINES AND TANGENTS.

34°				35°			
Sine	Cosine	Tan	Cotan.	Sine	Cosine	Tan.	Cotan.
0 9747562	9918574	9828987	10171013	9758591	9913365	9845227	10151773
1 9747719	9918489	9829260	10170740	9753772	9913276	9845436	10154504
2 9747936	9918404	9829532	10170468	9758952	9913187	9845761	10154236
3 9748123	9918318	9829805	10170195	9759132	9913099	9846033	10153967
4 9748310	9918233	9830077	10169923	9759312	9913010	9846302	10153698
5 9748497	9918147	9830349	10169651	9759492	9912922	9846570	10153430
6 9748683	9918062	9830621	10169379	9759672	9912833	9846839	10153161
7 9748870	9917976	9830893	10169107	9759852	9912744	9847108	10152892
8 9749056	9917891	9831165	10168835	9760031	9912655	9847376	10152624
9 9749243	9917805	9831437	10168563	9760211	9912566	9847644	10152356
10 9749429	9917719	9831709	10168291	9760390	9912477	9847913	10152087
11 9749615	9917634	9831981	10168019	9760569	9912388	9848181	10151819
12 9749801	9917548	9832253	10167747	9760748	9912299	9848449	10151551
13 9749987	9917462	9832525	10167475	9760927	9912210	9848717	10151283
14 9750172	9917376	9832796	10167204	9761106	9912121	9848986	10151014
15 9750358	9917290	9833068	10166932	9761285	9912031	9849254	10150746
16 9750543	9917204	9833339	10166661	9761464	9911942	9849522	10150478
17 9750729	9917118	9833611	10166389	9761642	9911853	9849790	10150210
18 9750914	9917032	9833882	10166118	9761821	9911763	9850057	10149943
19 9751099	9916946	9834154	10165846	9761999	9911674	9850325	10149675
20 9751284	9916859	9834425	10165575	9762177	9911584	9850593	10149407
21 9751469	9916773	9834696	10165304	9762356	9911495	9850861	10149139
22 9751654	9916687	9834967	10165033	9762534	9911405	9851129	10148871
23 9751839	9916600	9835238	10164762	9762712	9911315	9851396	10148604
24 9752023	9916514	9835509	10164491	9762889	9911226	9851664	10148336
25 9752208	9916427	9835780	10164220	9763067	9911136	9851931	10148069
26 9752392	9916341	9836051	10163949	9763245	9911046	9852199	10147801
27 9752576	9916254	9836322	10163678	9763422	9910956	9852466	10147534
28 9752760	9916167	9836593	10163407	9763600	9910866	9852733	10147267
29 9752944	9916081	9836864	10163136	9763777	9910776	9853001	10146999
30 9753128	9915994	9837134	10162866	9763954	9910686	9853268	10146732
31 9753312	9915907	9837405	10162595	9764131	9910596	9853535	10146465
32 9753495	9915820	9837675	10162325	9764308	9910506	9853802	10146198
33 9753679	9915733	9837946	10162054	9764485	9910415	9854069	10145931
34 9753862	9915646	9838216	10161784	9764662	9910325	9854336	10145664
35 9754046	9915559	9838487	10161513	9764838	9910235	9854603	10145397
36 9754229	9915472	9838757	10161243	9765015	9910144	9854870	10145130
37 9754412	9915385	9839027	10160973	9765191	9910054	9855137	10144863
38 9754595	9915297	9839297	10160703	9765367	9909963	9855404	10144596
39 9754778	9915210	9839568	10160432	9765544	9909873	9855671	10144329
40 9754960	9915123	9839838	10160162	9765720	9909782	9855938	10144062
41 9755143	9915035	9840108	10159892	9765896	9909691	9856204	10143796
42 9755326	9914948	9840378	10159622	9766072	9909601	9856471	10143529
43 9755508	9914860	9840648	10159352	9766247	9909510	9856737	10143263
44 9755690	9914773	9840917	10159083	9766423	9909419	9857004	10142996
45 9755872	9914685	9841187	10158813	9766598	9909328	9857270	10142730
46 9756054	9914598	9841457	10158543	9766774	9909237	9857537	10142463
47 9756236	9914510	9841727	10158273	9766949	9909146	9857803	10142197
48 9756418	9914422	9841996	10158004	9767124	9909055	9858069	10141931
49 9756600	9914334	9842266	10157734	9767300	9908964	9858336	10141664
50 9756782	9914246	9842535	10157465	9767475	9908873	9858602	10141398
51 9756963	9914158	9842805	10157195	9767649	9908781	9858868	10141132
52 9757144	9914070	9843074	10156926	9767824	9908690	9859131	10140866
53 9757326	9913982	9843343	10156657	9767999	9908599	9859400	10140600
54 9757507	9913894	9843612	10156388	9768173	9908507	9859666	10140334
55 9757688	9913806	9843882	10156118	9768348	9908416	9859932	10140068
56 9757869	9913718	9844151	10155849	9768522	9908324	9860198	10139802
57 9758050	9913630	9844420	10155580	9768697	9908233	9860464	10139536
58 9758230	9913541	9844689	10155311	9768871	9908141	9860730	10139270
59 9758411	9913453	9844958	10155042	9769045	9908049	9860995	10139005
60 9758591	9913365	9845227	10154773	9769219	9907958	9861261	10138739
Cosine	Sine	Cotan.	Tan.	Cosine	Sine	Cotan.	Tan.

36°					37°				
	Sine	Cosine	Tan.	Cotan.	Sine	Cosine	Tan.	Cotan.	
0	9769219	9907958	9861261	10138749	9779463	9902359	9874114	10122866	60
1	9769393	9907866	9861527	10138473	9779631	9902253	9877777	10122023	59
2	9769566	9907774	9861792	10138208	9779798	9902158	9877040	10122360	58
3	9769740	9907682	9862058	10137942	9779966	9902063	9877903	10122907	57
4	9769913	9907590	9862323	10137677	9780133	9901967	9878165	10121835	56
5	9770087	9907498	9862589	10137411	9780300	9901872	9878428	10121572	55
6	9770260	9907406	9862854	10137146	9780467	9901776	9878691	10121309	54
7	9770433	9907314	9863119	10136881	9780634	9901681	9878953	10121047	53
8	9770606	9907222	9863385	10136615	9780801	9901585	9879216	10120784	52
9	9770779	9907129	9863650	10136350	9780968	9901490	9879478	10120522	51
10	9770952	9907037	9863915	10136085	9781134	9901394	9879741	10120259	50
11	9771125	9906945	9864180	10135820	9781301	9901298	9880003	10119997	49
12	9771298	9906852	9864445	10135555	9781468	9901202	9880265	10119735	48
13	9771470	9906760	9864710	10135290	9781634	9901106	9880528	10119472	47
14	9771643	9906667	9864975	10135025	9781800	9901010	9880790	10119210	46
15	9771815	9906575	9865240	10134760	9781966	9900914	9881052	10118948	45
16	9771987	9906482	9865505	10134495	9782132	9900818	9881314	10118686	44
17	9772159	9906389	9865770	10134230	9782298	9900722	9881577	10118423	43
18	9772331	9906296	9866035	10133965	9782464	9900626	9881839	10118161	42
19	9772503	9906204	9866300	10133700	9782630	9900529	9882101	10117899	41
20	9772675	9906111	9866564	10133436	9782796	9900433	9882363	10117637	40
21	9772847	9906018	9866829	10133171	9782961	9900337	9882625	10117375	39
22	9773018	9905925	9867094	10132906	9783127	9900240	9882887	10117113	38
23	9773190	9905832	9867358	10132642	9783292	9900144	9883148	10116852	37
24	9773361	9905739	9867623	10132377	9783458	9900047	9883410	10116590	36
25	9773533	9905645	9867887	10132113	9783623	9899951	9883672	10116328	35
26	9773704	9905552	9868152	10131848	9783788	9899854	9883934	10116066	34
27	9773875	9905459	9868416	10131584	9783953	9899757	9884196	10115804	33
28	9774046	9905366	9868680	10131320	9784118	9899660	9884457	10115543	32
29	9774217	9905272	9868945	10131055	9784282	9899564	9884719	10115281	31
30	9774388	9905179	9869209	10130791	9784447	9899467	9884980	10115020	30
31	9774558	9905085	9869473	10130527	9784612	9899370	9885242	10114758	29
32	9774729	9904992	9869737	10130263	9784776	9899273	9885504	10114496	28
33	9774899	9904898	9870001	10129999	9784941	9899176	9885765	10114235	27
34	9775070	9904804	9870265	10129735	9785105	9899078	9886026	10113974	26
35	9775240	9904711	9870529	10129471	9785269	9898981	9886288	10113712	25
36	9775410	9904617	9870793	10129207	9785433	9898884	9886549	10113451	24
37	9775580	9904523	9871057	10128943	9785597	9898787	9886811	10113189	23
38	9775759	9904429	9871321	10128679	9785761	9898689	9887072	10112928	22
39	9775928	9904335	9871585	10128415	9785925	9898592	9887333	10112667	21
40	9776096	9904241	9871849	10128151	9786089	9898494	9887594	10112406	20
41	9776269	9904147	9872112	10127888	9786252	9898397	9887855	10112145	19
42	9776429	9904053	9872376	10127624	9786416	9898299	9888116	10111884	18
43	9776593	9903959	9872640	10127360	9786579	9898202	9888378	10111622	17
44	9776768	9903864	9872903	10127097	9786742	9898104	9888639	10111361	16
45	9776937	9903770	9873167	10126833	9786906	9898006	9888900	10111100	15
46	9777106	9903676	9873430	10126570	9787069	9897908	9889161	10110839	14
47	9777275	9903581	9873694	10126306	9787232	9897810	9889421	10110579	13
48	9777444	9903487	9873957	10126043	9787395	9897712	9889682	10110318	12
49	9777613	9903392	9874220	10125780	9787557	9897614	9889943	10110057	11
50	9777781	9903298	9874484	10125516	9787720	9897516	9890204	10109796	10
51	9777950	9903203	9874747	10125253	9787883	9897418	9890465	10109535	9
52	9778119	9903108	9875010	10124990	9788045	9897320	9890725	10109275	8
53	9778287	9903014	9875273	10124727	9788208	9897222	9890986	10109014	7
54	9778455	9902919	9875537	10124463	9788370	9897123	9891247	10108753	6
55	9778624	9902824	9875800	10124200	9788532	9897025	9891507	10108493	5
56	9778792	9902729	9876063	10123937	9788694	9896926	9891768	10108232	4
57	9778960	9902634	9876326	10123674	9788856	9896828	9892028	10107972	3
58	9779128	9902539	9876589	10123411	9789018	9896729	9892289	10107711	2
59	9779295	9902444	9876852	10123148	9789180	9896631	9892549	10107451	1
60	9779463	9902349	9877114	10122886	9789342	9896532	9892810	10107190	0
	Cosine	Sine	Cotan.	Tan.	Cosine	Sine	Cotan.	Tan.	

38°				39°			
Sine	Cosine	Tan.	Cotan.	Sine	Cosine	Tan.	Cotan.
0 9789312	9896532	9892818	10107190	9798872	9890503	9908369	10091631
1 9789504	9896433	9893070	10106930	9799028	9890400	9908628	10091372
2 9789605	9896333	9893331	10106669	9799184	9890296	9908886	10091114
3 9789687	9896236	9893591	10106409	9799339	9890195	9909144	10090856
4 9789988	9896137	9893851	10106149	9799495	9890093	9909402	10090598
5 9790149	9896038	9894111	10105889	9799651	9889990	9909660	10090340
6 9790310	9895930	9894372	10105628	9799806	9889888	9909918	10090082
7 9790471	9895830	9894632	10105368	9799962	9889785	9910177	10089823
8 9790632	9895741	9894892	10105108	9800117	9889682	9910435	10089565
9 9790793	9895641	9895152	10104848	9800272	9889579	9910693	10089307
10 9790954	9895542	9895412	10104588	9800427	9889477	9910951	10089049
11 9791115	9895443	9895672	10104328	9800582	9889374	9911209	10088791
12 9791275	9895343	9895932	10104068	9800737	9889271	9911467	10088533
13 9791436	9895244	9896192	10103808	9800892	9889168	9911725	10088275
14 9791596	9895145	9896452	10103548	9801047	9889064	9911982	10088017
15 9791757	9895045	9896712	10103288	9801201	9888961	9912240	10087760
16 9791917	9894945	9896971	10103029	9801356	9888858	9912498	10087502
17 9792077	9894846	9897231	10102769	9801511	9888755	9912756	10087244
18 9792237	9894746	9897491	10102509	9801665	9888651	9913014	10086986
19 9792397	9894646	9897751	10102249	9801819	9888548	9913271	10086729
20 9792557	9894546	9898010	10101990	9801973	9888444	9913529	10086471
21 9792716	9894446	9898270	10101730	9802128	9888341	9913787	10086213
22 9792876	9894346	9898530	10101470	9802282	9888237	9914044	10085956
23 9793035	9894246	9898789	10101211	9802436	9888134	9914302	10085698
24 9793195	9894146	9899049	10100951	9802589	9888030	9914560	10085440
25 9793354	9894046	9899308	10100692	9802743	9887926	9914817	10085183
26 9793514	9893946	9899568	10100432	9802897	9887822	9915075	10084925
27 9793673	9893846	9899827	10100173	9803050	9887718	9915332	10084668
28 9793832	9893745	9900087	10099913	9803204	9887614	9915590	10084410
29 9793991	9893645	9900346	10099654	9803357	9887510	9915847	10084153
30 9794150	9893544	9900605	10099395	9803511	9887406	9916104	10083896
31 9794308	9893444	9900864	10099136	9803664	9887302	9916362	10083638
32 9794467	9893343	9901124	10098876	9803817	9887198	9916619	10083381
33 9794626	9893243	9901383	10098617	9803970	9887093	9916877	10083123
34 9794784	9893142	9901642	10098358	9804123	9886989	9917134	10082866
35 9794942	9893041	9901901	10098099	9804276	9886885	9917391	10082609
36 9795101	9892940	9902160	10097840	9804428	9886780	9917648	10082352
37 9795259	9892839	9902420	10097580	9804581	9886676	9917906	10082094
38 9795417	9892739	9902679	10097321	9804734	9886571	9918163	10081837
39 9795575	9892638	9902938	10097062	9804886	9886466	9918420	10081580
40 9795733	9892536	9903197	10096803	9805039	9886362	9918677	10081323
41 9795891	9892432	9903456	10096544	9805191	9886257	9918934	10081066
42 9796049	9892334	9903714	10096286	9805343	9886152	9919191	10080809
43 9796206	9892233	9903973	10096027	9805495	9886047	9919448	10080552
44 9796364	9892132	9904232	10095768	9805647	9885942	9919705	10080295
45 9796521	9892030	9904491	10095509	9805799	9885837	9919962	10080038
46 9796679	9891929	9904750	10095250	9805951	9885732	9920219	10079781
47 9796836	9891827	9905008	10094992	9806103	9885627	9920476	10079524
48 9796993	9891726	9905267	10094733	9806255	9885522	9920733	10079267
49 9797150	9891624	9905526	10094474	9806406	9885416	9920990	10079010
50 9797307	9891523	9905785	10094215	9806557	9885311	9921247	10078753
51 9797464	9891421	9906043	10093957	9806709	9885205	9921503	10078497
52 9797621	9891319	9906302	10093698	9806860	9885100	9921760	10078240
53 9797777	9891217	9906560	10093440	9807011	9884994	9922017	10077983
54 9797934	9891115	9906819	10093181	9807163	9884889	9922274	10077726
55 9798091	9891013	9907077	10092923	9807314	9884783	9922530	10077470
56 9798247	9890911	9907336	10092664	9807465	9884677	9922787	10077213
57 9798403	9890809	9907594	10092406	9807615	9884572	9923044	10076956
58 9798560	9890707	9907853	10092147	9807766	9884466	9923300	10076700
59 9798716	9890605	9908111	10091889	9807917	9884360	9923557	10076443
60 9798872	9890503	9908369	10091631	9808067	9884254	9923814	10076186
Cosine	Sine	Cotan.	Tan.	Cosine	Sine	Cotan.	Tan.



40°				41°			
Sine	Cosine	Tan.	Cotan.	Sine	Cosine	Tan.	Cotan.
0 9°808067	9°884254	9°923814	10°076186	9°816943	9°877780	9°939163	10°060837
1 9°808218	9°884148	9°924070	10°075930	9°817084	9°877670	9°939418	10°060582
2 9°808368	9°884042	9°924327	10°075673	9°817233	9°877560	9°939673	10°060327
3 9°808519	9°883936	9°924583	10°075417	9°817379	9°877450	9°939928	10°060072
4 9°808669	9°883829	9°924840	10°075160	9°817524	9°877340	9°940183	10°059817
5 9°808819	9°883723	9°925096	10°074904	9°817668	9°877230	9°940439	10°059561
6 9°808969	9°883617	9°925352	10°074648	9°817813	9°877120	9°940694	10°059306
7 9°809119	9°883510	9°925609	10°074391	9°817958	9°877010	9°940949	10°059051
8 9°809269	9°883404	9°925865	10°074135	9°818103	9°876899	9°941204	10°058796
9 9°809419	9°883297	9°926122	10°073878	9°818247	9°876789	9°941459	10°058541
10 9°809569	9°883191	9°926378	10°073622	9°818392	9°876678	9°941713	10°058287
11 9°809718	9°883084	9°926634	10°073366	9°818536	9°876568	9°941968	10°058032
12 9°80 868	9°882977	9°926890	10°073110	9°818681	9°876457	9°942223	10°057777
13 9°810017	9°882871	9°927147	10°072853	9°818825	9°876347	9°942478	10°057522
14 9°810167	9°882764	9°927403	10°072597	9°818969	9°876236	9°942733	10°057267
15 9°810316	9°882657	9°927659	10°072341	9°819113	9°876125	9°942988	10°057012
16 9°810465	9°882550	9°927915	10°072085	9°819257	9°876014	9°943243	10°056757
17 9°810614	9°882443	9°928171	10°071829	9°819401	9°875904	9°943498	10°056502
18 9°810763	9°882336	9°928427	10°071573	9°819545	9°875793	9°943752	10°056248
19 9°810912	9°882229	9°928684	10°071316	9°819689	9°875682	9°944007	10°055993
20 9°811061	9°882121	9°928940	10°071060	9°819832	9°875571	9°944262	10°055738
21 9°811210	9°882014	9°929196	10°070804	9°819976	9°875459	9°944517	10°055483
22 9°811358	9°881907	9°929452	10°070548	9°820120	9°875348	9°944771	10°055229
23 9°811507	9°881799	9°929708	10°070292	9°820263	9°875237	9°945026	10°054974
24 9°811655	9°881692	9°929964	10°070036	9°820406	9°875126	9°945281	10°054719
25 9°811804	9°881584	9°930220	10°069780	9°820550	9°875014	9°945535	10°054465
26 9°811952	9°881477	9°930475	10°069525	9°820693	9°874903	9°945790	10°054210
27 9°812100	9°881369	9°930731	10°069269	9°820836	9°874791	9°946045	10°053955
28 9°812248	9°881261	9°930987	10°069013	9°820979	9°874680	9°946299	10°053701
29 9°812396	9°881153	9°931243	10°068757	9°821122	9°874568	9°946554	10°053446
30 9°812544	9°881046	9°931499	10°068501	9°821265	9°874456	9°946808	10°053192
31 9°812692	9°880938	9°931755	10°068245	9°821407	9°874344	9°947063	10°052937
32 9°812840	9°880830	9°932010	10°067990	9°821550	9°874232	9°947318	10°052682
33 9°812988	9°880722	9°932266	10°067734	9°821693	9°874121	9°947572	10°052428
34 9°813135	9°880613	9°932522	10°067478	9°821835	9°874009	9°947827	10°052173
35 9°813283	9°880505	9°932778	10°067222	9°821977	9°873896	9°948081	10°051919
36 9°813430	9°880397	9°933033	10°066967	9°822120	9°873784	9°948335	10°051665
37 9°813578	9°880289	9°933289	10°066711	9°822262	9°873672	9°948590	10°051410
38 9°813725	9°880180	9°933545	10°066455	9°822404	9°873560	9°948844	10°051156
39 9°813872	9°880072	9°933800	10°066200	9°822546	9°873448	9°949099	10°050901
40 9°814019	9°879963	9°934056	10°065944	9°822688	9°873335	9°949353	10°050647
41 9°814166	9°879855	9°934311	10°065689	9°822830	9°873223	9°949608	10°050392
42 9°814313	9°879746	9°934567	10°065433	9°822972	9°873110	9°949862	10°050138
43 9°814460	9°879637	9°934822	10°065178	9°823114	9°872998	9°950116	10°049884
44 9°814607	9°879529	9°935077	10°064922	9°823255	9°872885	9°950371	10°049629
45 9°814753	9°879420	9°935333	10°064667	9°823397	9°872772	9°950625	10°049375
46 9°814900	9°879311	9°935589	10°064411	9°823539	9°872659	9°950879	10°049121
47 9°815046	9°879202	9°935844	10°064156	9°823680	9°872547	9°951133	10°048867
48 9°815193	9°879093	9°936100	10°063900	9°823821	9°872434	9°951388	10°048612
49 9°815339	9°878984	9°936355	10°063645	9°823963	9°872321	9°951642	10°048358
50 9°815485	9°878875	9°936611	10°063389	9°824104	9°872208	9°951896	10°048104
51 9°815632	9°878766	9°936866	10°063134	9°824245	9°872095	9°952150	10°047850
52 9°815778	9°878656	9°937121	10°062879	9°824386	9°871981	9°952405	10°047595
53 9°815924	9°878547	9°937377	10°062623	9°824527	9°871868	9°952659	10°047341
54 9°816069	9°878438	9°937632	10°062368	9°824668	9°871755	9°952913	10°047087
55 9°816215	9°878328	9°937887	10°062113	9°824808	9°871641	9°953167	10°046832
56 9°816361	9°878219	9°938142	10°061858	9°824949	9°871528	9°953421	10°046579
57 9°816507	9°878109	9°938398	10°061602	9°825090	9°871414	9°953675	10°046325
58 9°816652	9°877999	9°938653	10°061347	9°825230	9°871301	9°953929	10°046071
59 9°816798	9°877890	9°938908	10°061092	9°825371	9°871187	9°954183	10°045817
60 9°816943	9°877780	9°939163	10°060837	9°825511	9°871073	9°954437	10°045563
Cosine	Sine	Cotan.	Tan.	Cosine	Sine	Cotan.	Tan.

42°				43°			
Sine	Cosine	Tan.	Cotan.	Sine	Cosine	Tan.	Cotan.
0 9°25511	9°871073	9°954437	10°04563	9°833783	9°864127	9°969656	10°030344
1 9°25651	9°870960	9°954691	10°045309	9°833919	9°864010	9°969909	10°030091
2 9°25791	9°870846	9°954946	10°045054	9°834054	9°863892	9°970162	10°029838
3 9°25931	9°870732	9°955200	10°044800	9°834189	9°863774	9°970416	10°029584
4 9°26071	9°870618	9°955454	10°044546	9°834325	9°863656	9°970669	10°029331
5 9°26211	9°870504	9°955708	10°044292	9°834460	9°863538	9°970922	10°029078
6 9°26351	9°870390	9°955961	10°044039	9°834595	9°863419	9°971175	10°028825
7 9°26491	9°870276	9°956215	10°043785	9°834730	9°863301	9°971429	10°028571
8 9°26631	9°870161	9°956469	10°043521	9°834865	9°863183	9°971682	10°028318
9 9°26770	9°870047	9°956723	10°043277	9°834999	9°863064	9°971935	10°028065
10 9°26910	9°869933	9°956977	10°043023	9°835134	9°862946	9°972188	10°027812
11 9°27049	9°869818	9°957231	10°042769	9°835269	9°862827	9°972441	10°027559
12 9°27189	9°869704	9°957485	10°042515	9°835403	9°862709	9°972695	10°027305
13 9°27328	9°869589	9°957739	10°042261	9°835538	9°862590	9°972948	10°027052
14 9°27467	9°869474	9°957993	10°042007	9°835672	9°862471	9°973201	10°026799
15 9°27606	9°869360	9°958247	10°041753	9°835807	9°862353	9°973454	10°026546
16 9°27745	9°869245	9°958500	10°041500	9°835941	9°862234	9°973707	10°026293
17 9°27884	9°869130	9°958754	10°041246	9°836075	9°862115	9°973960	10°026040
18 9°28023	9°869015	9°959008	10°040992	9°836209	9°861996	9°974213	10°025787
19 9°28162	9°868900	9°959262	10°040738	9°836343	9°861877	9°974466	10°025534
20 9°28301	9°868785	9°959516	10°040484	9°836477	9°861758	9°974720	10°025280
21 9°28439	9°868670	9°959769	10°040231	9°836611	9°861638	9°974973	10°025027
22 9°28578	9°868555	9°960023	10°039977	9°836745	9°861519	9°975226	10°024774
23 9°28716	9°868440	9°960277	10°039723	9°836879	9°861400	9°975479	10°024521
24 9°28855	9°868324	9°960530	10°039470	9°837012	9°861280	9°975732	10°024268
25 9°28993	9°868209	9°960784	10°039216	9°837146	9°861161	9°975985	10°024015
26 9°29131	9°868093	9°961038	10°038962	9°837279	9°861041	9°976238	10°023762
27 9°29269	9°867978	9°961292	10°038708	9°837412	9°860922	9°976491	10°023509
28 9°29407	9°867862	9°961545	10°038455	9°837545	9°860802	9°976744	10°023256
29 9°29545	9°867747	9°961799	10°038201	9°837679	9°860682	9°976997	10°023003
30 9°29683	9°867631	9°962052	10°037948	9°837812	9°860562	9°977250	10°022750
31 9°29821	9°867515	9°962306	10°037694	9°837945	9°860442	9°977503	10°022497
32 9°29959	9°867399	9°962560	10°037440	9°838078	9°860322	9°977756	10°022244
33 9°30097	9°867283	9°962813	10°037187	9°838211	9°860202	9°978009	10°021991
34 9°30234	9°867167	9°963067	10°036933	9°838344	9°860082	9°978262	10°021738
35 9°30372	9°867051	9°963320	10°036680	9°838477	9°859962	9°978515	10°021485
36 9°30509	9°866935	9°963574	10°036426	9°838610	9°859842	9°978768	10°021232
37 9°30646	9°866819	9°963828	10°036172	9°838742	9°859721	9°979021	10°020979
38 9°30784	9°866703	9°964081	10°035919	9°838875	9°859601	9°979274	10°020726
39 9°30921	9°866586	9°964335	10°035665	9°839007	9°859480	9°979527	10°020473
40 9°31058	9°866470	9°964588	10°035412	9°839140	9°859360	9°979780	10°020220
41 9°31195	9°866353	9°964842	10°035158	9°839272	9°859239	9°980033	10°019967
42 9°31332	9°866237	9°965095	10°034905	9°839404	9°859119	9°980286	10°019714
43 9°31469	9°866120	9°965349	10°034651	9°839536	9°858998	9°980538	10°019462
44 9°31606	9°866004	9°965602	10°034398	9°839668	9°858877	9°980791	10°019209
45 9°31742	9°865887	9°965855	10°034145	9°839800	9°858756	9°981044	10°018956
46 9°31879	9°865770	9°966108	10°033891	9°839932	9°858635	9°981297	10°018703
47 9°32015	9°865653	9°966362	10°033638	9°840064	9°858514	9°981550	10°018450
48 9°32152	9°865536	9°966616	10°033384	9°840196	9°858393	9°981803	10°018197
49 9°32288	9°865419	9°966869	10°033131	9°840328	9°858272	9°982056	10°017944
50 9°32425	9°865302	9°967123	10°032877	9°840459	9°858151	9°982309	10°017691
51 9°32561	9°865185	9°967376	10°032624	9°840591	9°858029	9°982562	10°017438
52 9°32697	9°865068	9°967629	10°032371	9°840722	9°857908	9°982814	10°017186
53 9°32833	9°864950	9°967883	10°032117	9°840854	9°857786	9°983067	10°016933
54 9°32969	9°864833	9°968136	10°031864	9°840985	9°857665	9°983320	10°016680
55 9°33105	9°864716	9°968389	10°031611	9°841116	9°857543	9°983573	10°016427
56 9°33241	9°864600	9°968643	10°031357	9°841247	9°857422	9°983826	10°016174
57 9°33377	9°864481	9°968896	10°031104	9°841378	9°857300	9°984079	10°015921
58 9°33512	9°864363	9°969149	10°030851	9°841509	9°857178	9°984332	10°015668
59 9°33648	9°864245	9°969403	10°030597	9°841640	9°857056	9°984584	10°015415
60 9°33783	9°864127	9°969656	10°030344	9°841771	9°856934	9°984837	10°015162
Cosine	Sine	Cotan.	Tan.	Cosine	Sine	Cotan.	Tan.

44°					
	Sine	Cosine	Tan.	Cotan.	
0	9°841771	9°856934	9°981837	10°015163	60
1	9°841902	9°856812	9°985090	10°014910	59
2	9°842033	9°856690	9°985343	10°014657	58
3	9°842163	9°856568	9°985596	10°014404	57
4	9°842294	9°856446	9°985848	10°014152	56
5	9°842424	9°856323	9°986101	10°013899	55
6	9°842555	9°856201	9°986354	10°013646	54
7	9°842685	9°856078	9°986607	10°013393	53
8	9°842815	9°855956	9°986860	10°013140	52
9	9°842946	9°855833	9°987112	10°012888	51
10	9°843076	9°855711	9°987365	10°012635	50
11	9°843206	9°855588	9°987618	10°012382	49
12	9°843336	9°855465	9°987871	10°012129	48
13	9°843466	9°855342	9°988123	10°011877	47
14	9°843595	9°855219	9°988376	10°011624	46
15	9°843725	9°855096	9°988629	10°011371	45
16	9°843855	9°854973	9°988882	10°011118	44
17	9°843984	9°854850	9°989134	10°010866	43
18	9°844114	9°854727	9°989387	10°010613	42
19	9°844243	9°854603	9°989640	10°010360	41
20	9°844372	9°854480	9°989893	10°010107	40
21	9°844502	9°854356	9°990145	10°009855	39
22	9°844631	9°854233	9°990398	10°009602	38
23	9°844760	9°854109	9°990651	10°009349	37
24	9°844889	9°853986	9°990903	10°009097	36
25	9°845018	9°853862	9°991156	10°008844	35
26	9°845147	9°853738	9°991409	10°008591	34
27	9°845276	9°853614	9°991662	10°008338	33
28	9°845405	9°853490	9°991914	10°008086	32
29	9°845533	9°853366	9°992167	10°007833	31
30	9°845662	9°853242	9°992420	10°007580	30
31	9°845790	9°853118	9°992672	10°007328	29
32	9°845919	9°852994	9°992925	10°007075	28
33	9°846047	9°852869	9°993178	10°006822	27
34	9°846175	9°852745	9°993431	10°006569	26
35	9°846304	9°852620	9°993683	10°006317	25
36	9°846432	9°852496	9°993936	10°006064	24
37	9°846560	9°852371	9°994189	10°005811	23
38	9°846688	9°852247	9°994441	10°005559	22
39	9°846816	9°852122	9°994694	10°005306	21
40	9°846944	9°851997	9°994947	10°005053	20
41	9°847071	9°851872	9°995199	10°004801	19
42	9°847199	9°851747	9°995452	10°004548	18
43	9°847327	9°851622	9°995705	10°004295	17
44	9°847454	9°851497	9°995957	10°004043	16
45	9°847582	9°851372	9°996210	10°003790	15
46	9°847709	9°851246	9°996463	10°003537	14
47	9°847836	9°851121	9°996715	10°003285	13
48	9°847964	9°850996	9°996968	10°003032	12
49	9°848091	9°850870	9°997221	10°002779	11
50	9°848218	9°850745	9°997473	10°002527	10
51	9°848345	9°850619	9°997726	10°002274	9
52	9°848472	9°850493	9°997979	10°002021	8
53	9°848599	9°850368	9°998231	10°001769	7
54	9°848726	9°850242	9°998484	10°001516	6
55	9°848852	9°850116	9°998737	10°001263	5
56	9°848979	9°849990	9°998989	10°001011	4
57	9°849106	9°849864	9°999242	10°000758	3
58	9°849232	9°849738	9°999495	10°000505	2
59	9°849358	9°849611	9°999747	10°000253	1
60	9°849485	9°849485	10.000000	10°000000	0
	Cosine	Sine	Cotan.	Tan.	
45°					

TRAVERSE TABLES,  
CALCULATED TO ANY NUMBER  
OF  
CHAINS, OR LINKS OF DISTANCE,  
AND TO  
**THREE MINUTES OF THE ANGLE OF BEARING.\***

*\* The principle and method of using these tables will be found in the Third Part, page 193.*



### TRAVERSE TABLES.

1		2		3		4		5		6		7		8		9		10		Chs.
at.	Dep.	Lat.	Dep.	Lat.	Dep.	Lat.	Dep.	Lat.	Dep.	Lat.	Dep.	Lat.	Dep.	Lat.	Dep.	Lat.	Dep.	Lat.	Dep.	
00	001	2 00	002	3 00	003	4 00	003	5 00	004	6 00	005	7 00	006	8 00	007	9 00	008	10 00	009	57
00	002	2 00	003	3 00	005	4 00	007	5 00	008	6 00	010	7 00	012	8 00	014	9 00	016	10 00	017	54
00	003	2 00	005	3 00	008	4 00	010	5 00	013	6 00	016	7 00	018	8 00	020	9 00	023	10 00	026	51
00	003	2 00	007	3 00	010	4 00	014	5 00	017	6 00	021	7 00	024	8 00	028	9 00	031	10 00	035	48
00	004	2 00	009	3 00	013	4 00	017	5 00	022	6 00	026	7 00	030	8 00	035	9 00	039	10 00	043	
00	005	2 00	010	3 00	016	4 00	021	5 00	026	6 00	031	7 00	037	8 00	042	9 00	047	10 00	052	2
00	006	2 00	012	3 00	018	4 00	024	5 00	030	6 00	037	7 00	043	8 00	049	9 00	055	10 00	061	39
00	007	2 00	014	3 00	021	4 00	028	5 00	035	6 00	042	7 00	049	8 00	056	9 00	063	10 00	070	36
00	008	2 00	016	3 00	023	4 00	031	5 00	039	6 00	047	7 00	055	8 00	063	9 00	071	10 00	078	33
00	009	2 00	018	3 00	027	4 00	036	5 00	045	6 00	053	7 00	062	8 00	071	9 00	079	10 00	086	30
00	010	2 00	019	3 00	029	4 00	038	5 00	048	6 00	058	7 00	067	8 00	077	9 00	086	10 00	096	27
00	011	2 00	021	3 00	032	4 00	042	5 00	053	6 00	063	7 00	074	8 00	084	9 00	095	10 00	105	24
00	011	2 00	023	3 00	034	4 00	045	5 00	056	6 00	068	7 00	079	8 00	090	9 00	102	10 00	113	21
00	012	2 00	024	3 00	036	4 00	049	5 00	061	6 00	073	7 00	085	8 00	098	9 00	110	10 00	122	18
00	013	2 00	026	3 00	039	4 00	052	5 00	065	6 00	079	7 00	092	8 00	105	9 00	118	10 00	131	15
00	014	2 00	028	3 00	042	4 00	056	5 00	070	6 00	084	7 00	098	8 00	112	9 00	126	10 00	140	12
00	015	2 00	030	3 00	044	4 00	059	5 00	074	6 00	089	7 00	104	8 00	118	9 00	133	10 00	148	9
00	016	2 00	031	3 00	047	4 00	063	5 00	078	6 00	094	7 00	109	8 00	126	9 00	141	10 00	157	6
00	017	2 00	033	3 00	050	4 00	066	5 00	083	6 00	100	7 00	116	8 00	123	9 00	139	10 00	166	3
00	017	2 00	035	3 00	052	4 00	070	5 00	087	6 00	105	7 00	122	8 00	140	9 00	157	10 00	175	89
00	018	2 00	036	3 00	055	4 00	073	5 00	091	6 00	110	7 00	128	8 00	146	9 00	165	10 00	183	57
00	019	2 00	038	3 00	057	4 00	076	5 00	095	6 00	115	7 00	134	8 00	153	9 00	172	10 00	191	54
00	020	2 00	040	3 00	060	4 00	080	5 00	100	6 00	120	7 00	140	8 00	160	9 00	180	10 00	200	51
00	021	2 00	042	3 00	063	4 00	084	5 00	105	6 00	126	7 00	147	8 00	168	9 00	189	10 00	210	48
00	022	2 00	044	3 00	065	4 00	087	5 00	109	6 00	131	7 00	153	8 00	174	9 00	196	10 00	218	45
00	022	2 00	045	3 00	068	4 00	091	5 00	113	6 00	136	7 00	150	8 00	182	9 00	204	10 00	227	42
00	023	2 00	047	3 00	070	4 00	094	5 00	117	6 00	141	7 00	161	8 00	188	9 00	211	10 00	235	39
00	024	2 00	049	3 00	073	4 00	097	5 00	122	6 00	146	7 00	171	8 00	195	9 00	220	10 00	244	36
00	025	2 00	051	3 00	076	4 00	101	5 00	127	6 00	152	7 00	177	8 00	202	9 00	228	10 00	253	3
00	026	2 00	052	3 00	079	4 00	105	5 00	131	6 00	157	7 00	183	8 00	210	9 00	236	10 00	262	30
00	027	2 00	054	3 00	081	4 00	108	5 00	135	6 00	162	7 00	189	8 00	216	9 00	243	10 00	270	27
00	028	2 00	056	3 00	084	4 00	112	5 00	140	6 00	168	7 00	190	8 00	224	9 00	252	10 00	280	24
00	029	2 00	058	3 00	086	4 00	115	5 00	144	6 00	173	7 00	202	8 00	230	9 00	259	10 00	288	21
00	030	2 00	059	3 00	089	4 00	119	5 00	148	6 00	178	7 00	208	8 00	238	9 00	267	10 00	297	18
00	031	2 00	061	3 00	092	4 00	122	5 00	153	6 00	183	7 00	214	8 00	244	9 00	275	10 00	305	15
00	031	2 00	063	3 00	094	4 00	126	5 00	157	6 00	188	7 00	220	8 00	251	9 00	283	10 00	314	12
99	032	1 99	065	2 99	097	3 99	129	4 99	161	5 99	194	6 99	226	7 99	258	8 99	291	9 99	323	9
99	033	1 99	066	2 99	099	3 99	132	4 99	165	5 99	199	6 99	232	7 99	265	8 99	298	9 99	331	6
99	034	1 99	068	2 99	102	3 99	136	4 99	170	5 99	204	6 99	238	7 99	272	8 99	306	9 99	340	3
99	035	1 99	070	2 99	105	3 99	140	4 99	175	5 99	209	6 99	241	7 99	279	8 99	316	9 99	349	88
99	036	1 99	072	2 99	107	3 99	143	4 99	179	5 99	215	6 99	251	7 99	286	8 99	322	9 99	358	57
99	037	1 99	073	2 99	110	3 99	146	4 99	183	5 99	220	6 99	256	7 99	288	8 99	329	9 99	366	54
99	037	1 99	075	2 99	112	3 99	150	4 99	188	5 99	225	6 99	263	7 99	300	8 99	338	9 99	375	51
99	038	1 99	077	2 99	115	3 99	154	4 99	192	5 99	230	6 99	269	7 99	307	8 99	346	9 99	384	48
99	039	1 99	078	2 99	118	3 99	157	4 99	196	5 99	235	6 99	274	7 99	314	8 99	353	9 99	392	45
99	040	1 99	080	2 99	120	3 99	160	4 99	201	5 99	241	6 99	281	7 99	321	8 99	361	9 99	401	42
99	041	1 99	082	2 99	123	3 99	164	4 99	205	5 99	246	6 99	287	7 99	328	8 99	369	9 99	410	39
99	042	1 99	084	2 99	126	3 99	168	4 99	210	5 99	251	6 99	293	7 99	335	8 99	377	9 99	419	36
99	043	1 99	085	2 99	128	3 99	171	4 99	214	5 99	256	6 99	299	7 99	342	8 99	384	9 99	427	33
99	044	1 99	087	2 99	131	3 99	174	4 99	218	5 99	262	6 99	305	7 99	349	8 99	392	9 99	436	30
99	045	1 99	089	2 99	134	3 99	178	4 99	223	5 99	267	6 99	312	7 99	356	8 99	401	9 99	445	27
99	045	1 99	091	2 99	136	3 99	182	4 99	227	5 99	272	6 99	318	7 99	363	8 99	409	9 99	454	24
99	046	1 99	092	2 99	139	3 99	185	4 99	231	5 99	277	6 99	323	7 99	370	8 99	416	9 99	462	21
99	047	1 99	094	2 99	141	3 99	188	4 99	236	5 99	283	6 99	330	7 99	377	8 99	424	9 99	471	18
99	048	1 99	096	2 99	144	3 99	192	4 99	240	5 99	288	6 99	336	7 99	384	8 99	432	9 99	480	15
99	049	1 99	098	2 99	147	3 99	196	4 99	245	5 99	293	6 99	342	7 99	391	8 99	440	9 99	489	12
99	050	1 99	099	2 99	149	3 99	199	4 99	249	5 99	298	6 99	348	7 99	398	8 99	447	9 99	497	9
98	051	1 99	101	2 99	152	3 99	202	4 99	253	5 99	304	6 99	354	7 99	405	8 99	455	9 99	506	6
98	052	1 99	103	2 99	155	3 99	206	4 99	258	5 99	309	6 99	360	7 99	412	8 99	463	9 99	515	3
ep	Lat.	Den	Lat.	Den	Lat.	Dep	Lat.	Dep	Lat.	Dep	Lat.	Dep	Lat.	Dep	Lat.	Dep	Lat.	Dep	Lat.	Dist.

### TRAVERSE TABLES.

Chs.	1		2		3		4		5		6		7		8		9		10		Chs.
	Lat.	Dep.	Lat.	Dep.	Lat.	Dep.	Lat.	Dep.	Lat.	Dep.	Lat.	Dep.	Lat.	Dep.	Lat.	Dep.	Lat.	Dep.	Lat.	Dep.	
3	99°00	02	199°105	299°157	399°209	499°261	599°314	699°366	799°418	899°471	999°523	87°									
3	99°053	199°106	299°160	399°213	499°266	599°319	699°372	799°426	899°479	999°532	57										
6	99°054	199°108	299°162	399°216	499°271	599°325	699°378	799°433	899°486	999°539	54										
9	99°055	199°110	299°165	399°220	499°275	599°330	699°385	799°440	899°493	999°546	51										
12	99°056	199°112	299°167	399°223	499°279	599°335	699°391	799°446	899°500	999°551	48										
15	99°057	199°113	299°170	399°227	499°283	599°340	699°397	799°454	899°506	999°558	45										
18	99°058	199°115	299°173	399°230	499°288	599°345	699°403	799°461	899°513	999°567	42										
21	99°059	199°117	299°175	399°234	499°292	599°350	699°409	799°467	899°520	999°574	39										
24	99°058	199°119	299°178	399°237	499°297	599°356	699°415	799°475	899°534	999°593	36										
27	99°060	199°120	299°180	399°240	499°300	599°361	699°421	799°481	899°541	999°601	33										
30	99°061	199°122	299°183	399°244	499°305	599°366	699°427	799°488	899°559	999°610	30										
33	99°062	199°124	299°186	399°248	499°310	599°371	699°433	799°495	899°567	999°619	27										
36	99°063	199°126	299°188	399°251	499°314	599°377	699°440	799°502	899°575	999°628	24										
39	99°064	199°127	299°191	399°255	499°318	599°382	699°447	799°509	899°582	999°636	21										
42	99°064	199°129	299°193	399°258	499°323	599°387	699°452	799°516	899°591	999°645	18										
45	99°065	199°131	299°196	399°262	499°327	599°392	699°458	799°523	899°599	999°654	15										
48	99°066	199°132	299°199	399°265	499°331	599°397	699°463	799°530	899°606	999°662	12										
51	99°067	199°134	299°201	399°268	499°336	599°403	699°470	799°537	899°614	999°671	9										
54	99°068	199°136	299°204	399°272	499°340	599°408	699°476	799°544	899°622	999°680	6										
57	99°069	199°138	299°207	399°276	499°345	599°413	699°482	799°551	899°630	999°689	3										
4	99°070	199°139	299°209	399°279	499°349	599°418	699°488	799°558	899°637	999°697	86°										
3	99°071	199°141	299°212	399°282	499°353	599°424	699°494	799°565	899°646	999°706	57										
6	99°071	199°143	299°214	399°286	499°357	599°429	699°500	799°572	899°653	999°715	54										
9	99°072	199°145	299°217	399°289	499°362	599°434	699°507	799°579	899°661	999°724	51										
12	99°073	199°146	299°220	399°293	499°366	599°439	699°513	799°586	899°669	999°732	48										
15	99°074	199°148	299°222	399°296	499°371	599°445	699°519	799°593	899°667	999°741	45										
18	99°075	199°150	299°225	399°300	499°375	599°450	699°525	799°600	899°675	999°750	42										
21	99°076	199°152	299°227	399°303	499°379	599°455	699°531	799°607	899°683	999°758	39										
24	99°077	199°153	299°230	399°307	499°384	599°460	699°537	799°614	899°690	999°767	36										
27	99°078	199°155	299°233	399°310	499°388	599°466	699°543	799°621	899°708	999°776	33										
30	99°078	199°157	299°235	399°314	499°392	599°470	699°549	799°627	899°709	999°784	30										
33	99°079	199°159	299°238	399°317	499°397	599°476	699°555	799°634	899°714	999°793	27										
36	99°080	199°160	299°241	399°321	499°401	599°481	699°561	799°642	899°722	999°802	24										
39	99°081	199°162	299°243	399°324	499°406	599°487	699°568	799°649	899°730	999°811	21										
42	99°082	199°164	299°246	399°328	499°410	599°491	699°573	799°655	899°737	999°819	18										
45	99°083	199°166	299°248	399°331	499°414	599°497	699°580	799°662	899°745	999°828	15										
48	99°084	199°167	299°251	399°335	499°419	599°502	699°586	799°670	899°753	999°837	12										
51	99°085	199°169	299°253	399°338	499°423	599°507	699°592	799°676	899°761	999°845	9										
54	99°085	199°171	299°256	399°342	499°427	599°512	699°598	799°683	899°766	999°854	6										
57	99°086	199°172	299°259	399°345	499°431	599°517	699°603	799°690	899°776	999°862	3										
3	99°087	199°174	299°261	399°349	499°436	599°523	699°610	799°697	899°784	999°872	85°										
3	99°088	199°176	299°263	399°352	499°440	599°528	699°616	799°704	899°792	999°880	57										
6	99°089	199°178	299°267	399°356	499°445	599°533	699°622	799°711	899°800	999°889	54										
9	99°090	199°179	299°269	399°359	499°448	599°538	699°628	799°718	899°807	999°897	51										
12	99°091	199°181	299°272	399°362	499°453	599°543	699°634	799°725	899°815	999°906	48										
15	99°091	199°183	299°274	399°366	499°457	599°549	699°640	799°732	899°823	999°915	45										
18	99°092	199°185	299°277	399°369	499°462	599°554	699°647	799°739	899°831	999°924	42										
21	99°093	199°186	299°280	399°373	499°466	599°559	699°652	799°746	899°839	999°932	39										
24	99°094	199°188	299°282	399°376	499°470	599°565	699°659	799°753	899°847	999°941	36										
27	99°095	199°190	299°285	399°380	499°475	599°570	699°665	799°760	899°855	999°950	33										
30	99°096	199°191	299°287	399°383	499°479	599°575	699°671	799°766	899°862	999°958	30										
33	99°097	199°193	299°290	399°387	499°483	599°580	699°677	799°774	899°870	999°967	27										
36	99°098	199°195	299°293	399°390	499°488	599°585	699°683	799°781	899°878	999°976	24										
39	99°098	199°197	299°295	399°393	499°492	599°591	699°689	799°787	899°886	999°984	21										
42	99°099	199°199	299°298	399°397	499°497	599°596	699°695	799°794	899°894	999°993	18										
45	99°100	199°200	299°300	399°400	499°501	599°601	699°701	799°801	899°902	999°1000	15										
48	99°101	199°202	299°303	399°404	499°505	599°606	699°707	799°808	899°909	999°1012	12										
51	99°102	199°204	299°306	399°408	499°510	599°612	699°714	799°816	899°918	999°1029	9										
54	99°103	199°205	299°308	399°411	499°513	599°616	699°719	799°822	899°924	999°1036	6										
57	99°104	199°207	299°311	399°415	499°518	599°622	699°726	799°830	899°933	999°1043	3										
1.	Dep.	Lat.	Dep.	Lat.	Dep.	Lat.	Dep.	Lat.	Dep.	Lat.	Dep.	Lat.	Dep.	Lat.	Dep.	Lat.	Dep.	Lat.	Dep.	Lat.	Dist.





### TRAVERSE TABLES.

Chs.	1		2		3		4		5		6		7		8		9		10		Chs.
/	Lat.	Dep	Lat.	Dep	Lat.	Dep	Lat	Dep	Lat	Dep	Lat.	Dep	Lat	Dep	Lat	Dep	Lat	Dep	Lat.	Dep	/
9°	988	156	197	313	296	469	395	626	494	782	593	938	691	109	790	125	889	141	988	156	81°
3	988	157	197	314	296	472	395	629	494	786	593	944	691	110	790	126	889	141	988	157	57
6	987	158	197	316	296	471	395	632	494	790	592	948	691	111	790	126	889	142	987	158	54
9	987	159	197	318	296	471	395	636	494	795	592	954	691	111	790	127	889	143	987	159	51
12	987	160	197	320	296	480	395	640	494	800	592	960	691	112	790	128	888	144	987	160	48
15	987	161	197	321	296	482	395	643	493	803	592	964	691	112	790	128	888	145	987	161	45
18	987	162	197	323	296	485	395	646	493	808	592	969	691	113	789	129	888	145	987	162	42
21	987	162	197	325	296	487	395	646	493	812	592	975	691	114	789	130	888	146	987	162	39
24	987	163	197	326	296	490	395	653	493	816	592	980	691	114	789	131	888	147	987	163	36
27	986	164	197	328	296	492	395	657	493	821	592	985	690	115	789	131	888	148	986	164	33
30	986	165	197	330	296	495	394	660	493	825	592	990	690	115	789	132	888	148	986	165	30
33	986	166	197	332	296	498	394	664	493	830	592	996	690	116	789	133	887	149	986	166	27
36	986	167	197	334	296	500	394	667	493	834	592	1000	690	117	789	133	887	150	986	167	24
39	986	168	197	335	296	503	394	670	493	838	591	1000	690	117	789	134	887	151	986	168	21
42	986	168	197	337	296	505	394	674	493	842	591	101	690	118	789	135	887	152	986	168	18
45	986	169	197	338	296	508	394	677	493	846	591	102	690	118	788	135	887	152	986	169	15
48	985	170	197	340	296	510	394	681	493	851	591	102	690	119	788	136	887	153	985	170	12
51	985	171	197	342	296	513	394	684	493	855	591	103	690	120	788	137	887	154	985	171	9
54	985	172	197	344	295	516	394	688	492	860	591	103	690	120	788	137	887	155	985	172	6
57	985	173	197	346	295	518	394	691	492	864	591	104	689	121	788	138	886	155	985	173	3
10°	985	174	197	347	295	521	394	694	492	868	591	104	689	121	788	139	886	156	985	174	80°
3	985	174	197	349	295	523	394	698	492	872	591	105	689	122	788	140	886	157	985	174	57
6	985	175	197	351	295	526	394	701	492	877	591	105	689	123	788	140	886	158	985	175	54
9	984	176	197	352	295	528	394	705	492	881	591	106	689	123	787	141	886	158	984	176	51
12	984	177	197	354	295	531	394	708	492	885	590	106	689	124	787	142	886	159	984	177	48
15	984	178	197	356	295	534	394	712	492	890	590	107	689	125	787	142	886	160	984	178	45
18	984	179	197	358	295	536	394	715	492	894	590	107	689	125	787	143	885	161	984	179	42
21	984	180	197	359	295	539	393	719	492	898	590	108	689	126	787	144	885	161	984	180	39
24	984	180	197	361	295	541	393	722	492	902	590	108	688	126	787	144	885	162	984	180	36
27	983	181	197	363	295	544	393	725	492	907	590	109	688	127	787	145	885	163	983	181	33
30	983	182	197	364	295	546	393	729	492	911	590	109	688	127	787	146	885	164	983	182	30
33	983	183	197	366	295	549	393	732	491	915	590	110	688	128	786	146	885	165	983	183	27
36	983	184	197	368	295	552	393	736	491	920	590	110	688	129	786	147	885	165	983	184	24
39	983	185	197	370	295	554	393	739	491	924	590	111	688	129	786	148	884	166	983	185	21
42	983	186	196	371	295	557	393	743	491	928	590	111	688	130	786	148	884	167	983	186	18
45	982	186	196	373	295	559	393	746	491	932	589	112	688	130	786	149	884	168	982	186	15
48	982	187	196	375	295	562	393	749	491	937	589	112	688	131	786	150	884	168	982	187	12
51	982	188	196	376	295	564	393	753	491	941	589	113	687	132	786	150	884	169	982	188	9
54	982	189	196	378	295	567	393	756	491	945	589	113	687	132	786	151	884	170	982	189	6
57	982	190	196	380	294	570	393	760	491	950	589	114	687	133	785	152	884	171	982	190	3
11°	982	191	196	382	294	572	393	763	491	954	589	114	687	133	785	153	883	172	982	191	79
3	982	192	196	383	294	575	393	767	491	958	589	115	687	134	785	153	883	172	982	192	77
6	981	192	196	385	294	577	393	770	491	962	589	115	687	135	785	154	883	173	981	192	74
9	981	193	196	387	294	580	392	773	491	967	589	116	687	135	785	155	883	174	981	193	71
12	981	194	196	388	294	582	392	777	490	971	588	116	687	136	784	155	883	175	981	194	68
15	981	195	196	390	294	585	392	780	490	975	588	117	687	136	784	156	883	175	981	195	65
18	981	196	196	392	294	588	392	784	490	980	588	117	686	137	784	157	883	176	981	196	62
21	980	197	196	394	294	590	392	787	490	984	588	118	686	138	784	157	882	177	980	197	59
24	980	198	196	395	294	593	392	790	490	988	588	118	686	138	784	158	882	178	980	198	56
27	980	198	196	397	294	595	392	794	490	992	588	119	686	139	784	159	882	179	980	198	53
30	980	199	196	399	294	598	392	797	490	997	588	120	686	139	784	159	882	179	980	199	50
33	980	200	196	400	294	600	392	801	490	1000	588	120	686	140	784	160	882	180	980	200	47
36	980	201	196	402	294	603	392	804	490	1000	588	120	686	141	784	160	882	181	980	201	44
39	979	202	196	404	294	606	392	808	490	1010	588	121	686	141	784	161	881	182	979	202	41
42	979	203	196	406	294	608	392	811	490	1010	588	122	685	142	783	162	881	182	979	203	38
45	979	204	196	407	294	611	392	814	490	102	587	122	685	143	783	163	881	183	979	204	35
48	979	204	196	409	294	613	391	818	489	102	587	123	685	143	783	164	881	184	979	204	32
51	979	205	196	410	294	616	391	821	489	102	587	123	685	144	783	164	881	185	979	205	29
54	979	206	196	412	293	618	391	825	489	103	587	124	685	144	783	165	881	185	979	206	26
57	978	207	196	414	293	621	391	828	489	103	587	124	685	145	783	165	880	186	978	207	23
Dist.	Dep	Lat.	Dep	Lat.	Dep	Lat.	Dep	Lat.	Dep	Lat.	Dep	Lat.	Dep	Lat.	Dep	Lat.	Dep	Lat.	Dep	Lat.	Dist.

Chs.	1		2		3		4		5		6		7		8		9		10		Chs.
	Lat.	Dep	Lat.	Dep	Lat.	Dep	Lat.	Dep	Lat.	Dep	Lat.	Dep	Lat.	Dep	Lat.	Dep	Lat.	Dep	Lat.	Dep	
12°	978	208	196	416	293	624	391	832	489	104	587	125	685	146	782	166	880	187	978	208	78°
3	978	209	196	418	293	626	391	833	489	104	587	125	685	146	782	167	880	188	978	209	57
6	978	210	196	419	293	629	391	838	489	105	587	126	684	147	782	168	880	189	978	210	54
9	978	210	196	421	293	633	391	842	489	105	587	126	684	147	782	168	880	190	978	210	51
12	977	211	195	421	293	634	391	843	489	106	586	127	684	148	782	169	880	190	977	211	48
15	977	212	195	424	293	637	391	849	489	106	586	127	684	149	782	170	880	191	977	212	45
18	977	213	195	426	293	639	391	852	489	107	586	128	684	149	782	170	879	192	977	213	42
21	977	214	195	428	293	642	391	856	488	107	586	128	684	150	781	171	879	192	977	214	39
24	977	215	195	429	293	644	391	859	488	107	586	129	684	150	781	172	879	193	977	215	36
27	977	216	195	431	293	647	391	862	488	108	586	129	684	151	781	172	879	194	976	216	33
30	976	216	195	433	293	649	391	866	488	108	586	130	683	152	781	173	879	195	976	216	30
33	976	217	195	435	293	651	390	869	488	109	586	130	683	152	781	174	878	196	976	217	27
36	976	218	195	436	293	654	390	873	488	109	586	131	683	153	781	175	878	196	976	218	24
39	976	219	195	438	293	657	390	876	488	109	585	131	683	153	781	175	878	197	976	219	21
42	976	220	195	440	293	660	390	879	488	110	585	132	683	154	780	176	878	198	976	220	18
45	975	221	195	441	293	662	390	883	488	110	585	132	683	154	780	177	878	199	975	221	15
48	975	222	195	443	293	665	390	886	488	111	585	133	683	155	780	177	878	199	975	222	12
51	975	222	195	446	292	667	390	890	487	111	585	133	682	156	780	178	877	200	975	222	9
54	975	223	195	447	292	670	390	893	487	112	585	134	682	156	780	179	877	201	975	223	6
57	975	224	195	448	292	672	390	896	487	112	585	134	682	157	780	179	877	202	975	224	3
13°	974	226	195	450	292	675	390	900	487	112	585	135	682	157	780	180	877	202	974	225	77°
3	974	226	195	452	292	677	390	903	487	113	584	135	682	158	779	181	877	203	974	226	57
6	974	227	195	453	292	680	390	907	487	113	584	136	682	159	779	181	876	204	974	227	54
9	974	227	195	455	292	682	390	910	487	114	584	136	682	159	779	182	876	205	974	227	51
12	974	228	195	457	292	685	389	913	487	114	584	137	681	160	779	183	876	206	974	228	48
15	974	229	195	458	292	688	389	917	487	115	584	138	681	160	779	183	876	206	973	229	45
18	973	230	195	460	292	690	389	920	487	115	584	138	681	161	778	183	876	207	973	230	42
21	973	231	195	462	292	693	389	924	486	115	584	138	681	162	778	185	876	208	973	231	39
24	973	232	195	463	292	695	389	927	486	116	584	138	681	162	778	185	875	208	973	232	36
27	973	233	194	466	292	699	389	931	486	116	583	140	681	163	778	186	875	210	973	233	33
30	972	233	194	467	292	700	389	934	486	117	583	140	681	163	778	186	875	210	972	233	30
33	972	234	194	468	292	702	389	937	486	117	583	141	681	164	778	187	875	211	972	234	27
36	972	235	194	470	292	705	389	940	486	118	583	141	680	165	778	188	875	212	972	235	24
39	972	236	194	472	291	708	389	944	486	118	583	142	680	165	777	189	874	212	972	236	21
42	972	237	194	474	291	710	389	947	486	118	583	142	680	166	777	189	874	213	972	237	18
45	971	238	194	475	291	713	389	951	486	119	583	143	680	166	777	190	874	214	971	238	15
48	971	239	194	478	291	716	388	955	486	119	583	143	680	167	777	191	874	215	971	239	12
51	971	239	194	479	291	718	388	957	485	120	582	144	680	167	777	191	874	215	971	239	9
54	971	240	194	480	291	721	388	960	485	120	582	144	679	168	777	192	874	216	971	240	6
57	971	241	194	482	291	723	388	964	485	121	582	145	679	169	776	193	873	217	970	241	3
14°	970	242	194	484	291	726	388	968	485	121	582	145	679	169	776	193	873	218	970	242	76°
3	970	243	194	485	291	728	388	971	485	121	582	146	679	170	776	191	873	218	970	243	57
6	970	243	194	487	291	730	388	974	485	122	582	146	679	170	776	195	873	219	970	244	54
9	970	244	194	489	291	733	388	978	485	122	582	147	679	171	776	196	873	220	970	244	51
12	970	245	194	491	291	736	388	981	485	123	582	147	679	172	776	196	873	221	969	245	48
15	969	246	194	492	291	738	388	985	485	123	582	148	678	172	775	197	872	222	969	246	45
18	969	247	194	494	291	741	388	988	485	123	581	148	678	173	775	197	872	222	969	247	42
21	969	248	194	496	291	743	387	991	484	124	581	149	678	173	775	198	872	223	969	248	39
24	969	249	194	497	291	746	387	995	484	124	581	149	678	174	775	199	872	224	969	249	36
27	968	250	194	499	290	749	387	998	484	125	581	150	678	175	775	200	872	225	968	250	33
30	968	250	194	500	290	751	387	1000	484	125	581	150	678	175	775	200	871	225	968	250	30
33	968	251	194	502	290	751	387	1000	484	126	581	151	678	176	774	201	871	226	968	251	27
36	968	252	194	504	290	756	387	101	484	126	581	151	677	176	774	202	871	227	968	252	24
39	967	253	194	506	290	759	387	101	484	126	580	152	677	177	774	202	871	228	967	253	21
42	967	254	193	507	290	761	387	101	484	127	580	152	677	178	774	203	871	228	967	254	18
45	967	255	193	509	290	761	387	102	484	127	580	153	677	178	774	204	870	229	967	255	15
48	967	255	193	511	290	766	387	102	483	128	580	153	677	179	773	204	870	230	967	255	12
51	967	256	193	513	290	768	387	103	483	128	580	154	677	179	773	205	870	231	967	256	9
54	966	257	193	514	290	771	387	103	483	129	580	154	676	180	773	206	870	231	967	257	6
57	966	258	193	516	290	774	386	103	483	129	580	155	676	181	773	206	870	232	966	258	3
Dist.	Lat.	Dep	Lat.	Dep	Lat.	Dep	Lat.	Dep	Lat.	Dep	Lat.	Dep	Lat.	Dep	Lat.	Dep	Lat.	Dep	Lat.	Dep	Dist.

TRAVERSE TABLES.

Chs.	1	2	3	4	5	6	7	8	9	10	Chs.
	Lat. Dep	Lat. Dep	Lat. Dep	Lat. Dep	Lat. Dep	Lat. Dep	Lat. Dep	Lat. Dep	Lat. Dep	Lat. Dep	
15°	066 259	193 516	290 776	386 104	483 129	580 155	676 181	773 207	869 233	966 259	75°
3	066 260	193 519	290 777	386 104	483 130	579 156	676 182	773 208	869 234	966 260	76
6	065 260	193 520	290 781	386 104	483 130	579 156	676 182	772 208	869 234	966 260	77
9	065 261	193 523	290 784	386 105	483 131	579 157	676 183	772 209	869 235	966 261	78
12	065 262	193 524	289 787	386 105	482 131	579 157	675 183	772 210	868 236	966 262	79
15	065 263	193 526	289 789	386 105	482 132	579 158	675 181	772 210	868 237	966 263	80
18	065 264	193 528	289 792	386 106	482 132	579 158	675 185	772 211	868 237	966 264	81
21	064 265	193 529	289 791	386 106	482 132	579 159	675 185	771 212	868 238	966 265	82
24	064 266	193 531	289 797	386 106	482 133	578 159	674 181	771 212	868 239	966 266	83
27	064 267	193 533	289 799	386 107	482 133	578 160	675 186	771 213	867 240	966 267	84
30	064 266	193 534	289 802	385 107	482 134	578 160	675 187	771 214	867 241	966 268	85
33	063 268	193 536	289 801	385 107	482 134	578 161	674 188	771 214	867 241	966 268	86
36	063 270	193 538	289 807	385 108	482 134	578 161	674 188	771 215	867 242	966 269	87
39	063 270	193 540	289 809	385 108	481 135	578 162	674 189	770 216	867 243	966 270	88
42	063 271	193 541	289 812	385 108	481 135	578 162	674 189	770 216	866 244	966 271	89
45	062 271	192 543	289 814	385 109	481 136	577 161	674 190	770 217	866 244	966 272	90
48	062 272	192 545	289 817	385 109	481 136	577 163	674 191	770 218	866 245	966 272	91
51	062 273	192 546	289 819	385 109	481 137	577 161	673 198	770 218	866 246	966 273	92
54	062 274	192 548	289 822	385 110	481 137	577 161	673 192	769 219	866 247	966 274	93
57	061 275	192 550	288 824	385 110	481 137	577 165	673 192	769 220	865 247	961 275	94
16°	061 276	192 551	288 827	385 110	481 138	577 165	673 193	769 220	865 248	961 276	95
3	061 277	192 552	288 829	384 110	480 138	577 166	673 194	769 221	865 249	961 277	96
6	061 277	192 554	288 831	384 111	480 139	576 169	673 194	769 222	865 250	961 277	97
9	061 278	192 556	288 834	384 111	480 139	576 167	672 195	768 222	864 251	961 278	98
12	060 279	192 558	288 837	384 112	480 139	576 167	672 195	768 223	864 251	960 279	99
15	060 280	192 560	288 839	384 112	480 140	576 168	672 196	768 224	864 252	960 280	100
18	060 281	192 561	288 842	384 112	480 140	576 168	672 196	768 224	864 253	960 281	101
21	060 281	192 563	288 844	384 113	480 141	576 169	672 197	768 225	864 253	960 281	102
24	059 282	192 565	288 847	384 113	480 141	576 159	672 198	767 226	863 251	959 282	103
27	059 283	192 566	288 850	384 113	480 142	575 170	671 198	767 226	863 253	959 283	104
30	059 284	192 568	288 852	384 114	479 142	575 170	671 199	767 227	863 256	959 284	105
33	059 285	192 570	288 854	383 114	479 142	575 171	671 199	767 228	863 256	959 285	106
36	058 286	192 571	287 857	383 114	479 143	575 171	671 200	767 228	862 257	958 286	107
39	058 287	192 573	287 860	383 115	479 143	575 172	671 201	767 230	862 258	958 287	108
42	058 287	192 575	287 862	383 115	479 144	575 172	670 201	766 230	862 259	958 287	109
45	058 288	192 576	287 865	383 115	479 144	575 173	670 202	766 231	862 259	958 288	110
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17°	056 292	191 585	287 877	383 117	478 146	574 175	669 205	765 234	861 263	956 292	73°
3	056 293	191 586	287 880	382 117	478 147	573 176	670 205	765 235	860 264	956 293	74
6	056 294	191 588	287 882	382 118	478 147	573 176	669 206	765 235	860 265	956 294	75
9	056 295	191 590	287 885	382 118	478 147	573 177	669 206	764 236	860 266	956 295	76
12	055 296	191 591	287 887	382 118	478 148	573 177	669 207	764 237	860 267	956 296	77
15	055 297	191 593	287 890	382 119	478 148	573 178	669 208	764 237	860 267	956 297	78
18	055 298	191 595	286 892	382 119	477 149	573 178	668 208	764 238	859 268	955 298	79
21	055 298	191 596	286 895	382 119	477 149	573 179	668 209	764 239	859 269	955 299	80
24	054 299	191 598	286 897	382 120	477 149	573 179	668 209	763 239	859 269	954 299	81
27	054 300	191 600	286 900	382 120	477 150	572 180	668 210	763 240	859 270	954 300	82
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33	053 301	191 603	286 905	381 121	477 151	572 181	667 211	763 241	858 271	953 301	84
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42	053 304	191 608	286 912	381 122	476 152	571 182	667 213	762 243	857 274	953 304	87
45	052 305	191 610	286 915	381 122	476 152	571 183	667 213	762 244	857 274	952 306	88
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54	052 307	190 615	285 922	381 123	476 154	571 184	666 215	761 246	856 277	952 308	91
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3	951	310	190	620	285	930	380	124	175	155	570	186	666	217	761	248	856	279	951	310	57
6	951	311	190	621	285	932	380	124	175	155	570	186	665	217	760	248	855	280	951	311	54
9	950	312	190	623	285	935	380	125	175	156	570	187	665	218	760	249	855	280	950	312	51
12	950	313	190	626	285	938	380	125	175	157	570	188	665	219	760	251	855	282	950	313	45
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24	949	316	190	633	285	949	379	127	174	158	569	190	664	222	759	253	854	285	949	316	33
27	948	317	190	635	284	952	379	127	174	159	569	190	664	222	759	254	853	286	948	317	30
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3	944	326	189	653	284	979	378	131	173	163	567	196	662	228	756	261	851	294	945	326	57
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9	944	328	189	656	283	984	378	131	172	164	567	197	661	230	756	262	850	295	945	328	51
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15	944	330	189	659	283	989	378	132	172	165	566	198	660	231	755	264	850	297	944	330	45
18	944	330	189	661	283	992	378	132	172	165	566	198	661	231	755	264	849	298	944	331	42
21	944	331	189	663	283	994	377	132	172	166	566	199	660	232	755	265	849	298	943	331	39
24	944	332	189	664	283	996	377	133	172	166	566	199	660	233	755	266	849	299	943	332	36
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6	939	344	188	687	282	103	376	137	170	172	563	206	657	241	751	275	845	309	939	344	54
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21	938	348	188	695	281	104	375	139	169	174	562	209	656	244	750	279	844	314	937	348	39
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3	933	359	1 87	718	2 80	1 08	3 73	1 44	4 67	1 80	5 60	2 16	6 53	2 51	7 47	2 87	8 40	3 23	9 33	3 89	57
6	933	360	1 87	720	2 80	1 08	3 73	1 44	4 66	1 80	5 60	2 16	6 53	2 52	7 46	2 88	8 40	3 24	9 33	3 90	54
9	933	361	1 87	722	2 80	1 08	3 73	1 44	4 66	1 80	5 60	2 16	6 53	2 53	7 46	2 89	8 39	3 25	9 33	3 61	51
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21	931	364	1 86	728	2 79	1 09	3 73	1 46	4 66	1 82	5 59	2 18	6 52	2 55	7 45	2 91	8 38	3 28	9 31	3 64	39
24	931	365	1 86	730	2 79	1 09	3 72	1 46	4 66	1 82	5 59	2 19	6 52	2 55	7 45	2 92	8 38	3 28	9 31	3 65	36
27	931	366	1 86	731	2 79	1 10	3 72	1 46	4 65	1 83	5 58	2 19	6 51	2 56	7 45	2 93	8 38	3 29	9 31	3 66	33
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39	916	401	1 83	802	2 75	1 20	3 66	1 60	4 58	2 01	5 50	2 41	6 41	2 81	7 33	3 21	8 24	3 61	9 16	4 01	28
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54	914	405	1 83	810	2 74	1 22	3 66	1 62	4 57	2 03	5 49	2 43	6 40	2 84	7 31	3 24	8 23	3 66	9 14	4 05	13
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6	913	408	180	817	274	122	365	163	456	204	548	245	639	286	730	327	822	367	913	408	68
9	912	409	180	818	274	122	365	164	456	205	547	245	639	286	730	327	821	368	912	409	69
12	912	410	180	820	274	123	365	164	456	205	547	246	638	287	730	328	821	369	912	410	70
15	912	411	180	821	274	123	365	164	456	205	547	246	638	287	729	328	821	370	912	411	71
18	911	412	180	823	273	123	365	165	456	206	547	247	638	288	729	329	820	370	911	412	72
21	911	412	180	825	273	124	364	165	456	206	547	247	638	289	729	330	820	371	911	412	73
24	911	413	180	826	273	124	364	165	455	207	546	248	637	289	729	330	820	372	911	413	74
27	910	414	180	828	273	124	364	166	455	207	546	248	637	290	728	331	819	372	910	414	75
30	910	415	180	829	273	124	364	166	455	207	546	249	637	290	728	332	819	373	910	415	76
33	910	415	180	831	273	125	364	166	455	208	546	249	637	291	728	332	819	374	910	415	77
36	909	416	180	833	273	125	364	166	455	208	546	250	636	291	727	333	818	375	909	416	78
39	909	417	180	834	273	125	364	167	454	209	545	250	636	292	727	334	818	375	909	417	79
42	909	418	180	836	273	125	363	167	454	209	545	251	636	293	727	334	818	376	909	418	80
45	908	419	180	837	272	126	363	167	454	209	545	251	636	293	727	335	817	377	908	419	81
48	908	419	180	839	272	126	363	168	454	210	545	252	635	294	727	336	817	378	908	419	82
51	907	420	180	840	272	126	363	168	454	210	544	252	635	294	726	336	817	378	907	420	83
54	907	421	180	842	272	126	363	168	454	211	544	253	635	295	726	337	816	379	907	421	84
57	907	422	180	844	272	127	363	169	453	211	544	253	635	295	725	337	816	380	907	422	85
25	906	423	180	845	272	127	363	169	453	211	544	254	634	296	725	338	816	380	906	423	86
3	906	423	180	847	272	127	362	169	453	211	544	254	634	296	725	339	815	381	906	423	87
6	906	424	180	848	272	127	362	170	453	212	543	255	634	297	724	339	815	382	906	424	88
9	905	425	180	850	272	127	362	170	453	212	543	255	634	297	724	340	815	382	905	425	89
12	905	426	180	852	271	128	362	170	452	213	543	255	633	298	724	341	814	383	905	426	90
15	905	427	180	853	271	128	362	171	452	213	543	256	633	299	724	341	814	384	905	427	91
18	904	427	180	855	271	128	362	171	452	214	542	256	633	299	723	342	814	385	904	427	92
21	904	428	180	856	271	128	361	171	452	214	542	257	633	300	723	343	813	385	904	428	93
24	903	429	180	858	271	129	361	172	452	214	542	257	632	300	723	343	813	386	903	429	94
27	903	430	180	860	271	129	361	172	451	215	542	258	632	301	722	344	813	387	903	430	95
30	903	430	180	861	271	129	361	172	451	215	542	258	632	301	722	344	812	387	903	430	96
33	902	431	180	863	271	129	361	173	451	216	541	259	632	302	722	345	812	388	902	431	97
36	902	432	180	864	271	130	361	173	451	216	541	259	631	302	721	346	812	389	902	432	98
39	901	433	180	866	270	130	361	173	451	216	541	260	631	303	721	346	811	390	902	433	99
42	901	434	180	867	270	130	360	173	451	217	541	260	631	304	721	347	811	390	902	434	100
45	901	434	180	869	270	130	360	174	450	217	540	261	630	304	721	348	811	391	902	435	101
48	900	435	180	870	270	131	360	174	450	218	540	261	630	305	720	348	810	392	900	435	102
51	900	436	180	872	270	131	360	174	450	218	540	262	630	306	720	349	810	393	900	436	103
54	900	437	180	874	270	131	360	175	450	219	540	263	629	306	719	350	809	394	900	437	104
57	899	438	180	875	270	131	360	175	450	219	540	263	629	306	719	350	809	394	899	438	105
26	899	438	180	877	270	132	360	175	449	219	539	263	629	307	719	351	809	395	899	438	106
3	898	439	180	878	270	132	359	176	449	220	539	263	629	307	719	351	809	395	898	439	107
6	898	440	180	880	269	132	359	176	449	220	539	264	628	308	718	352	808	396	898	440	108
9	898	441	180	881	269	132	359	176	449	220	539	264	628	308	718	353	808	397	898	441	109
12	897	441	179	883	269	132	359	177	449	221	538	265	628	309	718	353	808	397	897	441	110
15	897	442	179	885	269	133	359	177	448	221	538	265	628	310	717	354	807	398	897	442	111
18	896	443	179	886	269	133	359	177	448	222	538	266	628	310	717	354	807	399	896	443	112
21	896	444	179	888	269	133	358	178	448	222	538	266	627	311	717	355	806	399	896	444	113
24	896	447	179	889	269	133	358	178	448	222	537	267	627	311	717	356	806	400	896	446	114
27	895	445	179	891	269	134	358	178	448	223	537	267	627	312	716	356	806	401	895	445	115
30	895	446	179	892	268	134	358	178	447	223	537	268	626	312	716	357	805	402	895	446	116
33	895	447	179	894	268	134	358	179	447	223	537	268	626	313	716	358	805	402	895	447	117
36	894	448	179	896	268	134	358	179	447	224	536	269	626	313	715	358	805	403	894	448	118
39	894	449	179	897	268	135	358	179	447	224	536	269	626	314	715	359	804	404	894	449	119
42	893	449	179	899	268	135	357	180	447	225	536	270	625	315	715	359	804	404	893	449	120
45	893	450	179	900	268	135	357	180	446	225	536	270	625	315	714	360	804	405	893	450	121
48	893	451	179	902	268	135	357	180	446	226	536	271	625	316	714	361	803	406	893	451	122
51	892	452	178	903	268	135	357	181	446	226	535	271	625	316	714	361	803	406	892	452	123
54	892	453	178	905	268	136	357	181	446	226	535	271	624	317	714	362	803	407	892	453	124
57	891	453	178	906	267	136	357	181	446	227	535	272	624	317	713	363	802	408	891	453	125
Dist.	Lat.	Dep.	Lat.	Dep.	Lat.	Dep.	Lat.	Dep.	Lat.	Dep.	Lat.	Dep.	Lat.	Dep.	Lat.	Dep.	Lat.	Dep.	Lat.	Dep.	Dist.



# TRAVERSE TABLES.

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Chs.	1	2	3	4	5	6	7	8	9	10	Chs.
	Lat. Dep	Lat. Dep	Lat. Dep	Lat. Dep	Lat. Dep	Lat. Dep	Lat. Dep	Lat. Dep	Lat. Dep	Lat. Dep	
30°	846 500	1731 00	260 150	340 200	433 250	520 300	600 350	683 400	774 450	864 500	50°
3	860 501	1731 00	260 150	340 200	433 250	519 300	600 351	682 401	774 451	864 501	51°
6	865 502	1731 00	260 150	340 201	433 251	519 301	600 351	682 401	774 451	864 502	52°
9	865 502	1731 00	260 151	340 201	433 251	519 301	600 352	682 402	774 452	864 502	53°
12	865 503	1731 01	260 151	340 201	432 252	519 302	600 352	681 402	774 453	864 503	54°
15	864 504	1731 01	260 151	340 202	432 252	518 302	600 353	681 403	774 453	864 504	55°
18	863 505	1731 01	260 151	340 202	432 252	518 303	600 353	681 404	774 454	864 505	56°
21	863 505	1731 01	260 152	340 202	431 253	518 303	600 354	680 404	774 455	864 505	57°
24	862 506	1731 01	260 152	340 202	431 253	518 304	600 354	680 405	774 455	864 506	58°
27	862 507	1731 01	260 152	340 203	431 253	517 304	600 355	680 405	774 456	864 507	59°
30	862 508	1731 02	260 152	340 203	431 254	517 305	600 355	680 406	774 457	864 508	60°
33	861 508	1731 02	260 152	340 203	431 254	517 305	600 356	680 407	774 457	864 508	61°
36	861 509	1731 02	260 153	340 204	430 255	516 305	600 356	680 407	774 458	864 509	62°
39	860 510	1731 02	260 153	340 204	430 255	516 306	600 357	680 408	774 459	864 510	63°
42	860 511	1731 02	260 153	340 204	429 256	516 307	600 357	680 408	774 459	864 511	64°
45	859 512	1731 02	260 153	340 205	429 256	516 307	600 358	680 409	774 460	864 512	65°
48	859 512	1731 03	260 153	340 205	429 257	515 308	600 359	680 410	774 461	864 513	66°
51	858 514	1731 03	260 154	340 206	429 257	515 309	600 360	680 411	774 462	864 514	67°
54	858 514	1731 03	260 154	340 206	429 257	515 309	600 360	680 411	774 463	864 514	68°
57	858 514	1731 03	260 154	340 206	429 257	515 309	600 360	680 411	774 463	864 514	69°
31°	857 515	1711 03	267 153	343 206	429 258	514 309	600 361	680 412	774 464	864 515	50°
3	857 516	1711 03	267 153	343 206	429 258	514 309	600 361	680 413	774 464	864 516	51°
6	856 517	1711 03	267 153	343 207	428 259	513 310	599 362	680 414	774 465	864 517	52°
9	856 517	1711 03	267 155	343 207	428 259	513 310	599 362	680 414	774 466	864 517	53°
12	855 518	1711 04	267 155	343 207	428 259	513 311	599 363	680 415	774 466	864 518	54°
15	855 519	1711 04	267 156	343 208	427 260	513 311	598 363	680 415	774 467	864 519	55°
18	854 520	1711 04	267 156	343 208	427 260	513 312	598 364	680 416	774 468	864 520	56°
21	854 520	1711 04	267 156	343 208	427 260	513 312	598 364	680 416	774 468	864 520	57°
24	854 521	1711 04	267 156	341 208	427 261	512 313	597 365	680 417	774 469	864 521	58°
27	853 522	1711 04	267 157	341 209	427 261	512 313	597 365	682 417	774 470	864 522	59°
30	853 522	1711 04	267 157	341 209	426 261	512 313	597 366	682 418	774 470	864 523	60°
33	852 523	1701 05	267 157	341 209	426 262	511 314	597 366	682 419	774 471	864 523	61°
36	852 524	1701 05	267 157	341 210	426 262	511 314	596 366	681 419	774 472	864 524	62°
39	851 525	1701 05	267 157	341 210	426 262	511 315	596 367	681 420	774 473	864 525	63°
42	851 525	1701 05	267 158	340 210	425 263	510 315	596 368	681 421	774 474	864 526	64°
45	850 526	1701 05	267 158	340 211	425 263	510 316	596 369	680 422	774 475	864 527	65°
48	850 527	1701 05	267 158	340 211	425 264	510 317	596 369	680 422	774 476	864 528	66°
51	849 528	1701 06	267 159	340 211	424 264	510 317	594 370	679 423	774 477	864 529	67°
54	849 528	1701 06	267 159	340 212	424 265	509 318	594 371	679 424	774 478	864 530	68°
57	849 529	1701 06	267 159	340 212	424 265	509 318	594 371	679 425	774 479	864 531	69°
32°	848 530	1701 06	267 159	340 212	424 265	509 318	594 371	679 425	774 479	864 531	70°
3	848 531	1701 06	267 159	340 213	424 266	508 319	593 372	679 426	774 480	864 532	71°
6	847 531	1701 06	267 159	340 213	424 266	508 319	593 372	679 426	774 480	864 533	72°
9	847 532	1701 06	267 159	340 213	424 266	508 319	593 372	679 426	774 480	864 534	73°
12	846 533	1701 07	267 159	340 213	424 266	508 320	592 373	679 427	774 481	864 535	74°
15	846 534	1701 07	267 159	340 214	424 267	507 321	592 374	679 427	774 481	864 536	75°
18	845 534	1701 07	267 159	340 214	424 267	507 321	592 374	679 427	774 482	864 537	76°
21	845 535	1701 07	267 159	340 214	424 267	507 321	592 374	679 427	774 482	864 538	77°
24	844 536	1701 07	267 159	340 214	424 267	507 321	592 374	679 427	774 482	864 539	78°
27	844 536	1701 07	267 159	340 215	424 267	507 321	592 374	679 427	774 482	864 540	79°
30	843 537	1701 07	267 159	340 215	424 267	507 321	592 374	679 427	774 482	864 541	80°
33	843 537	1701 07	267 159	340 215	424 267	507 321	592 374	679 427	774 482	864 542	81°
36	842 538	1701 07	267 159	340 215	424 267	507 321	592 374	679 427	774 482	864 543	82°
39	842 538	1701 07	267 159	340 215	424 267	507 321	592 374	679 427	774 482	864 544	83°
42	841 539	1701 07	267 159	340 215	424 267	507 321	592 374	679 427	774 482	864 545	84°
45	841 539	1701 07	267 159	340 215	424 267	507 321	592 374	679 427	774 482	864 546	85°
48	841 539	1701 07	267 159	340 215	424 267	507 321	592 374	679 427	774 482	864 547	86°
51	840 540	1701 07	267 159	340 215	424 267	507 321	592 374	679 427	774 482	864 548	87°
54	840 540	1701 07	267 159	340 215	424 267	507 321	592 374	679 427	774 482	864 549	88°
57	840 540	1701 07	267 159	340 215	424 267	507 321	592 374	679 427	774 482	864 550	89°

Dist. Dep. Lat. Dep. Lat. Dep. Lat. Dep. Lat. Dep. Lat. Dep. Lat. Dep. Lat. Dist.



## TRAVERSE TABLE

Chs.	1	2	3	4	5	6	7	8	9	10	Chs.
Lat. Dep.	Lat. Dep.	Lat. Dep.	Lat. Dep.	Lat. Dep.	Lat. Dep.	Lat. Dep.	Lat. Dep.	Lat. Dep.	Lat. Dep.	Lat. Dep.	Lat. Dep.
32°	830-545	168-199	252-163	333-218	419-272	503-327	587-381	671-436	755-490	839-545	57°
33	832-546	168-199	251-164	333-218	419-273	503-328	587-382	671-436	755-491	839-546	58
34	834-546	168-199	251-164	333-218	419-273	503-328	587-382	670-437	754-491	838-546	59
35	837-547	167-199	251-164	333-219	419-273	502-328	586-383	670-437	754-492	837-547	60
36	837-548	167-199	251-164	333-219	418-274	502-329	586-383	669-438	753-493	837-548	61
37	839-548	167-199	251-164	333-219	418-274	502-329	585-384	669-439	753-493	836-548	62
38	839-549	167-199	251-165	334-220	418-275	501-329	585-384	669-439	752-494	836-549	63
39	839-550	167-199	251-165	334-220	418-275	501-330	585-385	668-440	752-495	835-550	64
40	839-550	167-199	250-165	334-220	417-275	501-330	584-385	668-440	751-495	835-550	65
41	839-551	167-199	250-165	334-220	417-276	501-331	584-386	667-441	751-496	834-551	66
42	839-552	167-199	250-166	334-221	417-276	500-331	584-386	667-442	750-497	834-552	67
43	839-553	167-199	250-166	333-221	417-276	500-332	583-387	667-442	750-497	833-553	68
44	839-553	167-199	250-166	333-221	416-277	500-332	583-387	666-443	750-498	833-553	69
45	839-554	166-199	250-166	333-222	416-277	499-332	583-388	666-443	749-499	833-554	70
46	839-555	166-199	250-166	333-222	416-277	499-333	582-389	666-444	749-500	832-555	71
47	839-556	166-199	249-167	333-222	416-278	499-333	582-389	665-444	748-500	832-556	72
48	839-556	166-199	249-167	332-222	415-278	499-334	582-389	665-445	748-501	831-556	73
49	839-557	166-199	249-167	332-223	415-279	498-334	581-390	664-446	747-501	831-557	74
50	839-558	166-199	249-167	332-223	415-279	498-335	581-390	664-446	747-502	830-558	75
51	839-558	166-199	249-168	332-223	415-279	498-335	581-391	664-447	747-503	830-558	76
52	839-559	166-199	249-168	332-224	415-280	497-336	580-391	663-447	746-503	829-559	77
53	839-560	166-199	249-168	331-224	414-280	497-336	580-392	663-448	746-504	829-560	78
54	839-561	166-199	248-168	331-224	414-280	497-337	580-392	663-448	745-505	828-561	79
55	839-561	166-199	248-168	331-225	414-281	497-337	579-393	663-449	745-505	828-561	80
56	839-562	165-199	248-169	331-225	414-281	496-337	579-393	663-449	744-506	827-562	81
57	839-563	165-199	248-169	331-225	413-281	496-338	579-394	663-449	744-507	827-563	82
58	839-563	165-199	248-169	330-225	413-282	496-338	578-394	663-449	743-508	826-564	83
59	839-564	165-199	248-169	330-226	413-282	495-339	578-395	663-449	743-508	826-564	84
60	839-565	165-199	248-169	330-226	413-282	495-339	577-395	663-449	742-509	825-565	85
61	839-566	165-199	247-170	330-227	412-283	494-340	577-396	663-449	742-509	825-566	86
62	839-567	165-199	247-170	329-227	412-284	494-340	577-396	663-449	741-510	824-567	87
63	839-568	165-199	247-170	329-227	412-284	494-341	576-397	663-449	741-511	824-568	88
64	839-568	165-199	247-171	329-227	411-284	494-341	576-398	663-449	740-512	823-569	89
65	839-569	164-199	247-171	329-228	411-285	493-342	575-398	663-449	740-512	823-569	90
66	839-569	164-199	247-171	329-228	411-285	493-342	575-399	663-449	739-513	822-570	91
67	839-570	164-199	246-171	328-228	411-285	493-342	575-399	663-449	739-514	821-571	92
68	839-571	164-199	246-171	328-229	410-286	492-343	574-400	663-449	739-514	821-571	93
69	839-572	164-199	246-172	328-229	410-286	492-343	574-401	663-449	738-515	820-572	94
70	839-573	164-199	246-172	328-229	410-286	492-344	574-401	663-449	738-516	820-573	95
71	839-574	164-199	246-172	328-229	410-287	491-344	573-402	663-449	737-516	819-574	96
72	839-574	164-199	246-172	327-230	409-287	491-345	573-402	663-449	737-517	819-574	97
73	839-575	164-199	245-173	327-230	409-288	491-345	573-403	663-449	736-518	818-575	98
74	839-575	164-199	245-173	327-230	409-288	491-345	572-403	663-449	736-518	818-575	99
75	839-576	163-199	245-173	327-231	409-289	490-346	572-404	663-449	735-519	817-576	100
76	839-577	163-199	245-173	327-231	408-288	490-346	572-404	663-449	735-519	817-576	101
77	839-578	163-199	245-173	326-231	408-289	490-347	571-404	663-449	735-520	816-578	102
78	839-579	163-199	245-174	326-231	408-289	489-347	571-405	663-449	734-521	816-579	103
79	839-580	163-199	244-174	326-232	407-290	489-348	570-406	663-449	734-521	815-580	104
80	839-581	163-199	244-174	326-232	407-290	488-348	570-406	663-449	733-522	814-581	105
81	839-581	163-199	244-174	325-233	407-291	488-349	569-407	663-449	733-523	814-581	106
82	839-582	163-199	244-175	325-233	407-291	488-349	569-407	663-449	732-524	813-582	107
83	839-583	163-199	244-175	325-233	406-292	487-350	568-408	663-449	732-524	813-582	108
84	839-584	163-199	244-175	325-234	406-292	487-351	568-409	663-449	731-525	812-583	109
85	839-585	163-199	244-175	325-234	406-292	487-351	568-409	663-449	731-525	812-583	110
86	839-586	163-199	243-176	324-235	405-293	486-351	567-410	663-449	730-526	811-584	111
87	839-587	163-199	243-176	324-235	405-293	486-352	567-410	663-449	730-526	811-584	112
88	839-588	163-199	243-176	324-235	405-294	486-352	567-411	663-449	729-527	810-585	113
89	839-589	163-199	243-176	324-235	405-294	486-352	567-411	663-449	729-528	810-585	114
90	839-590	163-199	243-176	324-235	405-294	486-352	567-411	663-449	729-528	810-585	115
91	839-591	163-199	243-176	324-235	405-294	486-352	567-411	663-449	729-528	810-585	116
92	839-592	163-199	243-176	324-235	405-294	486-352	567-411	663-449	729-528	810-585	117
93	839-593	163-199	243-176	324-235	405-294	486-352	567-411	663-449	729-528	810-585	118
94	839-594	163-199	243-176	324-235	405-294	486-352	567-411	663-449	729-528	810-585	119
95	839-595	163-199	243-176	324-235	405-294	486-352	567-411	663-449	729-528	810-585	120
96	839-596	163-199	243-176	324-235	405-294	486-352	567-411	663-449	729-528	810-585	121
97	839-597	163-199	243-176	324-235	405-294	486-352	567-411	663-449	729-528	810-585	122
98	839-598	163-199	243-176	324-235	405-294	486-352	567-411	663-449	729-528	810-585	123
99	839-599	163-199	243-176	324-235	405-294	486-352	567-411	663-449	729-528	810-585	124
100	839-600	163-199	243-176	324-235	405-294	486-352	567-411	663-449	729-528	810-585	125

## 61

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1	808 588	1 03 1 18	2 43 1 77	3 23 2 36	4 04 2 94	4 85 3 53	5 66 4 12	6 47 4 71	7 28 5 30	8 09 5 89	57
2	808 588	1 03 1 18	2 42 1 77	3 23 2 36	4 04 2 95	4 85 3 54	5 66 4 13	6 46 4 71	7 27 5 30	8 08 5 89	54
3	807 589	1 01 1 18	2 42 1 77	3 23 2 36	4 04 2 95	4 84 3 54	5 65 4 13	6 46 4 72	7 27 5 31	8 07 5 90	51
4	807 589	1 01 1 18	2 42 1 77	3 23 2 36	4 03 2 95	4 84 3 54	5 65 4 13	6 46 4 72	7 26 5 32	8 07 5 91	48
5	807 589	1 01 1 18	2 42 1 77	3 23 2 37	4 03 2 96	4 84 3 55	5 65 4 14	6 45 4 73	7 26 5 32	8 06 5 91	45
6	806 591	1 01 1 18	2 42 1 78	3 23 2 37	4 03 2 96	4 84 3 55	5 64 4 14	6 45 4 74	7 25 5 33	8 06 5 92	43
7	806 592	1 01 1 19	2 42 1 78	3 23 2 37	4 03 2 96	4 83 3 56	5 64 4 15	6 44 4 74	7 25 5 33	8 05 5 93	39
8	805 593	1 01 1 19	2 41 1 78	3 23 2 37	4 02 2 97	4 83 3 56	5 63 4 15	6 44 4 75	7 25 4 34	8 05 5 93	36
9	805 593	1 01 1 19	2 41 1 78	3 23 2 38	4 02 2 97	4 83 3 56	5 63 4 16	6 44 4 75	7 24 5 36	8 04 5 94	33
10	804 594	1 01 1 19	2 41 1 78	3 23 2 38	4 02 2 97	4 83 3 57	5 63 4 16	6 43 4 76	7 23 5 35	8 04 5 95	30
11	804 595	1 01 1 19	2 41 1 78	3 23 2 38	4 02 2 97	4 83 3 57	5 62 4 17	6 43 4 77	7 23 5 36	8 03 5 96	27
12	803 596	1 01 1 19	2 41 1 79	3 21 2 38	4 02 2 98	4 83 3 57	5 62 4 17	6 42 4 77	7 23 5 37	8 03 5 96	24
13	803 596	1 01 1 19	2 41 1 79	3 21 2 38	4 01 2 98	4 83 3 58	5 62 4 17	6 42 4 78	7 22 5 37	8 02 5 97	21
14	802 597	1 00 1 19	2 41 1 79	3 21 2 39	4 01 2 98	4 81 3 58	5 62 4 18	6 42 4 78	7 22 5 38	8 02 5 98	18
15	802 598	1 00 1 20	2 41 1 79	3 21 2 39	4 01 2 99	4 81 3 59	5 61 4 19	6 41 4 79	7 21 5 38	8 01 5 98	15
16	801 599	1 00 1 20	2 40 1 80	3 20 2 40	4 00 3 00	4 80 3 59	5 61 4 19	6 41 4 79	7 21 5 39	8 01 5 99	12
17	800 600	1 00 1 20	2 40 1 80	3 20 2 40	4 00 3 00	4 80 3 60	5 60 4 20	6 40 4 80	7 20 5 40	8 00 6 00	9
18	800 600	1 00 1 20	2 40 1 80	3 20 2 40	4 00 3 00	4 80 3 60	5 60 4 20	6 40 4 80	7 20 5 40	8 00 6 00	6
19	799 601	1 00 1 20	2 40 1 80	3 20 2 40	4 00 3 01	4 79 3 61	5 59 4 21	6 39 4 81	7 19 5 41	7 99 6 01	3
20	799 602	1 00 1 22	2 40 1 81	3 19 2 41	3 99 3 01	4 79 3 61	5 59 4 21	6 39 4 81	7 19 5 42	7 99 6 02	58
1	798 603	1 00 1 22	2 39 1 81	3 19 2 41	3 99 3 01	4 79 3 62	5 59 4 22	6 38 4 82	7 18 5 42	7 98 6 03	57
2	798 603	1 00 1 22	2 39 1 81	3 19 2 41	3 99 3 02	4 79 3 62	5 58 4 22	6 38 4 83	7 18 5 43	7 98 6 03	54
3	797 604	1 39 1 22	2 39 1 81	3 19 2 42	3 99 3 02	4 78 3 62	5 58 4 23	6 38 4 83	7 17 5 44	7 97 6 04	51
4	797 605	1 39 1 22	2 39 1 81	3 19 2 42	3 98 3 02	4 78 3 63	5 58 4 23	6 37 4 84	7 16 5 45	7 96 6 05	48
5	796 606	1 39 1 22	2 38 1 82	3 19 2 43	3 98 3 03	4 78 3 63	5 57 4 24	6 37 4 84	7 16 5 45	7 96 6 05	45
6	796 606	1 39 1 22	2 38 1 82	3 19 2 43	3 98 3 03	4 77 3 64	5 57 4 24	6 36 4 85	7 15 5 46	7 95 6 06	42
7	795 607	1 39 1 22	2 38 1 82	3 19 2 43	3 97 3 03	4 77 3 64	5 56 4 25	6 36 4 86	7 15 5 47	7 94 6 07	39
8	794 607	1 39 1 22	2 38 1 82	3 19 2 43	3 97 3 04	4 76 3 65	5 56 4 26	6 35 4 87	7 14 5 48	7 93 6 08	36
9	794 608	1 39 1 22	2 38 1 83	3 19 2 44	3 97 3 04	4 76 3 65	5 55 4 27	6 34 4 88	7 13 5 49	7 92 6 09	33
10	793 609	1 39 1 22	2 38 1 83	3 19 2 44	3 96 3 05	4 75 3 66	5 55 4 28	6 33 4 89	7 13 5 50	7 92 6 11	21
11	793 610	1 39 1 22	2 38 1 83	3 19 2 44	3 96 3 05	4 75 3 67	5 54 4 28	6 33 4 89	7 12 5 50	7 92 6 12	18
12	792 611	1 39 1 22	2 37 1 84	3 19 2 45	3 96 3 06	4 74 3 67	5 54 4 29	6 32 4 90	7 12 5 51	7 92 6 12	15
13	791 612	1 39 1 22	2 37 1 84	3 19 2 45	3 96 3 06	4 74 3 68	5 53 4 29	6 32 4 90	7 11 5 52	7 90 6 13	12
14	790 613	1 39 1 22	2 37 1 84	3 19 2 45	3 96 3 06	4 74 3 68	5 53 4 30	6 31 4 91	7 11 5 52	7 90 6 14	9
15	790 614	1 39 1 23	2 37 1 84	3 19 2 46	3 95 3 07	4 73 3 69	5 52 4 30	6 31 4 92	7 10 5 53	7 89 6 14	6
16	790 615	1 39 1 23	2 37 1 84	3 19 2 46	3 94 3 07	4 73 3 69	5 52 4 31	6 30 4 93	7 09 5 54	7 88 6 15	3
17	788 616	1 39 1 23	2 36 1 85	3 15 2 46	3 94 3 08	4 73 3 69	5 52 4 31	6 30 4 93	7 09 5 54	7 88 6 16	52
1	787 616	1 37 1 23	2 36 1 85	3 15 2 47	3 94 3 08	4 72 3 70	5 51 4 32	6 30 4 94	7 08 5 55	7 87 6 16	57
2	787 617	1 37 1 23	2 36 1 85	3 15 2 47	3 93 3 09	4 72 3 70	5 51 4 32	6 30 4 94	7 08 5 55	7 87 6 17	54
3	786 618	1 37 1 24	2 36 1 85	3 15 2 47	3 93 3 09	4 72 3 71	5 50 4 33	6 29 4 94	7 08 5 56	7 86 6 18	51
4	786 618	1 37 1 24	2 36 1 86	3 14 2 47	3 93 3 09	4 72 3 71	5 50 4 33	6 29 4 95	7 07 5 57	7 86 6 18	48
5	785 619	1 37 1 24	2 35 1 86	3 14 2 48	3 93 3 10	4 71 3 72	5 49 4 34	6 28 4 96	7 06 5 58	7 85 6 19	45
6	785 620	1 37 1 24	2 35 1 86	3 14 2 48	3 92 3 10	4 71 3 72	5 49 4 34	6 27 4 96	7 06 5 58	7 85 6 20	42
7	784 620	1 37 1 24	2 35 1 86	3 14 2 48	3 92 3 10	4 70 3 73	5 49 4 35	6 27 4 97	7 05 5 59	7 84 6 21	39
8	784 621	1 37 1 24	2 35 1 86	3 13 2 48	3 92 3 11	4 70 3 73	5 48 4 36	6 26 4 98	7 04 5 60	7 83 6 22	36
9	783 622	1 37 1 24	2 35 1 87	3 13 2 49	3 92 3 11	4 70 3 73	5 48 4 36	6 26 4 98	7 04 5 60	7 83 6 23	33
10	783 623	1 37 1 25	2 35 1 87	3 13 2 49	3 91 3 11	4 69 3 74	5 47 4 37	6 25 4 99	7 03 5 61	7 82 6 24	30
11	782 623	1 36 1 25	2 34 1 87	3 13 2 50	3 91 3 12	4 69 3 74	5 47 4 37	6 25 5 00	7 03 5 62	7 81 6 25	27
12	781 624	1 36 1 25	2 34 1 87	3 13 2 50	3 90 3 12	4 68 3 75	5 46 4 38	6 24 5 01	7 02 5 63	7 80 6 26	24
13	780 625	1 36 1 25	2 34 1 88	3 12 2 50	3 90 3 13	4 68 3 76	5 46 4 39	6 23 5 01	7 01 5 64	7 79 6 27	21
14	779 626	1 36 1 25	2 34 1 88	3 12 2 51	3 90 3 13	4 68 3 76	5 45 4 40	6 23 5 02	7 01 5 65	7 78 6 28	18
15	778 627	1 36 1 25	2 34 1 88	3 12 2 51	3 89 3 14	4 67 3 77	5 45 4 40	6 23 5 03	7 00 5 66	7 77 6 29	15
16	777 628	1 36 1 26	2 33 1 89	3 11 2 51	3 89 3 14	4 67 3 77	5 44 4 40	6 23 5 03	7 00 5 66	7 76 6 29	12
17	776 629	1 36 1 26	2 33 1 89	3 11 2 51	3 88 3 14	4 67 3 77	5 44 4 40	6 23 5 03	7 00 5 66	7 76 6 29	9
18	775 630	1 36 1 26	2 33 1 89	3 11 2 51	3 88 3 14	4 67 3 77	5 44 4 40	6 23 5 03	7 00 5 66	7 76 6 29	6
19	774 631	1 36 1 26	2 33 1 89	3 11 2 51	3 88 3 14	4 67 3 77	5 44 4 40	6 23 5 03	7 00 5 66	7 76 6 29	3
20	773 632	1 36 1 26	2 33 1 89	3 11 2 51	3 88 3 14	4 67 3 77	5 44 4 40	6 23 5 03	7 00 5 66	7 76 6 29	58
1	772 633	1 36 1 26	2 33 1 89	3 11 2 51	3 88 3 14	4 67 3 77	5 44 4 40	6 23 5 03	7 00 5 66	7 76 6 29	57
2	771 634	1 36 1 26	2 33 1 89	3 11 2 51	3 88 3 14	4 67 3 77	5 44 4 40	6 23 5 03	7 00 5 66	7 76 6 29	54
3	770 635	1 36 1 26	2 33 1 89	3 11 2 51	3 88 3 14	4 67 3 77	5 44 4 40	6 23 5 03	7 00 5 66	7 76 6 29	51
4	769 636	1 36 1 26	2 33 1 89	3 11 2 51	3 88 3 14	4 67 3 77	5 44 4 40	6 23 5 03	7 00 5 66	7 76 6 29	48
5	768 637	1 36 1 26	2 33 1 89	3 11 2 51	3 88 3 14	4 67 3 77	5 44 4 40	6 23 5 03	7 00 5 66	7 76 6 29	45
6	767 638	1 36 1 26	2 33 1 89	3 11 2 51	3 88 3 14	4 67 3 77	5 44 4 40	6 23 5 03	7 00 5 66	7 76 6 29	42
7	766 639	1 36 1 26	2 33 1 89	3 11 2 51	3 88 3 14	4 67 3 77	5 44 4 40	6 23 5 03	7 00 5 66	7 76 6 29	39
8	765 640	1 36 1 26	2 33 1 89	3 11 2 51	3 88 3 14	4 67 3 77	5 44 4 40	6 23 5 03	7 00 5 66	7 76 6 29	36
9	764 641	1 36 1 26	2 33 1 89	3 11 2 51	3 88 3 14	4 67 3 77	5 44 4 40	6 23 5 03	7 00 5 66	7 76 6 29	33
10	763 642	1 36 1 26	2 33 1 89	3 11 2 51	3 88 3 14	4 67 3 77	5 44 4 40	6 23 5 03	7 00 5 66	7 76 6 29	30
11	762 643	1 36 1 26	2 33 1 89	3 11 2 51	3 88 3 14	4 67 3 77	5 44 4 40	6 23 5 03	7 00 5 66	7 76 6 29	27
12	761 644	1 36 1 26	2 33 1 89	3 11 2 51	3 88 3 14	4 67 3 77	5 44 4 40	6 23 5 03	7 00 5 66	7 76 6 29	24
13	760 645	1 36 1 26	2 33 1 89	3 11 2 51	3 88 3 14	4 67 3 77	5 44 4 40	6 23 5 03	7 00 5 66	7 76 6 29	21
14	759 646	1 36 1 26	2 33 1 89	3 11 2 51	3 88 3 14	4 67 3 77	5 44 4 40	6 23 5 03	7 00 5 66	7 76 6 29	18
15	758 647	1 36 1 26	2 33 1 89	3 11 2 51	3 88 3 14	4 67 3 77	5 44 4 40	6 23 5 03	7 00 5 66	7 76 6 29	15
16	757 648	1 36 1 26	2 33 1 89	3 11 2 51	3 88 3 14	4 67 3 77	5 44 4 40	6 23 5 03	7 00 5 66	7 76 6 29	12
17	756 649	1 36 1 26	2 33 1 89	3 11 2 51	3 88 3 14	4 67 3 77	5 44 4 40	6 23 5 03	7 00 5 66	7 76 6 29	9
18	755 650	1 36 1 26	2 33 1 89	3 11 2 51	3 88 3 14	4 67 3 77	5 44 4 40	6 23 5 03	7 00 5 66	7 76 6 29	6
19	754 651	1 36 1 26	2 33 1 89	3 11 2 51	3 88 3 14	4 67 3 77	5 44 4 40	6 23 5 03	7 00 5 66	7 76 6 29	3
20	753 652	1 36 1 26	2 33 1 89	3 11 2 51	3 88 3 14	4 67 3 77	5 44 4 40	6 23 5 03	7 00 5 66	7 76 6 29	58
1	752 653	1 36 1 26	2 33 1 89	3 11 2 51	3 88 3 14	4 67 3 77	5				

### TRAVERSE TABLES.

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3	777 630	155 126	233 189	311 252	388 315	466 378	544 441	621 504	699 567	777 630	57										
6	776 631	155 126	233 189	310 253	388 316	466 378	543 441	621 505	698 568	776 631	54										
9	775 631	155 126	233 189	310 253	388 316	465 379	543 442	620 505	698 568	775 631	51										
12	775 632	155 126	232 190	310 253	387 316	465 379	542 442	620 506	697 569	775 632	48										
15	774 633	155 127	232 190	310 253	387 316	465 380	542 443	620 506	697 569	774 633	45										
18	774 633	155 127	232 190	310 253	387 317	464 380	542 443	620 507	696 570	774 633	42										
21	773 634	155 127	232 190	309 254	387 317	464 380	541 444	619 507	696 571	773 634	39										
24	773 635	155 127	232 190	309 254	386 317	464 381	541 444	618 508	695 571	773 635	36										
27	772 635	154 127	232 191	309 254	386 318	463 381	540 445	618 508	695 572	772 635	33										
30	772 636	154 127	231 191	309 254	386 318	463 382	540 445	617 509	694 572	772 636	30										
33	771 637	154 127	231 191	308 255	386 318	463 382	540 446	617 509	694 573	771 637	27										
36	771 637	154 127	231 191	308 255	385 319	462 382	539 446	616 510	693 574	771 637	24										
39	770 638	154 128	231 191	308 255	385 319	462 382	539 447	616 510	693 574	770 638	21										
42	769 639	154 128	231 192	308 256	385 319	462 383	539 447	616 511	692 575	769 639	18										
45	769 639	154 128	231 192	308 256	384 320	461 384	538 448	615 512	692 575	769 639	15										
48	768 640	154 128	230 192	307 256	384 320	461 384	538 448	615 512	691 576	768 640	12										
51	768 641	154 128	230 192	307 256	384 320	461 384	537 449	614 513	691 577	768 641	9										
54	767 641	153 128	230 192	307 257	383 321	460 385	537 449	613 514	690 577	767 641	6										
57	767 642	153 128	230 193	307 257	383 321	460 385	537 449	613 514	690 578	767 642	3										
40°	766 643	153 129	230 193	306 257	383 321	460 386	536 450	613 514	689 579	766 643	50°										
3	765 643	153 129	230 193	306 258	383 322	459 386	536 450	612 515	689 579	765 643	57										
6	765 644	153 129	229 193	306 258	382 322	459 386	535 451	612 515	688 580	765 644	54										
9	764 645	153 129	229 193	306 258	382 322	458 387	535 451	611 516	688 580	764 645	51										
12	764 645	153 129	229 194	306 258	382 323	458 387	535 452	611 516	687 581	764 645	48										
15	763 646	153 129	229 194	305 259	382 323	458 388	534 452	611 517	687 582	763 646	45										
18	763 647	153 129	229 194	305 259	381 324	458 388	534 453	610 517	686 582	763 647	42										
21	762 647	152 129	229 194	305 259	381 324	457 389	533 453	610 518	686 583	762 647	39										
24	762 648	152 130	228 194	305 259	381 324	457 389	533 454	609 518	685 583	762 648	36										
27	761 649	152 130	228 195	304 260	380 324	457 389	533 454	609 519	685 584	761 649	33										
30	760 649	152 130	228 195	304 260	380 325	456 390	532 455	608 520	684 585	760 649	30										
33	760 650	152 130	228 195	304 260	380 325	456 390	532 455	608 520	684 585	760 650	27										
36	759 651	152 130	228 195	304 260	380 325	456 390	531 456	607 520	683 586	759 651	24										
39	759 651	152 130	228 195	303 261	379 326	455 391	531 456	607 521	683 586	759 651	21										
42	758 652	152 130	227 196	303 261	379 326	455 391	531 457	607 522	682 587	758 652	18										
45	758 653	152 131	227 196	303 261	379 326	455 392	530 457	606 522	682 587	758 653	15										
48	757 653	151 131	227 196	303 261	378 327	454 392	530 457	606 523	681 588	757 653	12										
51	756 654	151 131	227 196	303 262	378 327	454 393	529 458	605 523	681 588	756 654	9										
54	756 655	151 131	227 196	302 262	378 327	454 393	529 458	605 524	680 589	756 655	6										
57	755 655	151 131	227 197	302 262	378 328	453 393	529 459	604 524	680 590	755 655	3										
41°	755 656	151 131	226 197	302 262	377 328	453 394	528 459	604 525	679 590	755 656	49°										
3	754 657	151 131	226 197	302 263	377 329	452 394	528 460	603 526	679 591	754 657	57										
6	754 657	151 131	226 197	301 263	377 329	452 394	527 460	603 526	678 592	754 657	54										
9	753 658	151 132	226 198	301 263	376 329	452 395	527 461	602 526	678 592	753 658	51										
12	752 659	150 132	226 198	301 263	376 329	451 395	527 461	602 527	677 593	752 659	48										
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21	751 661	150 132	225 198	300 264	375 330	450 396	525 462	601 529	676 595	751 661	39										
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27	750 662	150 132	225 199	300 265	375 331	450 397	525 463	600 530	675 596	750 662	33										
30	749 663	150 133	225 199	300 265	374 331	449 398	524 464	599 530	674 596	749 663	30										
33	748 663	150 133	225 199	299 265	374 332	449 398	524 464	599 531	674 597	748 663	27										
36	748 664	150 133	224 199	299 266	374 332	449 398	524 465	598 531	673 598	748 664	24										
39	747 665	149 133	224 199	299 266	374 332	448 399	523 465	598 532	673 598	747 665	21										
42	747 665	149 133	224 200	299 266	373 333	448 399	523 466	597 532	672 599	747 665	18										
45	746 666	149 133	224 200	298 267	373 333	448 400	522 466	597 533	671 599	746 666	15										
48	745 667	149 133	224 200	298 267	373 333	447 400	522 467	596 533	671 600	745 667	12										
51	745 667	149 133	223 200	298 267	372 334	447 400	521 467	596 534	670 600	745 667	9										
54	744 668	149 134	223 200	298 267	372 334	447 401	521 467	595 534	670 601	744 668	6										
57	744 668	149 134	223 201	297 267	372 334	446 401	520 468	595 535	669 602	744 668	3										
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3	743	670	1 49	1 34	2 23	2 01	2 97	2 68	3 71	3 35	4 45	1 02	5 20	1 69	5 94	5 36	6 68	6 03	7 43	6 70	57
6	742	670	1 48	1 34	2 23	2 01	2 97	2 68	3 71	3 36	4 45	1 02	5 19	1 69	5 94	5 36	6 68	6 03	7 42	6 70	54
9	741	671	1 48	1 34	2 22	2 01	2 97	2 68	3 71	3 36	4 45	1 03	5 19	1 70	5 93	5 37	6 67	6 04	7 41	6 71	51
12	741	672	1 48	1 34	2 22	2 02	2 96	2 69	3 70	3 36	4 44	1 03	5 18	1 70	5 93	5 37	6 67	6 05	7 41	6 72	48
15	740	672	1 48	1 34	2 22	2 02	2 96	2 69	3 70	3 36	4 44	1 03	5 18	1 71	5 92	5 38	6 66	6 05	7 40	6 72	45
18	740	673	1 48	1 35	2 22	2 02	2 96	2 69	3 70	3 37	4 44	1 04	5 18	1 72	5 91	5 39	6 66	6 06	7 40	6 73	42
21	739	674	1 48	1 35	2 22	2 02	2 96	2 69	3 70	3 37	4 43	1 04	5 17	1 72	5 91	5 39	6 65	6 06	7 39	6 74	39
24	738	674	1 48	1 35	2 22	2 02	2 95	2 70	3 69	3 37	4 43	1 05	5 17	1 72	5 91	5 39	6 65	6 07	7 38	6 74	36
27	738	675	1 48	1 35	2 21	2 02	2 95	2 70	3 69	3 37	4 43	1 05	5 17	1 72	5 90	5 40	6 64	6 07	7 38	6 75	33
30	737	676	1 47	1 35	2 21	2 03	2 95	2 70	3 69	3 38	4 42	1 05	5 16	1 73	5 90	5 40	6 64	6 08	7 37	6 76	30
33	737	676	1 47	1 35	2 21	2 03	2 95	2 70	3 68	3 38	4 42	1 06	5 16	1 73	5 89	5 41	6 63	6 09	7 37	6 76	27
36	736	677	1 47	1 35	2 21	2 03	2 94	2 70	3 68	3 38	4 42	1 06	5 15	1 74	5 89	5 42	6 62	6 09	7 36	6 77	24
39	736	678	1 47	1 36	2 21	2 03	2 94	2 71	3 68	3 39	4 41	1 07	5 15	1 74	5 88	5 42	6 62	6 10	7 36	6 78	21
42	735	678	1 47	1 36	2 20	2 03	2 94	2 71	3 67	3 39	4 41	1 07	5 14	1 75	5 88	5 43	6 61	6 10	7 35	6 78	18
45	734	679	1 47	1 36	2 20	2 04	2 94	2 72	3 67	3 39	4 41	1 07	5 14	1 75	5 87	5 43	6 61	6 11	7 34	6 79	15
48	733	679	1 47	1 36	2 20	2 04	2 93	2 72	3 67	3 40	4 40	1 08	5 14	1 76	5 87	5 44	6 60	6 11	7 34	6 79	12
51	733	680	1 47	1 36	2 20	2 04	2 93	2 72	3 67	3 40	4 40	1 08	5 13	1 76	5 87	5 44	6 60	6 12	7 33	6 80	9
54	733	681	1 47	1 36	2 20	2 04	2 93	2 72	3 66	3 40	4 40	1 08	5 13	1 77	5 86	5 45	6 59	6 13	7 33	6 81	6
57	732	681	1 46	1 36	2 20	2 04	2 93	2 73	3 66	3 41	4 39	1 09	5 12	1 77	5 86	5 45	6 59	6 13	7 32	6 81	3
47°	731	682	1 46	1 37	2 19	2 05	2 93	2 73	3 66	3 41	4 39	1 09	5 12	1 77	5 85	5 46	6 58	6 14	7 31	6 82	43°
3	731	683	1 46	1 37	2 19	2 05	2 93	2 73	3 65	3 41	4 38	1 10	5 12	1 78	5 85	5 46	6 58	6 14	7 31	6 83	57
6	730	683	1 46	1 37	2 19	2 05	2 92	2 73	3 65	3 42	4 38	1 10	5 11	1 78	5 84	5 47	6 57	6 15	7 30	6 83	54
9	730	684	1 46	1 37	2 19	2 05	2 92	2 74	3 65	3 42	4 38	1 10	5 11	1 79	5 84	5 47	6 57	6 16	7 30	6 84	51
12	729	685	1 46	1 37	2 19	2 05	2 92	2 74	3 64	3 42	4 37	1 11	5 10	1 79	5 83	5 48	6 56	6 16	7 29	6 85	48
15	728	685	1 46	1 37	2 19	2 06	2 91	2 74	3 64	3 43	4 37	1 11	5 10	1 80	5 83	5 48	6 56	6 17	7 28	6 85	45
18	728	686	1 46	1 37	2 18	2 06	2 91	2 74	3 64	3 43	4 37	1 11	5 10	1 80	5 82	5 49	6 55	6 17	7 28	6 86	42
21	727	686	1 46	1 37	2 18	2 06	2 91	2 75	3 64	3 43	4 36	1 12	5 09	1 81	5 82	5 49	6 54	6 18	7 27	6 86	39
24	727	687	1 45	1 37	2 18	2 06	2 91	2 75	3 63	3 44	4 36	1 12	5 09	1 81	5 81	5 50	6 53	6 19	7 26	6 88	36
27	726	688	1 45	1 38	2 18	2 06	2 90	2 75	3 63	3 44	4 36	1 13	5 08	1 81	5 81	5 50	6 53	6 19	7 26	6 88	33
30	725	688	1 45	1 38	2 18	2 07	2 90	2 75	3 63	3 44	4 35	1 13	5 08	1 82	5 80	5 51	6 53	6 20	7 25	6 88	30
33	725	689	1 45	1 38	2 17	2 07	2 90	2 76	3 62	3 44	4 35	1 13	5 07	1 82	5 80	5 51	6 52	6 20	7 25	6 89	27
36	724	689	1 45	1 38	2 17	2 07	2 90	2 76	3 62	3 45	4 35	1 14	5 07	1 83	5 79	5 52	6 52	6 21	7 24	6 90	24
39	723	690	1 45	1 38	2 17	2 07	2 89	2 76	3 62	3 45	4 34	1 14	5 07	1 83	5 79	5 52	6 51	6 21	7 24	6 90	21
42	723	691	1 45	1 38	2 17	2 07	2 89	2 76	3 61	3 45	4 34	1 15	5 06	1 84	5 78	5 53	6 51	6 22	7 23	6 91	18
45	722	692	1 44	1 38	2 17	2 07	2 89	2 77	3 61	3 46	4 33	1 15	5 06	1 84	5 78	5 53	6 50	6 22	7 22	6 92	15
48	722	692	1 44	1 38	2 17	2 08	2 88	2 77	3 61	3 46	4 33	1 16	5 05	1 85	5 77	5 54	6 49	6 23	7 21	6 93	12
51	721	693	1 44	1 39	2 16	2 08	2 88	2 77	3 60	3 47	4 32	1 16	5 04	1 86	5 76	5 55	6 48	6 24	7 21	6 93	9
54	721	693	1 44	1 39	2 16	2 08	2 88	2 78	3 60	3 47	4 32	1 16	5 04	1 86	5 76	5 55	6 48	6 24	7 20	6 94	6
57	720	694	1 44	1 39	2 16	2 08	2 88	2 78	3 60	3 47	4 32	1 17	5 04	1 86	5 75	5 56	6 47	6 25	7 19	6 95	3
46°	719	695	1 44	1 39	2 16	2 09	2 87	2 78	3 59	3 48	4 31	1 17	5 03	1 87	5 75	5 56	6 47	6 26	7 19	6 95	57
3	718	695	1 44	1 39	2 15	2 09	2 87	2 78	3 59	3 48	4 31	1 18	5 03	1 87	5 75	5 57	6 46	6 26	7 18	6 96	54
6	718	697	1 44	1 39	2 15	2 09	2 87	2 79	3 59	3 48	4 31	1 18	5 02	1 88	5 74	5 57	6 46	6 27	7 18	6 97	51
9	717	697	1 44	1 39	2 15	2 09	2 87	2 79	3 58	3 49	4 30	1 18	5 02	1 88	5 74	5 58	6 45	6 27	7 17	6 97	48
12	716	698	1 43	1 40	2 15	2 09	2 87	2 79	3 58	3 49	4 30	1 19	5 01	1 88	5 73	5 58	6 45	6 28	7 16	6 98	45
15	716	698	1 43	1 40	2 15	2 10	2 86	2 79	3 58	3 49	4 29	1 19	5 01	1 89	5 73	5 59	6 44	6 29	7 16	6 98	42
18	715	699	1 43	1 40	2 15	2 10	2 86	2 80	3 58	3 50	4 29	1 19	5 01	1 89	5 72	5 59	6 44	6 29	7 15	6 99	39
21	714	700	1 43	1 40	2 14	2 10	2 86	2 80	3 57	3 50	4 29	1 20	5 00	1 90	5 71	5 60	6 43	6 30	7 14	7 00	36
24	714	700	1 43	1 40	2 14	2 10	2 86	2 80	3 57	3 50	4 28	1 21	4 99	1 91	5 70	5 61	6 42	6 31	7 13	7 01	33
27	713	701	1 43	1 40	2 14	2 10	2 85	2 80	3 57	3 50	4 28	1 21	4 99	1 91	5 70	5 61	6 42	6 32	7 12	7 02	30
30	713	702	1 43	1 40	2 14	2 10	2 85	2 81	3 56	3 51	4 27	1 21	4 98	1 92	5 69	5 62	6 41	6 32	7 11	7 03	27
33	712	702	1 42	1 40	2 14	2 11	2 85	2 81	3 56	3 51	4 27	1 22	4 98	1 92	5 69	5 63	6 40	6 33	7 11	7 03	24
36	711	703	1 42	1 41	2 13	2 11	2 84	2 81	3 55	3 52	4 26	1 22	4 97	1 93	5 68	5 63	6 39	6 34	7 10	7 05	21
39	711	703	1 42	1 41	2 13	2 11	2 84	2 82	3 55	3 52	4 26	1 23	4 97	1 93	5 67	5 64	6 38	6 35	7 09	7 05	18
42	710	704	1 42	1 41	2 13	2 11	2 84	2 82	3 55	3 52	4 26	1 23	4 96	1 94	5 67	5 64	6 38	6 35	7 08	7 06	15
45	709	705	1 42	1 41	2 13	2 12	2 84	2 82	3 54	3 53	4 25	1 24	4 96	1 94	5 67	5 65	6 37	6 36	7 08	7 06	12
48	709	705	1 42	1 41	2 13	2 12	2 83	2 82	3 54	3 53	4 25	1 24	4 95	1 95	5 66	5 65	6 37	6 36	7 07	7 07	9
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54	708	706	1 42	1 41	2 12	2 12	2 83	2 83	3 54	3 53	4 24	1 24	4 95	1 95	5 66	5 66	6 36	6 36	7 07	7 07	3
57	707	707	1 41	1 41	2 12	2 12	2 83	2 83	3 54	3 53	4 24	1 24	4 95	1 95	5 66	5 66	6 36	6 36	7 07	7 07	45°
Dist.	Dep	Lat.	Dep	Lat.	Dep	Lat.	Dep	Lat.	Dep	Lat.	Dep	Lat.	Dep	Lat.	Dep	Lat.	Dep	Lat.	Dep	Lat.	Dist



